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ROBINSON'S

NEW

PRACTICAL ARITHMETIC

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ROBINSON'S

NEW

PRACTICAL ARITHMETIC

FOR

COMMON SCHOOLS AND ACADEMIES

NEW YORK ·:· CINCINNATI ·:· CHICAGO
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ROBINSON'S NEW ARITHMETICS

Graded to the wants of Primary, Intermediate, Grammar, Normal, and High Schools, Academies, and Colleges.

1 NEW PRIMARY ARITHMETIC.
2 NEW INTELLECTUAL ARITHMETIC. (*In preparation.*)
3 NEW RUDIMENTS OF ARITHMETIC.
4 NEW PRACTICAL ARITHMETIC.
KEY TO NEW PRACTICAL ARITHMETIC. (*In preparation.*)
5 HIGHER ARITHMETIC.

ROBINSON'S ARITHMETICS have for many years had a wide use among the best American schools, and the new edition here announced preserves, in a new dress, with careful editing and revision, the leading features which have proved so satisfactory both to teacher and to pupil.

In the revision, the order of subjects and the numbering of paragraphs have been preserved, and the new edition can be used without difficulty in the same classes with the old.

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ROB'S. NEW PRACT. AR.

Printed by
William Ivison
New York, U. S. A.

PREFACE.

ROBINSON'S Practical Arithmetic has been before the public so many years and its plan and features are so well known that it hardly seems necessary to call attention to them; but the new features of this revised edition are quite as worthy of mention as any in the older book. And here it behooves us to say that nothing of value or merit in the old Robinson's Practical Arithmetic has been discarded. The scheme of revision has been rather one of judicious addition than of omission, and yet by an economical adjustment and by an occasional dropping out of useless matter, it has been possible to add many valuable features and much new matter to the book without materially increasing its size.

The revision has not been in the nature of patchwork, as so many revisions are apt to be, but has been conducted upon a systematic plan, which runs through the entire book and has regulated the arrangement of the subjects and their order of sequence. Two great points have been borne in mind in the sequence of subjects. First, it is such that each subject leads naturally and by easy steps to the next; and second, the most important and most useful *applications* of the fundamental principles of arithmetic, as well as the principles themselves, have been placed at an early stage in the book. This matter is of far greater importance than might appear at first sight. More than one half of our children are obliged to leave school without completing any book on arithmetic, and when such important subjects as the measurement of land, lumber, etc., are placed at the end of a book as an afterthought, many pupils are obliged to battle with the intricacies of less practical operations, at the expense of this useful and necessary instruction.

The book, in its present shape, meets the requirements of all the standard examinations prescribed for admission to academic grade or for preparation for the best scientific schools of the country, but it meets them in a logical and sensible way.

For example, the additional proofs of the fundamental operations follow the last of these operations — Division. The chapter on Scales of Notation immediately precedes Denominate Numbers and naturally leads up to them, being the stepping-stone between the simple numbers of the decimal notation and the compound numbers with their varying scales found in Denominate Numbers. The Metric System has not been treated as a curiosity reserved for an Appendix, but each table of this system follows the corresponding table of the regular system. Immediately following Denominate Numbers are their practical applications — Measurements. Savings-Bank Accounts follow other banking business, Partitive Proportion precedes and leads up to Partnership, and so on with all the subjects.

The uniformity of plan in the book adds greatly to the ease with which the subjects are mastered. When once familiar with the style of the book, the pupil knows immediately where to look for and find any special point or detail in any subject. First he finds a clear and logical development of the subject, then a model example or two with the solution, and lastly the rule; but by the time he reaches the rule he knows it. He could frame it himself in his own way, and should be permitted to do so. The rule might, therefore, be omitted altogether were it not for the fact that a clear and brief summary, such as a pupil might not be able to express, aids the memory.

When two or three explanations or methods of working an example are equally good, they are all given. One method is easier for some children, another for others; and frequently one explanation elucidates another, to say nothing of the broadening influence of such a course upon the mind. The second method of working Compound Interest given on page 286 is, perhaps, the simplest (when a Compound Interest Table is not at hand) and the most easily understood and remembered; yet few, if any, of the arithmetics give it at all, or else they reserve it for treatment under Progressions.

There is so much valuable matter in this book that is not to be found in the old edition that it is hoped it will soon take the place of that book altogether; but nevertheless, for the convenience of those classes where the new book will be used side by side with the old, the old section numbering has been preserved as much as possible, and the new edition will seldom be found to vary from it more than two or three numbers.

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PRACTICAL ARITHMETIC.

DEFINITIONS.

1. **Quantity** is anything that can be increased, diminished, or measured.

2. **Mathematics** is the science of quantity.

3. A **Unit** is *one*, or a single thing.

4. A **Number** is a unit, or a collection of units.

5. An **Integer** is a whole number.

6. The **Unit of a Number** is *one* of the collection of units forming the number.

Thus, the unit of 23 is 1; of 23 dollars, 1 dollar.

7. **Like Numbers** are numbers that have the same kind of unit.

Thus, 74, 16, and 250; 7 dollars and 62 dollars; 4 ft. and 17 ft.

8. An **Abstract Number** is a number used without reference to any particular thing or quantity.

Thus, 17; 365; 8540.

9. A **Concrete Number** is a number used with reference to some particular thing or quantity.

Thus, 17 dollars; 365 days; 8540 men.

10. **Arithmetic** is the *science* of numbers, and the *art* of computation.

NOTATION AND NUMERATION.

11. **Notation** is a method of *writing* or expressing numbers by characters.

12. **Numeration** is a method of *reading* numbers expressed by characters.

13. Two systems of notation are in general use — the *Roman* and the *Arabic*.

The Roman Notation is supposed to have been first used by the Romans; hence its name. The Arabic Notation was introduced into Europe by the Arabs, by whom it was supposed to have been invented; but investigations have shown that it was adopted by them only 600 years ago, and that it has been in use among the Hindoos more than 2000 years. From this latter fact it is sometimes called the *Indian Notation*.

THE ROMAN NOTATION.

14. The Roman Notation employs seven capital letters to express numbers.

Thus,

LETTERS.	I	V	X	L	C	D	M
VALUES.	One.	Five.	Ten.	Fifty.	One hundred.	Five hundred.	One thousand.

15. The Roman Notation is founded upon five principles, as follows:

1. Repeating a letter repeats its value. Thus, II represents two, XX twenty, CCC three hundred.

2. If a letter is placed *after* one of greater value, its value is to be *added to* that of the greater. Thus, XI represents eleven, LX sixty, DC six hundred.

3. If a letter is placed *before* one of greater value, its value is to be *taken from* that of the greater. Thus, IX represents nine, XL forty, CD four hundred.

4. If a letter is placed *between* two letters, each of greater value, its value is to be *taken from* the *sum* of the other two. Thus, XIV represents fourteen, XXIX twenty-nine, XCIV ninety-four.

5. A bar or dash placed over a letter increases its value *one thousand times*. Thus, V signifies five, and $\overline{V}$ five thousand; L fifty, and $\overline{L}$ fifty thousand.

16. TABLE OF ROMAN NOTATION.

I = One.	XX = Twenty.
II = Two.	XXI = Twenty-one.
III = Three.	XXX = Thirty.
IV = Four.	XL = Forty.
V = Five.	L = Fifty.
VI = Six.	LX = Sixty.
VII = Seven.	LXX = Seventy.
VIII = Eight.	LXXX = Eighty.
IX = Nine.	XC = Ninety.
X = Ten.	C = One hundred.
XI = Eleven.	CC = Two hundred.
XII = Twelve.	D = Five hundred.
XIII = Thirteen.	DC = Six hundred.
XIV = Fourteen.	M = One thousand.
XV = Fifteen.	MC = One thousand one hundred.
XVI = Sixteen.	MM = Two thousand.
XVII = Seventeen.	$\overline{X}$ = Ten thousand.
XVIII = Eighteen.	$\overline{C}$ = One hundred thousand.
XIX = Nineteen.	$\overline{M}$ = One million.

The system of Roman Notation is not well adapted to the purposes of numerical calculation; it is principally confined to the numbering of chapters and sections of books, public documents, etc.

Express the following numbers by letters:

1. Eleven. *Ans.* XI.
2. Fifteen. *Ans.* XV.
3. Twenty-five. *Ans.* XXV.
4. Thirty-nine. *Ans.* XXXIX.
5. Forty-eight. *Ans.* XLVIII.
6. Seventy-seven. *Ans.* LXXVII.
7. One hundred fifty-nine.
8. Five hundred ninety-four.
9. One thousand five hundred thirty-eight.
10. One thousand nine hundred ten.
11. Express the present year.
12. One thousand eight hundred ninety-three.
13. Thirty-four.
14. One hundred eighty-nine.
15. Three hundred seventy-eight.
16. Six hundred thirty-nine.
17. Express the year in which Columbus discovered America.
18. Express the year in which the Declaration of Independence was made.
19. Eighty-four.
20. Six thousand eight hundred ninety-seven.
21. Four hundred forty-four.
22. Nineteen.
23. Three thousand seven hundred forty-eight.
24. Eighty-eight.
25. One thousand one hundred fifty.
26. Five thousand seventy-three.
27. Ten thousand five hundred.
28. One million ten thousand.
29. One million one hundred ten thousand.
30. One hundred thousand two hundred.
31. Five hundred thousand six hundred forty-three.

THE ARABIC NOTATION.

17. The Arabic Notation employs ten characters or figures to express numbers.

FIGURES.	0	1	2	3	4	5	6	7	8	9
NAMES.	Naught.	One.	Two.	Three.	Four.	Five.	Six.	Seven.	Eight.	Nine.

18. The first character is called *naught*, or *cipher*, because it has no value of its own. It is also called *nothing* and *zero*.

19. The other nine characters are called *significant figures*, because each has a value of its own. The significant figures are also called *digits*, a word derived from the Latin term *digitus*, which signifies *finger*.

20. The ten Arabic characters are the alphabet of arithmetic, and by combining them according to certain principles, all numbers can be expressed.

21. These ten characters, when standing singly, constitute the first order, or *order of units*.

22. Each of the nine digits has a value of its own; hence any number not greater than nine can be expressed by *one* figure.

23. As the Arabic Notation is a decimal notation* based on the number ten (*decem*), the number ten begins a second order which is expressed by two characters. This is called the *order of tens*. One ten is expressed by writing the unit 1 with the cipher, 0, at its right, 10.

* For other schemes of notation, see pp. 156–159.

In the same manner we represent

2 tens, or Twenty.	3 tens, or Thirty.	4 tens, or Forty.	5 tens, or Fifty.	6 tens, or Sixty.	7 tens, or Seventy.	8 tens, or Eighty.	9 tens, or Ninety.
20	30	40	50	60	70	80	90

We express the numbers between 10 and 20 by writing the 1 with each of the digits respectively at its right; thus

Eleven.	Twelve.	Thirteen.	Fourteen.	Fifteen.	Sixteen.	Seventeen.	Eighteen.	Nineteen.
11	12	13	14	15	16	17	18	19

In like manner we express the numbers between 20 and 30, between 30 and 40, and between any two successive tens.

Thus, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 47, 56, 68, 72, 89, 93.

24. The greatest number that can be expressed by *two* figures is 99.

25. The number one hundred begins the third order which is expressed by three characters. This is called the *order of hundreds*. One hundred is expressed by writing the unit 1 with two ciphers at its right.

Thus, 100.

In the same manner we represent

One hundred.	Two hundred.	Three hundred.	Four hundred.	Five hundred.	Six hundred.	Seven hundred.	Eight hundred.	Nine hundred.
100	200	300	400	500	600	700	800	900

We express the numbers between 100 and 999 by writing the figure of the hundred order with the proper figures of the tens' and units' orders at its right, remembering that when there are no tens or units to be represented, a cipher must be put in their place.

Thus, 101, 109, 120, 156.

26. The greatest number that can be expressed by *three* figures is 999.

27. PRINCIPLES. — 1. *When a number is expressed by one figure, it represents units.*

2. *When a number is expressed by two figures, the right-hand figure represents units and the left-hand figure tens.*

3. *When a number is expressed by three figures, the right-hand figure represents units, the next figure tens, and the left-hand figure hundreds.*

EXAMPLES.

1. Write one hundred twenty-five.
2. Write four hundred eighty-three.
3. Write seven hundred sixteen.
4. Express by figures nine hundred.
5. Express by figures two hundred ninety.
6. Write eight hundred nine.
7. Write five hundred five.
8. Write five hundred fifty-seven.
9. Write six hundred ninety-two.
10. Write three hundred twenty-six.
11. Write two hundred fourteen.
12. Write seven hundred seven.
13. Write eight hundred forty-one.

28. The number one thousand begins the fourth order, which is expressed by four characters. This is called the *order of thousands.* One thousand is expressed by writing the unit 1 with three ciphers at its right.

Thus, 1000.

In the same manner we represent

One thousand.	Two thousand.	Three thousand.	Four thousand.	Five thousand.
1000	2000	3000	4000	5000

Six thousand.	Seven thousand.	Eight thousand.	Nine thousand.
6000	7000	8000	9000

When a number is expressed by four figures, the places, commencing at the right hand, are *units*, *tens*, *hundreds*, *thousands*.

To express hundreds, tens, and units with thousands, we write in each place the figure indicating the number we wish to express in that place. To write four thousand two hundred sixty-nine, we write 4 in the place of thousands, 2 in the place of hundreds, 6 in the place of tens, and 9 in the place of units.

Thus, 4269.

Thousands.	Hundreds.	Tens.	Units.
4	2	6	9

29. The greatest number that can be expressed by *four* figures is 9999.

EXAMPLES.

Express the following numbers by figures:

1. One thousand two hundred.
2. Five thousand one hundred sixty.
3. Three thousand seven hundred forty-one.
4. Eight thousand fifty-six.
5. Two thousand ninety.
6. Seven thousand nine.
7. One thousand one.
8. Nine thousand four hundred twenty-seven.
9. Four thousand thirty-five.
10. One thousand nine hundred four.

Read the following numbers:

11. 76; 128; 405; 910; 116; 3416; 1025.
12. 2100; 5047; 7009; 4670; 3997; 1001.

30. Next to thousands come *tens* of thousands, the fifth order, and next to these come *hundreds* of thousands, the sixth order, as tens and hundreds come in

their order after units. Ten thousand is expressed by writing the unit 1 with four ciphers at its right.

Thus, 10000.

One hundred thousand is expressed by writing the unit 1 with five ciphers at its right.

Thus, 100000.

We can express thousands, tens of thousands, and hundreds of thousands in one number, in the same manner as we express units, tens, and hundreds in one number. To express five hundred twenty-one thousand eight hundred three, we write 5 in the sixth place, counting from units, 2 in the fifth place, 1 in the fourth place, 8 in the third place, 0 in the second place (because there are no tens), and 3 in the place of units.

Thus, 521803.

Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
5	2	1	8	0	3

31. The greatest number that can be expressed by *five* figures is 99999; and by *six* figures, 999999.

EXAMPLES.

Write the following numbers in figures:

1. Twenty thousand.
2. Forty-seven thousand.
3. Eighteen thousand one hundred.
4. Twelve thousand three hundred fifty.
5. Thirty-nine thousand five hundred twenty-two.
6. Fifteen thousand two hundred six.
7. Eleven thousand twenty-four.
8. Forty thousand ten.
9. Sixty thousand six hundred.
10. Two hundred twenty thousand.
11. One hundred fifty-six thousand.

12. Eight hundred forty thousand three hundred.

13. Five hundred one thousand nine hundred sixty-four.

14. One hundred thousand one hundred.

15. Three hundred thirteen thousand three hundred thirteen.

Read the following numbers:

16. 5006; 12304; 96071; 5470; 203410.

17. 36741; 400560; 13061; 49000; 100010.

18. 200200; 75620; 90402; 218094; 100101.

For convenience in reading large numbers, point them off, by commas, into periods of three figures each, counting from the right-hand or unit figure. This pointing enables us to read the hundreds, tens, and units in each period with facility.

32. Next above hundreds of thousands we have, successively, units, tens, and hundreds of millions, and then follow units, tens, and hundreds of each higher name, as seen in the following table:

NUMERATION TABLE.

Periods.	6th			5th			4th			3d			2d			1st		
Name.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Orders of Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
Number.	5	3	0,	0	4	5,	3	7	0,	0	3	6,	4	0	8,	0	6	0

The number is read 530 quadrillion, 45 trillion, 370 billion, 36 million, 408 thousand 60.

This is called the French method of pointing off the periods, and is the one in general use in this country.

EXAMPLES.

Write in figures, and read the following numbers:

1. One unit of the third order, four of the second.

2. Three units of the fifth order, two of the third, one of the first.

3. Eight units of the fourth order, five of the second.

4. Two units of the seventh order, nine of the sixth, four of the third, one of the second, seven of the first.

5. Three units of the sixth order, four of the second.

6. Nine units of the eighth order, six of the seventh, three of the fifth, seven of the fourth, five of the third, two of the second, nine of the first.

7. Four units of the tenth order, six of the eighth, four of the seventh, two of the sixth, one of the third, five of the second.

8. Eight units of the twelfth order, four of the eleventh, six of the tenth, nine of the seventh, three of the sixth, five of the fifth, three of the fourth, two of the third, seven of the second, eight of the first.

33. From the foregoing explanations and illustrations, the following important principles may be derived:

1. Figures have two values, **Simple** and **Local.**

The *Simple Value* of a figure is its value when taken alone.

Thus, 2, 5, 8.

The *Local Value* of a figure is its value when used with another figure or figures in the same number.

Thus, in 842 the simple values of the several figures are 8, 4, and 2; but the local value of the 8 is 800; of the 4 is 4 tens, or 40; and of the 2 is 2 units.

2. The local value of a figure depends upon its place from units of the first order, not upon the value of the figures at the right of it.

Thus, in 425 and 400, the value of the 4 is the same in both numbers, being 4 units of the third order, or 4 hundred.

Care should be taken not to mistake the local value of a figure for the value of the whole number. For, although the value of the 4 (hundreds) is the same in the two numbers, 425 and 400, the value of the whole of the first number is greater than that of the second.

3. A digit or figure, if used in the second place, expresses tens; in the third place, hundreds; in the fourth place, thousands; and so on.

4. Every *period* contains three figures (units, tens, and hundreds), except the last period to the left hand, which sometimes contains only one or two figures (units, or units and tens).

5. As ten units make 1 ten, 10 tens 1 hundred, 10 hundreds 1 thousand, and ten units of any order, or in any place, make 1 unit of the next higher order, or in the next place at the left, we readily see that the Arabic method of notation is based upon the following laws:

TWO GENERAL LAWS.

I. *The value of units of the different orders increases from right to left, and decreases from left to right, in a tenfold ratio.*

II. *Every removal of a figure one place to the left, increases its local value tenfold; and every removal of a figure one place to the right, diminishes its local value tenfold.*

Thus,

6 is 6 units.
60 is 10 times 6 units.
600 is 10 times 6 tens.
6000 is 10 times 6 hnndreds.
60000 is 10 times 6 thousands.

RULE FOR NOTATION. — I. *Beginning at the left hand, write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units of each successive period in their order, placing a cipher wherever an order of units is omitted.*

RULE FOR NUMERATION. — I. *Separate the number into periods of three figures each, commencing at the right hand.*

II. *Beginning at the left hand, read each period separately, and give the name to each period, except the last, or period of units.*

34. Until the pupils can write numbers readily, it may be well for them to write several periods of ciphers, point them off, and over each period write its name, and then write the given numbers underneath in their appropriate places.

Thus,

Trillions.	Billions.	Millions.	Thousands.	Units.
OOO,	OOO,	OOO,	OOO,	OOO.

EXERCISES IN NOTATION AND NUMERATION.

Express the following numbers by figures:

1. Four hundred thirty-six.
2. Seven thousand one hundred sixty-four.
3. Twenty-six thousand twenty-six.
4. Fourteen thousand two hundred eighty.
5. One hundred seventy-six thousand.
6. Four hundred fifty thousand thirty-nine.
7. Ninety-five million.
8. Four hundred thirty-three million eight thousand.
9. Nine hundred thousand ninety.
10. Ten million ten thousand ten hundred ten.
11. Sixty-one billion five million.
12. Five trillion eighty billion nine million one.

Point off, numerate, and read the following numbers:

13. 8240.	17. 1010.	21. 370005.
14. 400900.	18. 57468139.	22. 9400706342.
15. 308.	19. 5628.	23. 38429526.
16. 60720.	20. 850026800.	24. 74268113759.

25. Write seven million thirty-six.

26. Write five hundred sixty-three thousand four.

27. Write one million ninety-six thousand.

28. Numerate and read 9004082501.

29. Numerate and read 2584503962047.

30. A certain number contains 3 units of the seventh order, 6 of the fifth, 4 of the fourth, 1 of the third, 5 of the second, and 2 of the first; what is the number?

31. What orders of units are contained in the number 290648?

32. What orders of units are contained in the number 1037050?

33. What orders of units are contained in the number 23975630?

34. What orders of units are contained in the number 349765248?

35. What orders of units are contained in the number 987654321?

Numerate and read the following numbers:

36. 87641832.	44. 24318422.	52. 43567890.
37. 4307061.	45. 3141592.	53. 90807060.
38. 6840503.	46. 1000600.	54. 3456789.
39. 700007.	47. 87321302.	55. 98765432.
40. 9000009.	48. 61093432.	56. 89987543.
41. 333333.	49. 78956421.	57. 22334456.
42. 505050.	50. 30050053.	58. 9432197.
43. 6767676.	51. 21122121.	59. 19600000.

ADDITION.

ORAL EXERCISES.

35. 1. Henry gave $5 for a vest, and $7 for a coat. What did he pay for both?

SOLUTION. — He gave 5 dollars and 7 dollars, which are 12 dollars.

2. A farmer sold a pig for $3, and a calf for $8. What did he receive for both?

3. A drover bought 5 sheep of one man, 9 of another, and 3 of another. How many did he buy in all?

4. How many are 2 and 6? 2 and 7? 2 and 9? 2 and 8? 2 and 10?

5. How many are 4 and 5? 4 and 8? 4 and 7? 4 and 9?

6. How many are 6 and 4? 6 and 6? 6 and 9? 6 and 7?

7. How many are 7 and 7? 7 and 6? 7 and 8? 7 and 10? 7 and 9?

8. How many are 5 and 4 and 6? 7 and 3 and 8? 6 and 9 and 5?

36. **Addition** is the process of uniting *several* numbers of the same kind into *one* equivalent number.

37. The **Sum** or **Amount** is the result obtained by the addition.

38. The sign, +, is called *plus* (generally read *and*), which signifies *more*. When placed between two numbers, it denotes that they are to be *added*.

Thus, 6 + 4, shows that 6 and 4 are to be added.

39. The sign, =, is called the sign of *equality*. When placed between two numbers, or sets of numbers, it signifies that they are equal to each other.

Thus, the expression 6 + 4 = 10, is read 6 *plus* 4 are *equal to* 10, or 6 *and* 4 *are* 10, and denotes that the numbers 6 and 4, taken together, equal the number 10.

EXAMPLES.

40. When the amount of each column is less than 10.

1. A farmer sold some hay for 102 dollars, six cows for 162 dollars, and a horse for 125 dollars. What did he receive for all?

OPERATION.

	Hunds.	Tens.	Units.
	1	0	2
	1	6	2
	1	2	5
Amount,	3	8	9

SOLUTION. — We arrange the numbers so that units of like order shall stand in the same column. Then we add the columns separately, for convenience commencing at the right hand, and write each result under the column added. The sum of the units column is 9, the sum of the tens column is 8, the sum of the hundreds column is 3. The entire amount is 3 hundreds 8 tens and 9 units, or 389.

2.	3.	4.	5.
Pounds.	Rods.	Cents.	Days.
132	245	312	437
243	321	243	140
324	132	412	321
Ans. 699			

6. What is the sum of 144, 321, and 232? *Ans.* 697.
7. What is the amount of 122, 333, and 401? *Ans.* 856.

8. What is the sum of 42, 103, 321, and 32? *Ans.* 498.

9. A drover bought three droves of sheep. The first contained 230, the second 425, and the third 340. How many sheep did he buy in all? *Ans.* 995.

41. When the amount of any column equals or exceeds 10.

1. A merchant pays 725 dollars a year for the rent of a store, 475 dollars for a clerk, and 367 dollars for other expenses. What is the amount of his expenses?

OPERATION.

	Hunds.	Tens.	Units.
	7	2	5
	4	7	5
	3	6	7
Sum of the units,			17
Sum of the tens,		15	
Sum of the hundreds,	14		
Total amount,	1567		

SOLUTION. — Arranging the numbers as in 40, we first add the column of units, and find the sum to be 17 units, which is 1 ten and 7 units. We write the 7 units in the place of units, and the 1 ten in the place of tens. The sum of the figures in the column of tens is 15 tens, which is 1 hundred, and 5 tens. We write the 5 tens in the place of tens, and the 1 hundred in the place of hundreds. Next we add the column of hundreds, and find the sum to be 14 hundreds, which is 1 thousand and 4 hundreds. We write the 4 hundreds in the place of hundreds, and 1 thousand in the place of thousands. Lastly, by uniting the sum of the units with the sums of the tens and hundreds, we find the total amount to be 1 thousand 5 hundreds 6 tens 7 units, or 1567.

This example may be performed by another method, which is the common one in practice. Thus:

OPERATION.

725
475
367
1567

SOLUTION. — Arranging the numbers, as before, we add the first column and find the sum to be 17 units; writing the 7 units under the column of units, we add the 1 ten to the column of tens, and find the sum to be 16 tens; writing the 6 tens under the column of tens, we add the 1 hundred to the column of hundreds, and find the sum to be 15 hundreds; as this is the last column, we write down its amount, 15; and the *whole amount* is 1567, as before.

1. Units of the same order are written in the same column; and when the sum in any column is 10 or more than 10, it produces *one or more units* of a higher order, which must be added to the next column. This process is sometimes called "carrying the tens."

2. In adding, learn to pronounce the partial results without naming the numbers separately; thus, instead of saying 7 and 5 are 12, and 5 are 17, simply pronounce the results, 7, 12, 17, etc.

From the preceding examples and illustrations we deduce the following rules:

RULE. — I. *Write the numbers to be added so that the units of the same order stand in the same column; that is, units under units, tens under tens, etc.*

II. *Commencing with units, add each column separately, and write the sum underneath, if it is less than ten.*

III. *If the sum of any column is ten or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the entire sum of the last column.*

PROOF. — I. *Begin with the right hand or unit column, and add the figures in each column in an opposite direction to that in which they were first added; if the two results agree, the work is probably correct. Or,*

II. *Separate the numbers added into two sets by a horizontal line; find the sum of each set separately; add these sums, and if the amount is the same as that first obtained, the work may be presumed to be correct.*

42. By the methods of proof here given, the numbers are united in new combinations, which render it almost impossible for two precisely similar mistakes to occur.

The first method is the one commonly used in business.

2.	3.	4.	5.	6.
Miles.	Inches.	Tons.	Feet.	Bushels.
24	321	427	1342	3420
48	479	321	7306	7021
96	165	903	5254	327
82	327	278	8629	97
250	1292	1929	22531	10865

7.	8.	9.	10.
Hours.	Years.	Gallons.	Rods.
347	7104	3462	47637
506	3762	863	3418
218	9325	479	703
312	4316	84	26471
424	2739	57	84

11. $42 + 64 + 98 + 70 + 37 = ?$ *Ans.* 311.

12. $312 + 425 + 107 + 391 + 76 = ?$ *Ans.* 1311.

13. $1476 + 375 + 891 + 66 + 80 = ?$ *Ans.* 2888.

14. $37042 + 1379 + 809 + 127 + 40 = ?$ *Ans.* 39397.

15. What is the sum of one thousand six hundred fifty-six, eight hundred nine, three hundred ten, and ninety-four? *Ans.* 2869.

16. Add forty-two thousand two hundred twenty, ten thousand one hundred five, four thousand seventy-five, and five hundred seven. *Ans.* 56907.

17. Add two hundred ten thousand four hundred, one hundred thousand five hundred ten, ninety thousand six hundred eleven, forty-two hundred twenty-five, and eight hundred ten. *Ans.* 406556.

18. What is the sum of the following numbers: seventy-five, one thousand ninety-five, six thousand four hundred thirty-five, two hundred sixty-seven thousand, one thousand four hundred fifty-five, twenty-seven mil-

lion eighteen, two hundred seventy million twenty-seven thousand? *Ans.* 297303078.

19. A man on a journey traveled 37 miles the first day, 33 miles the second, 40 miles the third, and 35 miles the fourth. How far did he travel in the four days?

20. A merchant has in one cask 75 gallons of oil, in another 65, in a third 57, in a fourth 83, in a fifth 74, and in a sixth 67. How many gallons has he in all?

Ans. 421.

21. An estate is to be shared equally by four heirs, and the portion given to each heir is to be $3754. What is the amount of the estate? *Ans.* $15016.

22. How many men are there in an army consisting of 52714 infantry, 5110 cavalry, 6250 dragoons, 3927 light-horse, 928 artillery, 250 sappers, and 406 miners?

23. A merchant deposited $56 in a bank on Monday, $74 on Tuesday, $120 on Wednesday, $96 on Thursday, $170 on Friday, and $50 on Saturday. How much did he deposit during the week?

24. A merchant bought at public sale 746 yards of broadcloth, 650 yards of muslin, 2100 yards of flannel, and 250 yards of silk. How many yards did he buy in all?

25. Five persons deposited money in the same bank; the first, $5897; the second, $12980; the third, $65973; the fourth, $37345; and the fifth as much as the first and second together. How many dollars did they all deposit?

Ans. $141072.

26. A man willed his estate to his wife, two sons, and four daughters; to his daughters he gave $2630 apiece, to his sons, each $4647, and to his wife $3595. What was the value of his estate? *Ans.* $23409.

27. A man commenced farming in the west, and raised, the first year, 724 bushels of corn; the second

year, 3498 bushels; the third year, 9872 bushels; the fourth year, 9964 bushels; and the fifth year, 11078 bushels. How many bushels did he raise in the five years? *Ans.* 35136 bushels.

28.	29.	30.	31.	32.
476	908	126	443	180
390	371	324	298	976
915	569	503	876	209
207	245	891	569	314
841	703	736	137	563
632	421	517	910	842
234	127	143	347	175
143	354	274	256	224
536	781	531	324	135
245	436	275	463	253

33. A has $3648, B has $7035, C has $429 more than A and B together, and D has as many dollars as all the rest. How many dollars has D? How many have they all? *Ans.* D has $21795. All have $43590.

34. A man bought three houses and lots for $15780, and sold them so as to gain $695 on each lot. For how much did he sell them? *Ans.* $17865.

35. At the battle of Waterloo, which took place June 18, 1815, the estimated loss of the French was 40000 men; of the Prussians, 38000; of the Belgians, 8000; of the Hanoverians, 3500; and of the English, 12000. What was the entire loss of life in this battle?

36. The expenditures for certain purposes in New England for a certain year were as follows: Maine, $380623; New Hampshire, $221146; Vermont, $246604; Massachusetts, $1424873; Rhode Island, $136729; and Connecticut, $430826. What was the total expenditure?

Ans. $2840801.

37. The eastern continent contains 35413799 square miles; the western continent, 16237535; Australia, Polynesia, and other islands, 3709781. What is the entire area of the land surface of the globe?

38. The population of New York in 1890 was 1515301; of Boston, 448447; of Philadelphia, 1046964; of Chicago, 1099850; of St. Louis, 451770; of New Orleans, 242039. What was the entire population of these cities?

39. The population of the globe is estimated as follows: North America, 88005695; South America, 33565882; Europe, 360580788; Asia, 823155251; Africa, 168497091; Australia and Polynesia, 5683968. What is the total population of the globe according to this estimate?

Ans. 1479488675.

40. The railroad distance from New York to Albany is 143 miles; from Albany to Buffalo, 297; from Buffalo to Cleveland, 183; from Cleveland to Toledo, 113; from Toledo to Decatur, 323; and from Decatur to St. Louis, 113 miles. What is the distance from New York to St. Louis? *Ans.* 1172 miles.

41. A man owns farms valued at $56800; city lots valued at $86760; a house worth $12500, and other property to the amount of $6785. What is the entire value of his property? *Ans.* $162845.

42. Three persons bought a hotel. The first agreed to pay $7375, the second agreed to pay twice as much, and the third, $13555. What was the value of the hotel? *Ans.* $35680.

43. In January, 1876, a merchant bought goods to the amount of $2675; in February, $4375; and in March, $1897. How much did he owe for the goods?

44. I owe three notes, one note being for $2509, another for $6563, and a third for $3925. How much do I owe for the three notes?

45. A man bought a house for $15675; he paid $5096 for furniture, $3.40 for repairs, and $35 for insurance. How much did he pay in all for the house?

46. Lake Superior covers an area of 32290 square miles; Lake Erie, 9493; and Lake Ontario, 7654. What is the area covered by them all?

47.	48.	49.	50.
15038	26881	41919	93808
7404	12173	19577	41371
34971	39665	74736	110525
30359	33249	66768	102936
6293	6318	12673	17087
2875	4318	7193	13251
16660	34705	51365	112110
64934	80597	155497	220619
80901	95299	183134	225255
7444	8624	16845	68940
57068	53806	111139	176974
17255	18647	35902	86590
32543	41609	82182	149162
40022	35077	75153	109355
56063	46880	132936	283910
33860	41842	82939	112511
17548	26876	44424	72908
28944	36642	65586	157672
16147	29997	52839	86160
38556	44305	83211	119557
234882	262083	522294	839398
39058	39744	78861	117787
152526	169220	353428	471842
179122	198568	386214	571778
7626	8735	17005	41735
1218099	1395860		

SUBTRACTION.

ORAL EXERCISES.

43. 1. A farmer, having 14 cows, sold 6 of them. How many had he left?

SOLUTION. — Fourteen cows less 6 cows are 8 cows. He had 8 cows left.

2. Stephen, having 9 marbles, lost 4 of them. How many had he left?

3. If a man earns 10 dollars a week, and spends 6 dollars for provisions, how many dollars has he left?

4. A merchant, having 16 barrels of flour, sells 9 of them. How many barrels has he left?

5. Charles had 18 cents, and gave 10 of them for a book. How many cents had he left?

6. James is 17 years old, and his sister Julia is 5 years younger. How old is Julia?

7. A grocer, having 20 boxes of lemons, sold 11 boxes. How many boxes had he left?

8. From a cistern containing 25 barrels of water, 15 barrels leaked out. How many barrels remained?

9. I paid $16 for a coat, and $7 for a vest. How much more did the coat cost than the vest?

10. How many are 18 less 5? 17 less 8? 12 less 7?

11. How many are 20 less 14? 18 less 12? 19 less 11?

12. How many are 11 less 3? 16 less 11? 19 less 8? 20 less 9? 22 less 20?

44. Subtraction is the process of taking away part of a number or of finding the difference between two numbers of the same kind.

45. The **Minuend** is the number from which another is to be subtracted.

46. The **Subtrahend** is the number to be subtracted.

47. The **Remainder** or **Difference** is the result obtained by the process of subtraction.

The minuend and subtrahend must be *like numbers;* thus, 5 dollars taken from 9 dollars leaves 4 dollars; 5 apples taken from 9 apples leaves 4 apples; but it would be absurd to try to take 5 apples from 9 dollars, or 5 dollars from 9 apples.

48. The sign, −, is called *minus*, which signifies *less*. When placed between two numbers, it denotes that the number following it is to be taken from the one preceding it.

Thus, $8 - 6 = 2$ is read 8 *minus* 6 *equals* 2, or 8 less 6 is 2, and shows that 6, the *subtrahend*, taken from 8, the *minuend*, equals 2, the *remainder*.

EXAMPLES.

49. When no figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 574 take 323.

OPERATION.

Minuend,	574
Subtrahend,	323
Remainder,	251

SOLUTION. — We write the less number under the greater, with units under units, tens under tens, etc., and draw a line underneath. Then, beginning at the right hand, we subtract separately each figure of the subtrahend from the figure above it in the minuend. Thus, 3 taken from 4 leaves 1, which is the difference of the units; 2 taken from 7 leaves 5, the difference of the tens; 3 taken from 5 leaves 2, the difference of the hundreds. The whole difference is 2 hundreds 5 tens and 1 unit, or 251.

	2.	3.	4.	5.
Minuend,	876	676	367	925
Subtrahend,	334	415	152	213
Remainder,	542	261	215	712

	6.	7.	8.	9.
From	876	732	987	498
Take	523	522	782	178

		Remainders.
10.	From 3276 take 2143.	1133.
11.	From 7634 take 3132.	4502.
12.	From 41763 take 11521.	30242.
13.	From 18346 take 5215.	13131.
14.	From 397631 take 175321.	222310.
15.	Subtract 47321 from 69524.	22203.
16.	Subtract 16330 from 48673.	32343.
17.	Subtract 291352 from 895752.	604400.
18.	Subtract 84321 from 397562.	313241.

19. A farmer paid $645 for a span of horses and a carriage, and sold them for $522. What did he lose?

20. A man bought a mill for $3724, and sold it for $4856. What did he gain? *Ans.* $1132.

21. A drover bought 1566 sheep, and sold 435 of them. How many had he left? *Ans.* 1131 sheep.

22. A piece of land was sold for $2945, which was $832 more than it cost. What did it cost?

23. A man willed to his son $15768, and to his daughter $4537. How much more did he will to his son than to his daughter? *Ans.* $11231.

24. A merchant sold goods to the amount of $6742, and by so doing gained $2540. What did the goods cost him?

25. If I borrow $15475 of a person, and pay him $4050, what do I still owe him?

50. When any figure in the subtrahend is greater than the corresponding figure in the minuend.

1. From 846 take 359.

OPERATION.

	Hunds.	Tens.	Units.
Minuend,	8	4	6
Subtrahend,	3	5	9
Remainder,	4	8	7

SOLUTION. — Since we cannot take 9 units from 6 units, we add 10 units to 6 units, making 16 units; 9 units taken from 16 units leaves 7 units. But as we have added 10 units, or 1 ten, to the minuend, we shall have a remainder 1 ten too large, to avoid which, we add 1 ten to the 5 tens in the subtrahend, making 6 tens. We cannot take 6 tens from 4 tens; so we add 10 tens to 4, making 14 tens; 6 tens taken from 14 tens leaves 8 tens. Now, having added 10 tens, or 1 hundred, to the minuend, we shall have a remainder 1 hundred too large, unless we add 1 hundred to the 3 hundreds in the subtrahend, making 4 hundreds; 4 hundreds taken from 8 hundreds leaves 4 hundreds, and we have for the total remainder, 487, the same as before.

51. The following method is preferred by some, as being simpler.

OPERATION.

	(7)	(13)	(16)
Minuend,	8	4	6
Subtrahend,	3	5	9
Remainder,	4	8	7

SOLUTION. — In this example we cannot take 9 units from 6 units. From the 4 tens we take 1 ten, which equals 10 units, and add it to the 6 units, making 16 units; 9 units taken from 16 units leaves 7 units, which we write in the remainder in units' place. As we have taken 1 ten from the 4 tens, only 3 tens are left. We cannot take 5 tens from 3 tens; so from the 8 hundreds we take 1 hundred, which equals 10 tens, and add it to the 3 tens, making 13 tens; 5 tens from 13 tens leaves 8 tens, which we write in the remainder in tens' place. As we have taken 1 hundred from the 8 hundreds, only 7 hundreds are left; 3 hundreds taken from 7 hundreds leaves 4 hundreds, which we write in the remainder in hundreds' place, and we have the whole remainder, 487.

The numbers written over the minuend are used simply to explain more clearly the method of subtracting; in practice the process should be performed mentally, and these numbers should be omitted.

From the preceding illustrations we derive the following rule:

RULE. — I. *Write the less number under the greater, placing units of the same order in the same column.*

II. *Begin at the right hand, and take each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure in the subtrahend is greater than the corresponding figure in the minuend, add* 10 *to that figure in the minuend before subtracting, and then add* 1 *to the next left-hand figure of the subtrahend; or add* 10 *to that figure in the minuend and take* 1 *from the next left-hand figure of the minuend.*

PROOF. — *Add the remainder to the subtrahend, and if their sum is equal to the minuend, the work may be presumed to be right.*

52. The process of adding 10 to the minuend is sometimes called *borrowing* 10, and that of adding 1 to the next figure of the subtrahend, *carrying* 1.

	2.	3.	4.	5.
Minuend,	873	7432	1969	8146
Subtrahend,	538	6711	1408	4377
Remainder,	335			

	6. Gallons.	7. Bushels.	8. Miles.	9. Days.
From	3176	9076	7320	5097
Take	2907	4567	3871	3809

	10. Dollars.	11. Rods.	12. Acres.	13. Feet.
From	76377	67777	900076	767340
Take	45761	46699	899934	5039

14. $479 - 382 = ?$ *Ans.* 97.
15. $6593 - 1807 = ?$ *Ans.* 4786.
16. $17380 - 3417 = ?$ *Ans.* 13963.
17. $80014 - 43190 = ?$ *Ans.* 36824.
18. $282731 - 90756 = ?$ *Ans.* 191975.
19. From 234100 take 9970.
20. From 345673 take 124799.
21. From 4367676 take 256569. *Ans.* 4111107.
22. From 3467310 take 987631. *Ans.* 2479679.
23. From 941000 take 5007. *Ans.* 935993.
24. From 1970000 take 1361111. *Ans.* 608889.
25. From 290017 take 108045.
26. Take 3077097 from 7045676. *Ans.* 3968579.
27. Take 9999999 from 60000000. *Ans.* 50000001.
28. Take 220202 from 4040053. *Ans.* 3819851.
29. Take 2199077 from 3000001. *Ans.* 800924.
30. Take 377776 from 8000800. *Ans.* 7623024.
31. Take 501300347 from 1030810040.
32. Subtract nineteen thousand nineteen from twenty thousand ten. *Ans.* 991.
33. From one million nine thousand six take twenty thousand four hundred. *Ans.* 988606.
34. What is the difference between two million seven thousand eighteen, and one hundred five thousand seventeen?
35. Take 67092 from 91307.
36. Take 4193286 from 6431842.
37. Take 3141592 from 5000000.
38. Take 8913456 from 9000000.
39. What is the difference between 19553068 and 3204313?
40. The population of Great Britain in 1891 was 37888153, and that of England alone, 27482104. What was the difference?

EXAMPLES COMBINING ADDITION AND SUBTRACTION.

1. A merchant gave his note for $5200. He paid at one time $2500, and at another $175. What remained due? *Ans.* $2525.

2. A traveler who was 1300 miles from home, traveled homeward 235 miles in one week, in the next 275 miles, in the next 325 miles, and in the next 280 miles; how far had he still to go before he would reach home?

3. A man deposited in a bank $8752; he drew out at various times $4234, $1700, $962, and $49. How much had he remaining in the bank?

4. A man bought a farm for $4765, and paid $750 for fencing and other improvements; he then sold it for $384 less than it cost him. What did he receive for it?

5. A forwarding merchant had in his warehouse 7520 barrels of flour; he shipped at one time 1224 barrels, at another time 1500 barrels, and at another time 1805 barrels. How many barrels remained?

6. A had 450 sheep, B had 175 more than A, and C had as many as A and B together minus 114. How many sheep had C? *Ans.* 961 sheep.

7. A farmer raised 1575 bushels of wheat, and 900 bushels of corn. He sold 807 bushels of wheat, and 391 bushels of corn to A, and the remainder to B. How much of each did he sell to B?

8. A man traveled 6784 miles; 2324 miles by railroad, 1570 miles in a stage coach, 450 miles on horseback, 175 miles on foot, and the remainder by steamboat. How many miles did he travel by steamboat?

Ans. 2265 miles.

9. I borrowed of my neighbor at one time $750, at another time $379, and at another $450. Having paid him $1000, how much do I still owe him? *Ans.* $579.

MULTIPLICATION.

ORAL EXERCISES.

53. 1. What will 4 pounds of sugar cost, at 8 cents a pound?

SOLUTION. — Four pounds will cost 4 times 8 cents, or 32 cents.

2. If a ream of paper costs 3 dollars, what will 2 reams cost?

3. At 7 cents a quart, what will 4 quarts of cherries cost?

4. At $12 a ton, what will 3 tons of hay cost? 4 tons? 5 tons?

5. There are 7 days in one week. How many days are there in 6 weeks? In 8 weeks?

6. What will 9 quarts of berries cost, at 10 cents a quart?

7. If Henry earns 12 dollars in one month, how much can he earn in 5 months? In 7 months? In 9 months?

8. What will 11 dozen eggs cost, at 9 cents a dozen? At 10 cents? At 12 cents?

9. When flour is $7 a barrel, what must be paid for 7 barrels? For 9 barrels? For 12 barrels?

10. At $9 a week, what will 4 weeks' board cost? 7 weeks' board? 9 weeks' board?

11. At 9 cents a foot, what will 4 feet of lead pipe cost? 7 feet? 10 feet?

12. If I deposit $12 in a savings bank every month, how many dollars will I deposit in 6 months?

13. When hay is $8 a ton, how much will 3 tons cost? 4 tons? 7 tons? 9 tons? 11 tons?

14. What will be the cost of 11 barrels of apples at $2 a barrel? At $3?

15. At 5 cents a pound, what will 9 pounds of sugar cost? 11 pounds? 12 pounds?

16. John is 12 years old and his father is 5 times as old. How old is his father?

54. Multiplication is the process of taking one of two given numbers as many times as there are units in the other.

55. The **Multiplicand** is the number to be multiplied.

56. The **Multiplier** is the number by which to multiply, and shows how many times the multiplicand is to be taken.

57. The **Product** is the result obtained by the process of multiplication.

1. Factors are producers, and the multiplicand and multiplier are factors of the product because they produce it.

2. Multiplication is a short method of performing addition when the numbers to be added are equal.

58. The sign $\times$, placed between two numbers, denotes that one is to be multiplied by the other.

Thus $9 \times 6 = 54$, is read 9 *times* 6 *equals* 54, or 6 multiplied by 9 equals 54.

59. The *multiplier* is always *regarded* as an *abstract number.*

Either factor of a product may be *used* abstractly, even though it be concrete, hence either factor may be used as a multiplier.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXAMPLES.

60. When the multiplier consists of one figure.

1. Multiply 374 by 6.

OPERATION.

	Hunds.	Tens.	Units.
Multiplicand,	3	7	4
Multiplier,			6
Units,		2	4
Tens,		4	2
Hundreds,	1	8	
Product,	2	2	4 4

SOLUTION. — In this example it is required to take 374 *six* times. If we take the units of each order 6 times, we shall take the entire number 6 times. Therefore, writing the multiplier under the unit figure of the multiplicand, we proceed as follows: 6 times 4 units are 24 units; 6 times 7 tens are 42 tens; 6 times 3 hundreds are 18 hundreds; and adding these partial products, we obtain the entire product, 2244.

The operation in this example may be performed in another way, which is the one in common use.

OPERATION.

374
6
2244

SOLUTION. — Writing the numbers as before, we begin at the right hand or unit figure, and say: 6 times 4 units are 24 units, which are 2 *tens* and 4 *units;* we write the 4 units in the product in units' place, and reserve the 2 tens to add to the next product; 6 times 7 tens are 42 tens, and the two tens reserved in the

last product added, are 44 tens, which are 4 *hundreds* and 4 *tens;* we write the 4 tens in the product in tens' place, and reserve the 4 hundreds to add to the next product; 6 times 3 hundreds are 18 hundreds, and 4 hundreds added are 22 hundreds, which being written in the product in the places of hundreds and thousands, gives, for the entire product, 2244.

61. The unit value of a number is not changed by repeating the number.

62. When both the factors of a product are abstract, the product will be abstract. When one of the factors is concrete, the product will be of the same unit value as that factor.

1. When, as explained in the note under 59, a concrete number is *used* as a multiplier, the product will be of the same denomination as the multiplier. When the multiplicand is the concrete number, the product will be of the same denomination as the multiplicand.

2. In multiplying, learn to pronounce the partial results, as in addition, without naming the numbers separately; thus, in the last example, instead of saying 6 times 4 are 24, 6 times 7 are 42 and 2 to carry are 44, 6 times 3 are 18 and 4 to carry are 22, pronounce only the results, 24, 44, 22, performing the operations mentally. This will greatly facilitate the process of multiplying.

	2.	3.	4.
Multiplicand,	7324	6812	34651
Multiplier,	4	6	5
Product,	29296	40872	173255

5.	6.	7.	8.
82456	92714	28093	46247
3	7	8	9

9. Multiply 32746 by 5. *Ans.* 163730.
10. Multiply 840371 by 7. *Ans.* 5882597.
11. Multiply 137629 by 8. *Ans.* 1101032.
12. Multiply 93762 by 3. *Ans.* 281286.
13. Multiply 543272 by 4. *Ans.* 2173088.
14. Multiply 703164 by 9. *Ans.* 6328476.
15. What will be the cost of 344 cords of wood, at $4 a cord? *Ans.* $1376.

16. How much will an army of 7856 men receive in one week, if each man receives $6? *Ans.* $47136.

17. In one day there are 86400 seconds. How many seconds are there in 7 days? *Ans.* 604800 seconds.

18. What will 7640 pounds of prunes cost, at 9 cents a pound? *Ans.* $687.60.

19. At $5 an acre, what will 2487 acres of land cost? *Ans.* $12435.

20. In one mile there are 5280 feet. How many feet are there in 8 miles? *Ans.* 42240 feet.

21. What will 125 barrels of apples cost, at $4 a barrel?

22. There are 24 hours in a day. How many hours are there in 365 days?

63. When the multiplier consists of two or more figures.

1. Multiply 746 by 23.

OPERATION.

Multiplicand,	746	
Multiplier,	23	
	2238	3 {times the multiplicand.
	1492	20 {times the multiplicand.
Product,	17158	23 {times the multiplicand.

SOLUTION. — Writing the multiplicand and multiplier as in **60**, we first multiply each figure in the multiplicand by the unit figure of the multiplier, precisely as in **60**. Then we multiply by the 2 tens. 2 tens times 6 units, or 6 times 2 tens, are 12 tens, equal to 1 hundred, and 2 tens; we place the 2 tens under the tens' figure in the product already obtained, and add the 1 hundred to the next hundreds produced. 2 tens times 4 tens are 8 hundreds, and the 1 hundred of the last product added are 9 hundreds; we write the 9 in the hundreds' place in the product. 2 tens times 7 hundreds are 14 thousands, equal to 1 ten thousand and 4 thousands, which we write in their appropriate places in the product. Then adding the two products, the entire product is 17158. When there are ciphers between the significant figures of the multiplier, pass over them, and multiply by the significant figures only.

64. When the multiplier contains two or more figures, the several results obtained by multiplying by each figure are called *partial products.*

From the preceding examples and illustrations we deduce the following general rule:

RULE. — I. *Write the multiplier under the multiplicand, placing units of the same order under each other.*

II. *Multiply the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.*

III. *If there are partial products, add them, and their sum will be the product required.*

PROOF. — I. *Multiply the multiplier by the multiplicand, and if the product is the same as the first result, the work is probably correct. Or,*

II. *Multiply the multiplicand by the multiplier diminished by* 1, *and to the product add the multiplicand; if the sum is the same as the product by the whole of the multiplier, the work is probably correct.*

	2.	3.	4.
Multiply	4732	8721	17605
By	36	47	204
	28392	61047	70420
	14196	34884	35210
Ans.	170352	409887	3591420

	5.	6.	7.
	7648	81092	37967
	328	194	426

8. How many yards of linen are there in 759 pieces, each piece containing 25 yards? *Ans.* 18975 yards.

9. Sound is known to travel about 1142 feet in a second of time. How far will it travel in 69 seconds?

10. A man bought 36 lots at $475 each. What did they all cost him? *Ans.* $17100.

11. What would be the value of 867 shares of railroad stock, at $97 dollars a share? *Ans.* $84099.

12. How many pages are there in 3475 books, if there are 362 pages in each book? *Ans.* 1257950 pages.

13. In a garrison of 4507 men, each man receives annually $208. What do they all receive?

14. Multiply 7198 by 216. *Ans.* 1554768.

15. Multiply 31416 by 175. *Ans.* 5497800.

16. Multiply 7071 by 556. *Ans.* 3931476.

17. Multiply 75649 by 579. *Ans.* 43800771.

18. Multiply 15607 by 3094. *Ans.* 48288058.

19. Multiply 79094451 by 76095. *Ans.* 6018692248845.

20. Multiply five hundred forty thousand six hundred nine, by seventeen hundred fifty. *Ans.* 946065750.

21. Multiply four million twenty-five thousand three hundred ten, by seventy-five thousand forty-six.

Ans. 302083414260.

22. Multiply eight hundred seventy-seven million five hundred ten thousand eight hundred sixty-four, by five hundred forty-five thousand three hundred fifty-seven.

Ans. 478556692258448.

23. If one mile of railroad requires 116 tons of iron, worth $65 a ton, what will be the cost of sufficient iron to construct a road 128 miles in length? *Ans.* $965120.

24. The area of the United States is 3,668,167 square miles, and according to the census of 1890 there was an average of 17 people to the square mile. What was the population of the United States according to that census?

CONTRACTIONS.

65. A **Composite Number** is one that is the product of two or more other numbers.

Thus, 18 is a composite number, since $6 \times 3 = 18$; or, $9 \times 2 = 18$; or, $3 \times 3 \times 2 = 18$.

66. The **Component Factors** of a number are the several numbers which, when multiplied by one another, produce the given number.

Thus, the component factors of 20 are 10 and 2 ($10 \times 2 = 20$); or, 4 and 5 ($4 \times 5 = 20$); or, 2 and 2 and 5 ($2 \times 2 \times 5 = 20$).

The pupil must not confound the *factors* with the *parts* of a number. Thus, the *factors* of which 12 is composed, are 4 and 3 ($4 \times 3 = 12$); while the *parts* of which 12 is composed are 8 and 4 ($8 + 4 = 12$), or 10 and 2 ($10 + 2 = 12$). The *factors* are *multiplied* while the *parts* are *added*, to produce the number.

EXAMPLES.

67. When the multiplier is a composite number.

1. What will 32 horses cost, at \$174 apiece?

OPERATION.

Multiplicand,	\$ 174	cost of 1 horse.
1st factor,	4	
	\$ 696	cost of 4 horses.
2d factor,	8	
Product,	\$ 5568	cost of 32 horses.

SOLUTION. — The factors of 32 are 4 and 8. If we multiply the cost of 1 horse by 4, we obtain the cost of 4 horses; and by multiplying the cost of 4 horses by 8, we obtain the cost of 8 times 4 horses, or 32 horses, the number bought.

RULE. — I. *Separate the composite number into two or more factors. Multiply the multiplicand by one of these factors, and the product by another, and so on until all the factors have been used successively; the last product will be the product required.*

68. The product of any number of factors will be the same in whatever order they are multiplied.

Thus, $4 \times 3 \times 5 = 60$, and $5 \times 4 \times 3 = 60$, and $3 \times 4 \times 5 = 60$.

2. Multiply 3472 by 48, (6 × 8). *Ans.* 166656.
3. Multiply 14761 by 64, (8 × 8).
4. Multiply 87034 by 81, (3 × 3 × 9). *Ans.* 7049754.
5. Multiply 47326 by 120, (6 × 5 × 4).
6. Multiply 60315 by 96. *Ans.* 5790240.
7. Multiply 291042 by 125. *Ans.* 36380250.
8. If a vessel sails 436 miles in 1 day, how far will she sail in 56 days? *Ans.* 24416 miles.
9. What will 72 acres of land cost, at $124 an acre? *Ans.* $8928.
10. There are 5280 feet in a mile. How many feet are there in 84 miles? *Ans.* 443520 feet.
11. What will 120 yoke of cattle cost, at $125 a yoke?

69. When the multiplier is 10, 100, 1000, etc.

If a cipher is annexed to the multiplicand, each figure is removed *one* place toward the left, and consequently the value of the whole number is increased tenfold (**33**). If two ciphers are annexed, each figure is removed *two* places toward the left, and the value of the number is increased one hundred fold; and every additional cipher increases the value tenfold.

RULE. — *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier; the number so formed will be the product required.*

1. Multiply 347 by 10. *Ans.* 3470.
2. Multiply 4731 by 100. *Ans.* 473100.
3. Multiply 13071 by 1000.
4. Multiply 89017 by 10000.
5. If 1 acre of land costs $36, what will 10 acres cost? *Ans.* $360.
6. If 1 bushel of corn costs 65 cents, what will 1000 bushels cost? *Ans.* $650.

70. When there are ciphers at the right hand of one or both of the factors.

1. Multiply 1200 by 60.

OPERATION.

Multiplicand,	1200
Multiplier,	60
Product,	72000

SOLUTION. — Both multiplicand and multiplier may be resolved into their component factors; 1200 into 12 and 100, and 60 into 6 and 10. If these several factors are multiplied by one another they will produce the same product as the given numbers (**67**). Thus, $12 \times 6 = 72$, and $72 \times 100 = 7200$, and $7200 \times 10 = 72000$, which is the same result as in the operation.

RULE. — *Multiply the significant figures of the multiplicand by those of the multiplier, and to the product annex as many ciphers as there are ciphers on the right of both factors.*

	2.	3.
Multiply	20	10340000
By	340000	105000
	1888	5170
	1416	1034
	1604800000	1085700000000

4. Multiply 70340 by 800400. *Ans.* 56300136000.
5. Multiply 3400900 by 207000. *Ans.* 703986300000.
6. Multiply 634003000 by 40020.
Ans. 25372800060000.
7. Multiply 10203070 by 50302000.
Ans. 513234827140000.
8. Multiply 30090800 by 600080.
Ans. 18056887264000.
9. Multiply eighty million seven thousand six hundred, by eight million seven hundred sixty.
Ans. 640121605776000.

10. Multiply fifty million ten thousand seventy, by sixty-four thousand. *Ans.* 3200644480000.

11. Multiply ten million three hundred fifty thousand one hundred, by eighty thousand nine hundred.

Ans. 837323090000.

EXAMPLES COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. A dealer bought 45 cords of wood at $4 a cord, and 9 loads of hay at $13 a load. What was the cost of the wood and hay? *Ans.* $297.

2. A merchant bought 6 hogsheads of sugar at $31 a hogshead, and sold it for $39 a hogshead. What did he gain?

3. I bought 288 barrels of flour for $1875, and sold the same for $9 a barrel. What was the gain?

Ans. $717.

4. If a young man receives $500 a year salary and pays $240 for board, $125 for clothing, $75 for books, and $50 for other expenses, how much will he have left at the end of the year? *Ans.* $10.

5. A farmer sold 184 bushels of wheat at $2 a bushel, for which he received 67 yards of cloth at $4 a yard, and the balance in groceries. What did his groceries cost him?

6. A sold a farm of 320 acres at $36 an acre; B sold one of 244 acres at $48 an acre. Which received the greater sum, and how much? *Ans.* B, $192.

7. A merchant bought 14 bales of cloth, each bale containing 26 pieces, and each piece 43 yards. How many yards of cloth did he buy? *Ans.* 15652 yards.

8. If a man has an income of $3700 a year, and his daily expenses are $4, what will he save in a year, or 365 days? *Ans.* $2240.

9. Two persons start from the same point and travel in opposite directions, one at the rate of 35 miles a day, and the other 29 miles a day. How far apart will they be in 16 days? *Ans.* 1024 miles.

10. A man sold three houses; for the first he received $2475, for the second $840 less than he received for the first, and for the third as much as for the other two. What did he receive for the three? *Ans.* $8220.

11. A man sets out to travel from Albany to Buffalo, a distance of 336 miles, and walks 28 miles a day. How far is he from Buffalo when he has walked 10 days?

12. Mr. C bought 14 cows at $23 each, 7 horses at $96 each, 34 oxen at $57 each, and 300 sheep at $2 each; he sold them all for $3842. What did he gain? *Ans.* $310.

13. A drover bought 164 head of cattle at $36 a head, and 850 sheep at $3 a head. What did he pay for all?

14. A banker has an income of $14760 a year; he pays $1575 for house rent, and four times as much for family expenses. What does he save annually? *Ans.* $6885.

15. A flour merchant bought 936 barrels of flour at $9 a barrel; he sold 480 barrels at $10 a barrel, and the remainder at $8 a barrel. What did he gain or lose? *Ans.* Gained $24.

16. What will 24 acres of land cost, at $124 an acre? *Ans.* $2976.

17. How far will a ship sail in 56 weeks, at the rate of 1512 miles per week? *Ans.* 84672 miles.

18. How many pounds of iron are there in 54 loads, each weighing 2873 pounds? *Ans.* 155142 pounds.

19. Multiply 2874 by 72. *Ans.* 206928.

20. Multiply 8074 by 108. *Ans.* 871992.

DIVISION.

ORAL EXERCISES.

71. 1. How many hats, at $4 apiece, can be bought for $20?

SOLUTION. — Twenty dollars will buy as many hats as 4 is contained times in 20, which is 5 times. Five hats, at $4 apiece, can be bought for $20.

2. A man gave $16 for 8 barrels of apples. What was the cost of each barrel?

3. If 1 cord of wood costs $3, how many cords can be bought for $15?

4. At $6 a barrel, how many barrels of flour can be bought for $24?

5. When flour is $5 a barrel, how many barrels can be bought for $30?

6. If a man can dig 7 rods of ditch in a day, how many days will it take him to dig 28 rods?

7. If an orchard contains 56 trees, and 7 trees in a row, how many rows are there?

8. I bought 6 barrels of flour for $42. What was the cost of 1 barrel?

9. If a farmer divides 21 bushels of potatoes equally among 7 laborers, how many bushels will each receive?

10. How many oranges, at 3 cents each, can be bought for 27 cents?

11. A farmer paid $35 for sheep, at $5 apiece. How many did he buy?

12. How many times 4 in 28? In 16? In 36?
13. How many times 8 in 40? In 56? In 64?
14. How many times 9 in 36? In 63? In 81?
15. How many times 7 in 49? In 70? In 84?

72. Division is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

Division may also be regarded as a short method of performing several subtractions of a number.

73. The **Dividend** is the number to be divided, and the **Divisor** is the number by which to divide.

74. The **Quotient** is the result obtained by the process of division, and shows how many times the divisor is contained in the dividend.

1. When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the *remainder*, and it must be less than the divisor.
2. As the remainder is always a part of the dividend, it is always of the same name or kind.
3. When there is no remainder, the division is said to be *exact*.

75. The sign, $\div$, placed between two numbers, denotes division, and shows that the number on the *left* is *to be divided* by the number on the *right*.

Thus, $20 \div 4 = 5$, is read, 20 *divided by* 4 is *equal* to 5.

Division is also indicated by writing the dividend *above*, and the divisor *below*, a short horizontal line.

Thus, $\frac{12}{3} = 4$, shows that 12 *divided by* 3 equals 4.

76. In finding how many times one number is contained in another, the *dividend* and *divisor* are *like numbers*, and the *quotient* is an *abstract number*.

In finding one of the equal parts of a number the *dividend* and *quotient* are *like numbers*, and the *divisor* is an *abstract number*.

EXAMPLES.

77. When the divisor consists of one figure.

1. How many times is 4 contained in 848?

OPERATION.

Dividend.
Divisor. 4)848
Quotient. 212

SOLUTION. — After writing the divisor on the left of the dividend, with a line between them, we begin at the left hand and say: 4 is contained in 8 hundreds, 2 hundreds times, and we write 2 in hundreds' place in the quotient; 4 is contained in 4 tens 1 ten times; we write the 1 in tens' place in the quotient; 4 is contained in 8 units 2 units times; we write the 2 in units' place in the quotient, and the entire quotient is 212.

2. How many times is 4 contained in 2884?

OPERATION.

4)2884
721

SOLUTION. — As we cannot divide 2 thousands by 4, we take the 2 thousands and the 8 hundreds together. 4 is contained in 28 hundreds 7 hundreds times, which we write in hundreds' place in the quotient; 4 is contained in 8 tens 2 tens times, which we write in tens' place in the quotient; and 4 is contained in 4 units 1 unit time, which we write in units' place, and we have the entire quotient, 721.

3. How many times is 6 contained in 1824?

OPERATION.

6)1824
304

SOLUTION. — Beginning as in the last example, we say, 6 is contained in 18 hundreds 3 hundreds times, which we write in hundreds' place in the quotient; 6 is contained in 2 tens no times, and we write a cipher in tens' place in the quotient; taking the 2 tens and 4 units together, 6 is contained in 24 units 4 units times, which we write in units' place in the quotient, and we have 304 for the entire quotient.

4. How many times is 4 contained in 943?

OPERATION.

4)943
235 . . . 3 Rem.

SOLUTION. — Here 4 is contained in 9 hundreds 2 hundreds times, and 1 hundred over, which, united to the 4 tens, makes 14 tens; 4 in 14 tens, 3 tens times and 2 tens over, which, united to the 3 units, makes 23 units; 4 in 23 units 5 units times and 3 units over. The 3 which is left after performing the division, should be divided by 4; but we merely indicate the division by placing the divisor under the dividend, thus, $\frac{3}{4}$. The entire quotient is written $235\frac{3}{4}$, which may be read, two hundred thirty-five and *three divided by four*, or, two hundred thirty-five and *three fourths*, or, two hundred thirty-five and a *remainder* of three.

Hence the following rule:

RULE. — I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Beginning at the left hand, divide each figure of the dividend by the divisor, and write the result under the dividend.*

III. *If there is a remainder after dividing any figure, regard it as prefixed to the figure of the next lower order in the dividend, and divide as before.*

IV. *Should any figure or part of the dividend be less than the divisor, write a cipher in the quotient, prefix the number to the figure of the next lower order in the dividend, and divide as before.*

V. *If there is a remainder after dividing the last figure, place it over the divisor at the right hand of the quotient.*

PROOF. — *Multiply the quotient by the divisor, and to the product add the remainder, if any; if the result is equal to the dividend, the work is correct.*

1. This method of proof depends on the fact that division is the reverse of multiplication. The *dividend* corresponds to the *product*, the *divisor* to one of the *factors*, and the *quotient* to the *other*.

2. In multiplication the two factors are given, to find the product; in division, the product and one of the factors are given, to find the other factor.

5. Divide 7824 by 6.

OPERATION.		PROOF.	
Divisor. 6)7824	Dividend.	1304	Quotient.
1304	Quotient.	6	Divisor.
		7824	Dividend.

6.	7.	8.
4)65432	5)89135	6)178932
9.	**10.**	**11.**
7)4708935	8)1462376	9)7468542

Quotients.

12. Divide 3102455 by 5. 620491.

13. Divide 1762891 by 4. $440722\frac{3}{4}$.

		Quotients.
14.	Divide 546215747 by 11.	49655977.
15.	Divide 30179624 by 12.	$2514968\frac{8}{12}$.
16.	Divide 9254671 by 9.	$1028296\frac{7}{9}$.

Find the quotients and remainders of:

17.	$7341568 \div 7$.	**20.**	$84201763 \div 9$.
18.	$3179632 \div 5$.	**21.**	$2947691 \div 12$.
19.	$19038716 \div 8$.	**22.**	$42084796 \div 6$.

The sums of quotients and remainders of examples 17 to 22 are 20680083. 28.

23. Divide $47645 equally among 5 men. What will each receive? *Ans.* $9529.

24. There are seven days in one week. How many weeks are there in 17675 days? *Ans.* 2525 weeks.

25. How many barrels of flour, at $6 a barrel, can be bought for $6756? *Ans.* 1126 barrels.

26. Twelve things make a dozen. How many dozen are there in 46216464? *Ans.* 3851372 dozen.

27. How many barrels of flour can be made from 347560 bushels of wheat, if it takes 5 bushels to make one barrel? *Ans.* 69512 barrels.

28. If there are 3240622 acres of land in 11 townships, how many acres are there in each township?

29. A man left his estate, worth $38470, to be shared equally by his wife and 4 children. What did each receive? *Ans.* $7694.

30. At $5 an acre, how many acres of land can be bought for $3875?

31. If a man walks 4 miles an hour, in how many hours will he walk 352 miles?

32. I paid $1792 for 7 horses. What did each cost?

33. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

78. When the divisor consists of two or more figures.

To illustrate more clearly the method of operation, we will first take an example usually performed by Short Division.

1. How many times is 8 contained in 2528?

OPERATION.

Divisor. Divid'd. Quotient.

```
8 ) 2528 ( 316
    24
    ---
     12
      8
     --
     48
     48
     --
```

SOLUTION. — As 8 is not contained in 2 thousands, we take 2 and 5 as one number, and consider how many times 8 is contained in this *partial dividend*, 25 hundreds, and find that it is contained 3 hundreds times, and a remainder. To find this remainder, we multiply the divisor, 8, by the quotient figure, 3 hundreds, and subtract the product, 24 hundreds, from the partial dividend, 25 hundreds, and there remains 1 hundred. To this remainder we bring down the 2 tens of the dividend, and consider the 12 tens as a *second* partial dividend. Then, 8 is contained in 12 tens 1 ten time and a remainder; 8 multiplied by 1 ten produces 8 tens, which, subtracted from 12 tens, leave 4 tens. To this remainder we bring down the 8 units, and consider the 48 units as the *third* partial dividend. Then, 8 is contained in 48 units 6 units times. Multiplying and subtracting as before, we find that nothing remains, and the entire quotient is 316.

2. How many times is 23 contained in 4807?

OPERATION.

Divisor. Divid'd. Quotient.

```
23 ) 4807 ( 209
     46
     --
      207
      207
      ---
```

SOLUTION. — We first find how many times 23 is contained in 48, the first partial dividend, and place the result in the quotient on the right of the dividend. We then multiply the divisor, 23, by the quotient figure, 2, and subtract the product, 46, from the part of the dividend used, and to the remainder bring down the next figure of the dividend, which is 0, making 20, for the second partial dividend. Then, since 23 is contained in 20 no times, we place a cipher in the quotient, and bring down the next figure of the dividend, making a third partial dividend, 207; 23 is contained in 207, 9 times; multiplying and subtracting as before, nothing remains, and the entire quotient is 209.

79. When the process of division is performed mentally, and the results only are written, the operation is termed *Short Division.*

When the whole process of division is written, the operation is termed *Long Division.*

Short Division is generally used when the divisor is a number that will allow the division to be performed mentally.

From the preceding illustrations we derive the following general rule:

RULE. — I. *Write the divisor at the left of the dividend, as in short division.*

II. *Divide the least number of the left hand figures in the dividend that will contain the divisor one or more times, and place the quotient at the right of the dividend, with a line between them.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder bring down the next figure of the dividend.*

IV. *Divide as before, until all the figures of the dividend have been brought down and divided.*

V. *If any partial dividend will not contain the divisor, place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.*

VI. *If there is a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.*

1. If any remainder is *equal to*, or *greater* than the divisor, the quotient figure is too *small*, and must be increased.

2. If the product of the divisor by the quotient figure is *greater* than the partial dividend, the quotient figure is too *large*, and must be diminished.

PROOF. — 1. *The same as in short division. Or,*

2. *Subtract the remainder, if any, from the dividend, and divide the difference by the quotient; if the result is the same as the given divisor, the work is correct.*

80. The operations in long division consist of five principal steps, viz.:

1. Write down the numbers.
2. Find how many times the divisor is contained in the partial dividend.
3. Multiply.
4. Subtract.
5. Bring down another figure.

3. Find how many times 36 is contained in 11798.

OPERATION.

Dividend.
Divisor. 36)11798(327 Quotient.
108
99
72
278
252
26 Remainder.

PROOF BY MULTIPLICATION.

327 Quotient.
36 Divisor.
1962
981
11772
26 Remainder.
11798 Dividend.

4. Find how many times 82 is contained in 89634.

OPERATION.

82)89634(1093
82
763
738
254
246
8

Quotient. 1093

PROOF BY DIVISION.

89634 Dividend.
8 Remainder.
89626(82 Divisor.
8744
2186
2186

5. Find how many times 154 is contained in 32740.
6. Divide 32572 by 34. *Ans.* 958.
7. Divide 1554768 by 216. *Ans.* 7198.

8. Divide 5497800 by 175. *Ans.* 31416.
9. Divide 3931476 by 556. *Ans.* 7071.
10. Divide 10983588 by 132. *Ans.* 83209.
11. Divide 73484248 by 19. *Ans.* 3867592.
12. Divide 8121918 by 21. *Ans.* 386758.
13. Divide 10557312 by 16. *Ans.* 659832.
14. Divide 93840 by 63. *Rem.* 33.
15. Divide 352417 by 29. *Rem.* 9.
16. Divide 51846734 by 102. *Rem.* 32.
17. Divide 1457924651 by 1204. *Rem.* 1051.
18. Divide 729386 by 731. *Rem.* 579.
19. Divide 4843167 by 3605. *Rem.* 1652.
20. Divide 49816657 by 9101. *Rem.* 6884.
21. Divide 75867308 by 10115. *Rem.* 4808.

	Quotients.	Rem.
22. Divide 28101418481 by 1107.	25385201.	974.
23. Divide 65358547823 by 2789.	23434402.	645.
24. Divide 102030405060 by 123456.	826451.	70404.
25. Divide 48659910 by 54001.	901.	5009.
26. Divide 2331883961 by 6739549.	346.	7.

27. A railroad cost one million eight hundred fifty thousand four hundred dollars, and was divided into eighteen thousand five hundred and four shares. What was the value of each share? *Ans.* $100.

28. If a tax of seventy-two million three hundred twenty thousand sixty dollars is equally assessed on ten thousand seven hundred thirty-five towns, what amount of tax must each town pay? *Ans.* 6736\frac{9100}{10735}$.

29. If 213 college libraries contain 942321 volumes, what is the average number of volumes to each library?

Ans. 4424$\frac{9}{213}$ volumes.

30. A man bought 240 acres of land at $15 an acre, giving in payment horses valued at $180 apiece. How many horses did he give?

CONTRACTIONS.

EXAMPLES.

81. When the divisor is a composite number.

1. If 3270 dollars are divided equally among 30 men, how many dollars will each receive?

OPERATION.

5 | 3270
6 | 654
109 *Ans.*

SOLUTION. — If 3270 dollars are divided equally among 30 men, each man will receive as many dollars as 30 is contained times in 3270 dollars. 30 may be resolved into the factors 5 and 6; and we may suppose the 30 men divided into 5 groups of 6 men each; dividing the 3270 dollars by 5, the number of groups, we have 654, the number of dollars to be given to each group; and dividing the 654 dollars by 6, the number of men in each group, we have 109, the number of dollars that each man will receive.

Hence we have the following rule:

RULE. — *Divide the dividend by one of the factors, and the quotient thus obtained by another, and so on if there are more than two factors, until every factor has been made a divisor. The last quotient will be the quotient required.*

2. Divide 3690 by 15, (3 × 5). *Ans.* 246.
3. Divide 3528 by 24, (4 × 6). *Ans.* 147.
4. Divide 7280 by 35, (5 × 7). *Ans.* 208.
5. Divide 6228 by 36, (6 × 6). *Ans.* 173.
6. Divide 33642 by 27, (3 × 9). *Ans.* 1246.
7. Divide 153160 by 56, (7 × 8). *Ans.* 2735.
8. Divide 15625 by 125, (5 × 5 × 5). *Ans.* 125.
9. Divide 34692 by 42, (6 × 7).
10. Divide 401472 by 64, (8 × 8).
11. Divide 139104 by 72, (3 × 4 × 6).
12. Divide 9078120 by 90, (3 × 5 × 6).
13. Divide 18730560 by 120, (4 × 5 × 6).

14. Divide 7360416 by 96, $(2 \times 6 \times 8)$.
15. Divide 24726170 by 70, $(2 \times 5 \times 7)$.
16. Divide 796950 by 175, $(7 \times 5 \times 5)$.

82. When the divisor is 10, 100, 1000, etc.

1. Divide 374 acres of land equally among 10 men. How many acres will each have?

OPERATION.

1|0)37|4

Quotient. 37 . . . 4 Rem.

or, $37\frac{4}{10}$ acres.

SOLUTION. — As has been shown, to remove a figure one place toward the left by annexing a cipher increases its value tenfold, or multiplies it by 10 (**69**). On the contrary, by cutting off or taking away the right hand figure of a number, each of the other figures is removed one place toward the right, and, consequently, the value of each is diminished tenfold, or divided by 10 (**33**).

83. For similar reasons, if we cut off *two* figures, we divide by 100, if *three*, we divide by 1000, and so on.

RULE. — *From the right hand of the dividend cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.*

2. Divide 47600 by 10. By 100.
3. Divide 1306321 by 1000.
4. Divide 9760347 by 10000.
5. Divide 2037160310 by 100000.

84. When there are ciphers on the right hand of the divisor.

1. Divide 437661 by 800.

OPERATION.

8|00)4376|61

547 . . . 61 Rem.

SOLUTION. — In this example we resolve 800 into the factors 8 and 100, and divide first by 100, by cutting off two right hand figures of the dividend (**82**), and we have a quotient of 4376, and a remainder of 61. We next divide by 8, and obtain 547 for a quotient; and the entire quotient is $547\frac{61}{800}$.

2. Divide 34716 by 900.

OPERATION.

$9|00)\underline{347|16}$

38 Quotient. 5 2d Rem.

$5 \times 100 + 16 = 516$ True Rem.

$38\frac{5}{9}\frac{1}{0}\frac{6}{0}$. *Ans.*

SOLUTION. — Dividing as in the last example, we have a quotient of 38, and two remainders, 16 and 5; but the 5 being in hundreds' place represents 500, to which we add 16, and we have 516 for the true remainder.

85. When there is a remainder after dividing by the significant figures, it must be prefixed to the figures cut off from the dividend to give the true remainder; if there is no other remainder, the figures cut off from the dividend will be the true remainder.

		Quotients.	Rem.
3.	Divide 34716 by 900.	38.	516.
4.	Divide 1047634 by 2400.	436.	1234.
5.	Divide 47321046 by 45000.	1051.	26046.
6.	Divide 2037903176 by 140000.		63176.
7.	Divide 976031425 by 92000.		3425.
8.	Divide 80013176321 by 700000.		376321.
9.	Divide 19070367428 by 4160000.	4584.	927428.
10.	Divide 379025644319 by 554000000.		89644319.
11.	Divide 897654321234 by 940000000.		
12.	Divide 394298765984 by 898000000.		
13.	Divide 647281909085 by 102030400.		

14. The circumference of the earth at the equator is 24898 miles. How many hours would a train of cars require to travel that distance, going at the rate of 50 miles an hour? *Ans.* $497\frac{4}{5}\frac{8}{0}$.

15. The sum of $350000 is paid to an army of 14000 men. What does each man receive? *Ans.* $25.

16. If 800 shares of railroad stock are valued at *$840999, what is* the value of each share?

GENERAL PRINCIPLES OF DIVISION.

86. The quotient in Division depends upon the relative values of the dividend and divisor. Hence any change in the value of either dividend or divisor must produce a change in the value of the quotient. But some changes may be produced upon *both* dividend and divisor, *at the same time*, that will not affect the quotient.

The laws which govern these changes are called *General Principles of Division.* They are as follows:

$$54 \div 9 = 6.$$

I. Multiplying the dividend by 3, we have

$$54 \times 3 \div 9 = 162 \div 9 = 18,$$

and 18 = the quotient, 6, multiplied by 3. Hence,

Multiplying the dividend by any number, multiplies the quotient by the same number.

Using the same example, $54 \div 9 = 6$.

II. Dividing the dividend by 3, we have

$$\tfrac{54}{3} \div 9 = 18 \div 9 = 2,$$

and 2 = the quotient, 6, divided by 3. Hence,

Dividing the dividend by any number, divides the quotient by the same number.

III. Multiplying the divisor by 3, we have

$$54 \div 9 \times 3 = 54 \div 27 = 2,$$

and 2 = the quotient, 6, divided by 3. Hence,

Multiplying the divisor by any number, divides the quotient *by the same number.*

IV. Dividing the divisor by 3, we have

$$54 \div \tfrac{9}{3} = 54 \div 3 = 18,$$

and 18 = the quotient, 6, multiplied by 3. Hence,

Dividing the divisor by any number, multiplies the quotient by the same number.

V. Multiplying both dividend and divisor by 3, we have $54 \times 3 \div 9 \times 3 = 162 \div 27 = 6$. Hence,

Multiplying both dividend and divisor by the same number, does not alter the value of the quotient.

VI. Dividing both dividend and divisor by 3, we have $\frac{54}{3} \div \frac{9}{3} = 18 \div 3 = 6$. Hence,

Dividing both dividend and divisor by the same number, does not alter the value of the quotient.

These examples illustrate all the changes we ever have occasion to make upon the dividend and divisor. The principles upon which these changes are based may be stated as follows:

PRINCIPLES. — I. *Multiplying the dividend multiplies the quotient; dividing the dividend divides the quotient.* (I. and II.)

II. *Multiplying the divisor divides the quotient; dividing the divisor multiplies the quotient.* (III. and IV.)

III. *Multiplying or dividing both dividend and divisor by the same number, does not alter the quotient.* (V. and VI.)

These three principles may be embraced in one law.

GENERAL LAW. — *A change in the* DIVIDEND *produces a* LIKE *change in the quotient; but a change in the* DIVISOR *produces an* OPPOSITE *change in the quotient.*

If a number is multiplied and the product divided by the same number, the quotient will be equal to the number multiplied. Thus, $15 \times 4 = 60$, and $60 \div 4 = 15$.

EXAMPLES IN THE PRECEDING RULES.

1. George Washington was born in 1732, and lived 67 years. In what year did he die? *Ans.* In 1799.

2. How many dollars a day must a man spend, to use an income of $1095 a year? *Ans.* $3.

3. If I give $141 for a piece of cloth containing 47 yards, for what must I sell it in order to gain $1 a yard? *Ans.* $188.

4. A speculator who owned 500 acres, 17 acres, 98 acres, and 121 acres of land, sold 325 acres. How many acres had he left? *Ans.* 411 acres.

5. A dealer sold a cargo of salt for $2300, and gained $625. What did the cargo cost him? *Ans.* $1675.

6. If a man earns $60 a month, and spends $45 in the same time, how long will it take him to save $900 from his earnings?

7. If 9 persons use a barrel of flour in 87 days, how many days will a barrel last 1 person at the same rate? *Ans.* 783 days.

8. The first of three numbers is 4, the second is 8 times the first, and the third is 9 times the second. What is their sum? *Ans.* 324.

9. If 2, 2, and 7 are three factors of 364, what is the other factor? *Ans.* 13.

10. A man has 3 farms; the first contains 78 acres, the second 104 acres, and the third as many acres as both the others. How many acres are there in the 3 farms?

11. If the expenses of a boy at school are $90 for board, $30 for clothes, $12 for tuition, $5 for books, and $7 for pocket money, what would be the expenses of 27 boys at the same rate? *Ans.* $3888.

12. Two men travel in opposite directions, one at the rate of 35 miles a day, and the other at the rate of 40

miles a day. How far apart are they at the end of 6 days?

13. Four children inherited $2250 each; but one dying, the remaining three inherited the whole. What was the share of each? *Ans.* $3000.

14. Two men travel in the same direction, one at the rate of 35 miles a day, and the other at the rate of 40 miles a day. How far apart are they at the end of 6 days?

15. A man was 45 years old, and he had been married 19 years. How old was he when married?

Ans. 26 years old.

16. Upon how many acres of ground can the entire population of the globe stand, supposing that 25000 persons can stand upon one acre, and that the population is 1000000000? *Ans.* 40000 acres.

17. Add 384, 1562, 25, and 946; subtract 2723 from the sum; divide the remainder by 97; and multiply the quotient by 142. What is the result? *Ans.* 284.

18. How many steps of three feet each would a man take in walking a mile, or 5280 feet? *Ans.* 1760 steps.

19. A man purchased a house for $2375, and expended $340 in repairs; he then sold it for railroad stock worth $867, and 235 acres of western land valued at $8 an acre. What did he gain by the trade? *Ans.* $32.

20. The salary of a clergyman is $800 a year, and his yearly expenses are $450. If he is worth $1350 now, in how many years will he be worth $4500? *Ans.* 9 years.

21. How many bushels of oats, at 40 cents a bushel, must be given for 1600 bushels of wheat at 75 cents a bushel? *Ans.* 3000 bushels.

22. If you deposit $2.25 each week in a savings bank, and take out 75 cents a week, how much money will you have left at the end of the year? *Ans.* $78.

23. I bought 325 loads of wheat, each load containing 50 bushels, at $2 a bushel. What did the wheat cost?

24. The product of two numbers is 31383450, and one of the numbers is 4050. What is the other number?

25. A certain railroad is 700 miles long, and cost $31647000. What did it cost per mile? *Ans.* $45210.

26. What number is that, which being divided by 7, the quotient multiplied by 3, the product divided by 5, and this quotient increased by 40, the sum will be 100?

Ans. 700.

27. How many cows, at $27 apiece, must be given for 54 tons of hay at $17 a ton?

28. A mechanic receives $56 for 26 days' work, and spends $2 a day for the whole time. How many dollars has he left? *Ans.* $4.

29. If 7 men can build a house in 98 days, how long would it take one man to build it? *Ans.* 686 days.

30. If the cash value of 11137 school-houses is $5301212, what is the average value? *Ans.* $476.

31. A cistern whose capacity is 840 gallons has two pipes; through one pipe 60 gallons run into it in an hour and through the other 39 gallons run out in the same time. In how many hours will the cistern be filled?

Ans. 40 hours.

32. The average beat of the pulse of a man at middle age is about 4500 times in an hour. How many times does it beat in 24 hours? *Ans.* 108000 times.

33. How many years is it from the discovery of America, in 1492, to the year 1900? What year is the 400th centennial of the discovery of America?

34. A man bought 40 acres of ground at $15 an acre, and 80 acres at $25 an acre. He sold 90 acres for $4500, and the remainder at $60 an acre. For how much did he sell the whole land? How much did he gain?

35. What is the remainder after dividing 62530000 by 87900? *Ans.* 33100.

36. A pound of cotton has been spun into a thread 8 miles in length; allowing 235 pounds for waste, how many pounds will it take to spin a thread to reach round the earth, supposing the distance to be 25000 miles?

Ans. 3360 pounds.

37. John has $8546, which is $342 less than 4 times as much as Charles has. How many dollars has Charles?

Ans. $2222.

38. The quotient of one number divided by another is 37, the divisor 245, and the remainder 230. What is the dividend? *Ans.* 9295.

39. What number multiplied by 72084 will produce 5190048? *Ans.* 72.

40. There are two numbers, the greater of which is 73 times 109, and their difference is 17 times 28. What is the less number? *Ans.* 7481.

41. The sum of two numbers is 360, and the less is 114. What is the product of the two numbers?

Ans. 28044.

42. What number added to 2473248 makes 2568754?

43. A farmer sold 35 bushels of wheat at $2 a bushel, and 18 cords of wood at $3 a cord; he received 9 yards of cloth at $4 a yard, and the balance in money. How many dollars did he receive? *Ans.* $88.

44. A farmer receives $684 a year for produce from his farm, and his expenses are $375 a year. What will he save in 5 years?

45. If a salt manufacturer pays 58 cents for wood to boil one barrel of salt, 10 cents for boiling, 5 cents for the brine, 28 cents for the barrel, and 3 cents for packing and weighing, and receives $1.25 from the purchaser, what does he make on a barrel? *Ans.* 21 cents.

PROOFS.

CASTING OUT NINES.

87. The remainder arising from dividing any number by 9 is always the same as the remainder that arises from dividing the sum of all its digits by 9.

This arises from the fact that every 10, 100, or 1000 of a number, when divided by 9, has a remainder of 1. Thus, 9 into 10, once and 1 over; 9 into 100, 11 times and 1 over, etc.; 9 into 20, twice and 2 over; into 200, 2000, etc., 2 over; into 30, 300, 3000, etc., 3 over. Hence, if we divide by 9 separately the parts of a number, the remainders will be expressed by the digits of the number. Thus, if we divide 2345 by 9, first dividing the 2000, remainder 2; dividing 300, remainder 3; dividing 40, remainder 4; dividing 5, remainder 5. Total remainder, $2+3+4+5=14$; excess of 9's in $14 = 5$, which is the same as the excess of 9's in 2345.

Methods of proving addition, subtraction, multiplication, and division have already been given. The casting out of 9's furnishes another proof, briefer but not so sure.

EXAMPLES.

1. Prove that $3523 + 6414 + 1894 + 2129 = 13960$.

OPERATION.

3523, excess of 9's =	4
6414, excess of 9's =	6
1894, excess of 9's =	4
2129, excess of 9's =	5
13960	19

Exc. of 9's = 1. Exc. of 9's = 1.

SOLUTION. — Adding the digits in the first number, the excess of 9's is 4; in the second, 6; in the third, 4; and in the fourth, 5. The sum of the excesses is 19 = two 9's and 1. Dividing 13960 by 9, we find the remainder to be 1. Since the excess in both cases is the same, the answer is probably correct.

2. Prove that $15964 - 9432 = 6532$.

OPERATION.

15964, excess of 9's = 7
9432, excess of 9's = 0
6532 7

Exc. of 9's = 7. Exc. of 9's = 7.

SOLUTION. — Casting out the 9's from the minuend, the remainder is 7, and from the subtrahend, the remainder is 0. The difference, 7, is the same as the excess of 9's in 6532; hence the answer is probably correct.

3. Prove that $62 \times 42 = 2646$.

OPERATION.

62, excess of 9's = 8
42, excess of 9's = 6
2604, Exc. of 9's = 3. 48, Exc. = 3.

SOLUTION. — Casting out the 9's from the multiplicand, the remainder is 8; from the multiplier, 6. Their product is 48, in which the excess of 9's is 3. The excess of 9's in 2604 is 3; hence the answer is probably correct.

4. Prove that $207 \div 23 = 9$.

OPERATION.

23) 207 (9
Exc. = 5. Exc. = 0. Exc. = 0.

SOLUTION. — Casting out the 9's from divisor and quotient, the excess is 5 and 0. Their product is 0. Casting out the 9's from the dividend, the excess is 0. Hence the answer is probably correct.

RULE. — I. *In Addition, find the excess of 9's in the sum of the digits in each addend; add these excesses, and find the excess of 9's in their sum. If it agrees with the excess of 9's in the answer, the answer is probably correct.*

II. *In Subtraction, subtract the excess of 9's in the minuend from that in the subtrahend, casting out the 9's from the remainder. Compare the excess with excess in answer.*

NOTE. — If the excess of 9's in the subtrahend is greater than that in the minuend, add one 9 to the minuend and then subtract.

III. *In Multiplication, from the product of the excess of 9's in the multiplicand and multiplier, cast out the 9's. Compare the excess with excess in answer.*

IV. *In Division, find the excess of 9's in the quotient and divisor, and take their product. Cast out the 9's from this product. Compare this excess with excess in dividend.*

CASTING OUT ELEVENS.

88. When a number expressed by a digit in an odd place is divided by 11, the remainder is equal to that digit; and a number expressed by a digit in an even place, lacks that digit of being a multiple of 11. Therefore, if a number expressed by two figures is divided by 11, the remainder equals the digit in the odd place minus the digit in the even place.

Thus, in 4500, 500 represented by 5 in the third place, divided by 11, has a remainder of 5. 4000 represented by 4 in the fourth place, lacks 4 of being a multiple of 11. $5 - 4 = 1$. 4500 divided by 11 has a remainder of 1.

If the digit in the even place is greater than that in the odd place, it cannot be subtracted, so we add one 11 to it, and then proceed to subtract.

Hence, any number divided by 11 *will have a remainder equal to the sum of the digits in the odd places minus the sum of those in the even places.*

From these principles, we deduce proofs of the fundamental processes by casting out 11's, similar to the proofs by casting out 9's.

NOTE. — The following examples are the same as those used to illustrate the proofs by casting out 9's.

EXAMPLES.

1. Prove that 3523, 6414, 1894, and 2129 = 13960.

3523, $8 - 5 = 3$
6414, $8 - 7 = 1$
1894, $12 - 10 = 2$
2129, $10 - 4 = 6$

13960, 12, Exc. = 1.
$10 - 9 = 1$.
Exc. = 1.

SOLUTION. — The sums of the digits in the odd places minus those in the even places in the various addends are $3 + 1 + 2 + 6 = 12$; excess of 11's in $12 = 1$. The sum of the digits in the odd places of 13960 minus those in the even places = 1; excess of 11's = 1. Hence, the answer is probably correct.

2. Prove that $15964 - 9432 = 6532$.

15964, $14 - 11 = 3 (+11)$
9432, $6 (+11) - 12 = 5$
6532, 9, Exc.
$7 (+11) - 9 = 9$, Exc.

SOLUTION. — In the minuend the sum of the digits in the odd places is 14, and in the even places 11. Their difference is 3. In the subtrahend the sum of the digits in the odd places is 6, and in the even places 12. Since 12 cannot be subtracted from 6, we add one 11 to 6, making 17, and $17 - 12 = 5$. We subtract the results; but since 5 cannot be subtracted from 3, we add one 11 to 3, making 14, and $14 - 5 = 9$. In the same way we find the excess of 11's in 6532 to be 9. Since the two excesses of 11 agree, the answer is probably correct.

3. Prove that $62 \times 42 = 2646$.

62, $2 (+11) - 6 = 7$
42, $2 (+11) - 4 = 9$
2604 63
$10 - 2 = 8$, Exc. Exc. $= 8$

SOLUTION. — The excess of 11's in 62 is 7; in 42, 9. Their product 63 has an excess of 8. In 2604, the excess of 11's is 8. Since the two excesses are the same, the answer is probably correct.

4. Prove that $207 \div 23 = 9$.

$23) 207 (9$
Exc. $3 - 2 = 1$ 9 9

SOLUTION. — Excess of 11's in divisor is 1, in quotient, 9. Their product is 9. The excess of 11's in the dividend is 9. Since the excesses are the same, the answer is probably correct.

RULE. — I. *In Addition, subtract the sum of the digits in the even places from the sum of those in the odd places in each addend. Add the results and find the excess of* 11's *in their sum. If this agrees with the excess of* 11's *in the answer, the answer is probably correct.*

II. *In Subtraction, subtract the excess of* 11's *in subtrahend from that in minuend. Compare with answer.*

III. *In Multiplication, cast out* 11's *from the product of the excess of* 11's *in the multiplicand and multiplier, and compare with answer.*

IV. *In Division, find the product of excess of* 11's *in the divisor and quotient; cast out the* 11's. *Compare with excess in dividend.*

FACTORS.

DIVISIBILITY OF NUMBERS.

89. An **Even Number** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are *even*.

90. An **Odd Number** is a number that is not exactly divisible by 2.

All numbers whose unit figure is 1, 3, 5, 7, or 9, are *odd*.

91. A **Prime Number** is a number that has no integral factors except unity and itself.

Thus, 2, 3, 5, 11, 23, etc., are prime numbers.
2 is the only even prime number.

92. A **Composite Number** is a number that has other *integral* factors besides unity and itself.

Thus, 21 is a composite number, since $21 = 7 \times 3$.

93. The **Factors** of a number are the numbers whose product is the given number.

Thus, 7 and 8 are factors of 56; 3, 4, and 7, of 84.

A **Prime Factor** is a prime number used as a *factor*.

The *prime factors* of a number are also the *prime divisors* of it.

94. Numbers are **prime to each other** when they have no *common integral factors, or divisors*.

Thus, 9 and 14, 16 and 25, are prime to each other.

95. An **Exact Divisor** of a number is one that will divide that number without a remainder.

Thus, 6 is an exact divisor of 48, and 9 is an exact divisor of 72.

The *exact divisors* of a number are also the *factors* of that number.

An exact divisor of a number is sometimes called the *measure* of that number.

When a number is a factor, or divisor, of each of *two* or *more* numbers, it is called a *common* factor, or divisor of those numbers.

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2 is an exact divisor of all even numbers.

3 is an exact divisor of any number, the sum of whose digits is divisible by 3. Thus, 3 is an exact divisor of 696, 3735, 840.

4 is an exact divisor if its two right hand figures are ciphers, or express a number divisible by 4. Thus, 4 is an exact divisor of 200, 756, 1284.

5 is an exact divisor of every number whose unit figure is 0 or 5. Thus, 5 is an exact divisor of 20, 955, and 2840.

6 is an exact divisor of an even number if the sum of its digits is divisible by 3. Thus, 6 is an exact divisor of 549, 678, 399.

8 is an exact divisor when it will exactly divide the hundreds, tens, and units of a number. Thus, 8 is an exact divisor of 1728, 5280, and 213560.

9 is an exact divisor when it will exactly divide the sum of the digits of a number. Thus, in 2486790, the sum of the digits $2 + 4 + 8 + 6 + 7 + 9 + 0 = 36$, and $36 \div 9 = 4$.

10 is an exact divisor when 0 occupies units' place.

11 is an exact divisor when the difference between the sum of its even digits and the sum of its odd digits is exactly divisible by 11, or when it is 0. Thus, 11 is an exact divisor of 4554, 91322.

100 is an exact divisor when 00 occupy the places of units and tens.

1000 is an exact divisor when 000 occupy the places of units, tens, and hundreds, etc.

A *composite* number is an exact divisor of any number, when all its factors are exact divisors of the same number. Thus, 2, 2, and 3 are exact divisors of 12; so also are 4 ($=2 \times 2$) and 6 ($=2 \times 3$).

An *even* number is never an exact divisor of an odd number.

If an *odd* number is an exact divisor of an even number, the quotient will be an even number.

If an *odd* number is an exact divisor of an even number, twice that odd number is also an exact divisor of the even number. *Thus, 7 is an exact* divisor of 42; so also is 7×2, or 14.

For reference, and to aid in determining the prime factors of composite numbers, we give the following table:

TABLE OF PRIME NUMBERS FROM 1 TO 1000.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

FACTORING.

EXAMPLES.

96. To resolve any composite number into its prime factors.

1. What are the prime factors of 2772?

OPERATION.

2	2772
2	1386
3	693
3	231
7	77
11	11
	1

SOLUTION. — We divide the given number by 2, the least prime factor, and the result by 2; this gives an odd number for a quotient, divisible by the prime factor 3, and the quotient resulting from this division is also divisible by 3. The next quotient, 77, we divide by its least prime factor, 7, and obtain the quotient 11; this being a prime number, the division cannot be carried further. The divisors and last quotient, 2, 2, 3, 3, 7, and 11, are all the prime factors *of the given* number, 2772.

RULE. — *Divide the given number by any prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.*

PROOF. — *The product of all the prime factors will be the given number.*

2. What are the prime factors of 1140?

Ans. 2, 2, 3, 5, 19.

3. What are the prime factors of 29925?
4. What are the prime factors of 2431?
5. Find the prime factors of 12673.
6. Find the prime factors of 2310.
7. Find the prime factors of 2205.
8. What are the prime factors of 13981?
9. What are the prime factors of 532?
10. What are the prime factors of 2500?

97. To resolve a number into all the different sets of factors possible.

1. In 36 how many sets of factors are there, and what are they?

OPERATION.

$$36 = \begin{cases} 2 \times 18 \\ 3 \times 12 \\ 4 \times 9 \\ 6 \times 6 \\ 2 \times 2 \times 9 \\ 2 \times 3 \times 6 \\ 3 \times 3 \times 4 \\ 2 \times 2 \times 3 \times 3 \end{cases}$$

SOLUTION. — Writing the 36 at the left of the sign =, we arrange all the different sets of factors into which it can be resolved, under each other, as shown in the operation, and we find that 36 can be resolved into 8 sets of factors.

2. How many sets of factors are there in the number 24? What are they? *Ans.* 6 sets.

3. In 125 how many sets of factors are there? What are they? *Ans.* 2 sets.

CANCELLATION.

98. Cancellation is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

It has been shown (**77**) that the dividend is equal to the product of the divisor multiplied by the quotient. Hence, if the dividend can be resolved into two factors, one of which is the divisor, the other factor will be the quotient.

EXAMPLES.

1. Divide 63 by 7.

OPERATION.

Divisor, $7)\overline{7 \times 9}$ Dividend.

9 Quotient.

SOLUTION. — We see in this example that 63 is composed of the factors 7 and 9, and that the factor 7 is equal to the divisor. Therefore we reject the factor 7, and the remaining factor, 9, is the quotient.

99. Whenever the dividend and divisor are each composite numbers, the factors common to both may first be rejected without altering the final result. (**86,** Prin. III.)

2. What is the quotient of 24 times 56 divided by 7 times 48?

OPERATION.

$$\frac{24 \times 56}{7 \times 48} = \frac{4 \times \cancel{6} \times \cancel{7} \times \cancel{8}}{\cancel{7} \times \cancel{6} \times \cancel{8}} = 4, \textit{ Ans.}$$

SOLUTION. — We first *indicate* the operation to be performed by writing the numbers which constitute the dividend above a line, and those which constitute the divisor below it. Instead of multiplying 24 by 56, in the dividend, we resolve 24 into the factors 4 and 6, and 56 into the factors 7 and 8; and 48 in the divisor into the factors 6 and 8. We next cancel the factors 6, 7, and 8, which are common to the dividend and divisor, and we have left the factor 4 in the dividend, which is the quotient.

When all the factors or numbers in the dividend are canceled, 1 should be retained.

100. If any two numbers, one in the dividend and one in the divisor, contain a common factor, we may reject that factor.

3. In 54 times 77, how many times 63 are there?

OPERATION.

$$\frac{\cancel{54}^{6} \times \cancel{77}^{11}}{\cancel{63}_{9}\ \cancel{7}_{1}}$$

SOLUTION.— In this example we see that 9 will divide 54 and 63; so we reject 9 as a factor of 54, and retain the factor 6, and we also reject 9 as a factor of 63, and retain the factor 7. Again, 7 will divide 7 in the divisor, and 77 in the dividend. Dividing both numbers by 7, 1 will be retained in the divisor, and 11 in the dividend: Finally, the product of $6 \times 11 = 66$, the quotient.

4. Divide $25 \times 16 \times 12$ by $10 \times 4 \times 6 \times 7$.

OPERATION.

$$\frac{\cancel{25}^{5} \times \cancel{16}^{4} \times \cancel{12}^{2}}{\cancel{10}_{2} \times \cancel{4} \times \cancel{6} \times 7} = \frac{5 \times 4}{7} = \frac{20}{7} = 2\tfrac{6}{7}.$$

SOLUTION.— In this, as in the preceding example, we reject all the factors that are common to both dividend and divisor, and we have remaining the factor 7 in the divisor, and the factors 5 and 4 in the dividend. Completing the work, we have $\frac{20}{7} = 20 \div 7$ or $2\frac{6}{7}$, *Ans.*

5. Divide the product of $120 \times 44 \times 6 \times 7$ by $72 \times 33 \times 14$.

OPERATION.

$$\frac{\cancel{120}^{10} \times \cancel{44}^{4} \times \cancel{6} \times 7}{\cancel{72}_{6} \times \cancel{33}_{3} \times \cancel{14}_{2}} = \frac{10 \times 2}{3} = \frac{20}{3} = 6\tfrac{2}{3}.$$

SOLUTION.—Rejecting all the factors common to both dividend and divisor, we have remaining the factor 3 in the divisor, and the factors 10 and 2 in the dividend. Completing the work, we have $\frac{20}{3} = 20 \div 3$ or $6\frac{2}{3}$. *Ans.*

From the preceding examples and illustrations we derive the following rule:

RULE.— I. *Write the numbers composing the dividend above a horizontal line, and the numbers composing the divisor below it.*

II. *Cancel all the factors common to both dividend and divisor.*

III. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

1. Rejecting a factor from any number is dividing the number by that factor.
2. When a factor is canceled, the unit, 1, is supposed to take its place.
3. *One* factor in the dividend will cancel only *one equal* factor in the divisor.
4. If all the factors or numbers of the divisor are canceled, the product of the remaining factors of the dividend will be the quotient.

6. What is the quotient of $16 \times 5 \times 4$ divided by 20×8? *Ans.* 2.

7. Divide the product of $90 \times 66 \times 8$ by $4 \times 11 \times 30$. *Ans.* 36.

8. Divide the product of $33 \times 35 \times 28$ by $11 \times 15 \times 14$. *Ans.* 14.

9. What is the quotient of $21 \times 11 \times 26$ divided by 14×13? *Ans.* 33.

10. Divide the product of the numbers 48, 72, 28, and 5, by the product of the numbers 84, 15, 7, and 6, and give the result. *Ans.* $9\frac{1}{7}$.

11. Divide $140 \times 39 \times 13 \times 7$ by $30 \times 7 \times 26 \times 21$. *Ans.* $4\frac{1}{8}$.

12. What is the quotient of $66 \times 9 \times 18 \times 5$ divided by $22 \times 6 \times 40$? *Ans.* $10\frac{1}{8}$.

13. Divide the product of $200 \times 36 \times 30 \times 21$ by $270 \times 40 \times 15 \times 14$. *Ans.* 2.

14. Multiply 240 by 56, and divide the product by 60 multiplied by 28. *Ans.* 8.

15. The product of the numbers 18, 6, 4, and 42 is to be divided by the product of the numbers 4, 9, 3, 7, and 6. What is the result? *Ans.* 4.

16. How many tons of hay, at \$12 a ton, must be given for 30 cords of wood, at \$4 a cord? *Ans.* 10 tons.

GREATEST COMMON DIVISOR.

101. A **Common Divisor** of two or more numbers is a number that will exactly divide each of them.

102. The **Greatest Common Divisor** of two or more numbers is the *greatest* number that will exactly divide each of them.

Numbers prime to each other have no common divisor.

A common divisor is sometimes called a Common Measure; and the greatest common divisor, the Greatest Common Measure.

EXAMPLES.

103. When the numbers are readily factored.

1. What is the greatest common divisor of 6 and 10?

OPERATION.

$$\begin{array}{r|rr} 2 & 6 & 10 \\ \hline & 3 & 5 \end{array}$$

2, *Ans.*

SOLUTION. — We readily find by inspection that 2 will divide both the given numbers; hence, 2 is a common divisor; and since the quotients 3 and 5 have no common factor, but are prime to each other, the common divisor, 2, must be the greatest common divisor.

2. What is the greatest common divisor of 42, 63, and 105?

OPERATION.

$$\begin{array}{r|rrr} 3 & 42 & 63 & 105 \\ \hline 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

$3 \times 7 = 21$, *Ans.*

SOLUTION. — We observe that 3 will exactly divide each of the given numbers, and that 7 will exactly divide each of the resulting quotients. Hence, each of the given numbers can be exactly divided by 3 times 7; and these numbers must be *component factors* of the greatest common divisor. Now, if there were *any other* component factor of the greatest common divisor, the quotients, 2, 3, 5, would be exactly divisible by it. But these quotients are prime to each other. Hence, 3 and 7 are *all* the component factors of the greatest common divisor sought.

3. What is the greatest common divisor of 28, 140, and 280?

OPERATION.

4	28	140	280
7	7	35	70
	1	5	10

$4 \times 7 = 28$, *Ans.*

SOLUTION. — We first divide by 4; then we divide the quotients by 7. The resulting quotients, 1, 5, and 10, are prime to each other. Hence, 4 and 7 are all the component factors of the greatest common divisor.

From these examples and analyses we derive the following rule:

RULE. — I. *Write the numbers in a line, with a vertical line at the left, and divide by any factor common to all the numbers.*

II. *Divide the quotients in like manner, and continue the division till a set of quotients is obtained that have no common factor.*

III. *Find the product of all the divisors. This will be the greatest common divisor sought.*

4. What is the greatest common divisor of 12, 36, 60, 72? *Ans.* 12.

5. What is the greatest common divisor of 18, 24, 30, 36, 42? *Ans.* 6.

6. What is the greatest common divisor of 72, 120, 240, 384? *Ans.* 24.

7. What is the greatest common divisor of 36, 126, 72, 216? *Ans.* 18.

8. What is the greatest common divisor of 42 and 112? *Ans.* 14.

9. What is the greatest common divisor of 32, 80, and 256? *Ans.* 16.

10. What is the greatest common divisor of 210, 280, 350, 630, and 840? *Ans.* 70.

11. What is the greatest common divisor of 300, 525, 225, and 375? *Ans.* 75.

12. What is the greatest common divisor of 252, 630, 1134, and 1386? *Ans.* 126.

13. What is the greatest common divisor of 96 and 544? *Ans.* 32.

14. What is the greatest common divisor of 468 and 1184? *Ans.* 4.

15. What is the greatest common divisor of 200, 625, and 150? *Ans.* 25.

104. When the numbers cannot be readily factored.

As the analysis of the method under this case depends upon three properties of numbers which have not been introduced, we present them in this place.

I. *An exact divisor divides any number of times its dividend.*

II. *A common divisor of two numbers is an exact divisor of their* SUM.

III. *A common divisor of two numbers is an exact divisor of their* DIFFERENCE.

1. What is the greatest common divisor of 84 and 203?

OPERATION.

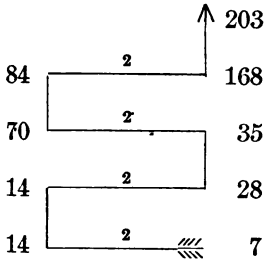
		203
84	2	168
70	2	35
14	2	28
14	2	7, *Ans.*
0		

SOLUTION. — We draw two vertical lines, and place the larger number on the right, and the smaller number on the left, one line lower down. Then we divide 203, the larger number, by 84, the smaller, and write 2, the quotient, between the verticals, the product, 168, opposite, under the greater number, and the remainder, 35, below. Next we divide 84 by this remainder, writing the quotient, 2, between the verticals, the product, 70, on the left, and the new remainder, 14, below the 70. Again we divide the last divisor, 35, by 14, and obtain 2 for a quotient, 28 for a product, and 7 for a remainder, all

of which we write in the same order as in the former steps. Finally we divide the last divisor, 14, by the last remainder, 7, and we have no remainder. 7, the last divisor, is the greatest common divisor of the given numbers.

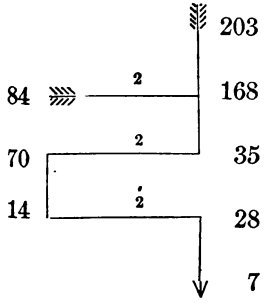
In order to show that the last divisor in such a process is the greatest common divisor, we will first trace the work in the reverse order, as indicated by the arrow line below.

OPERATION.



7 divides the 14, as proved by the last division; it will also divide two times 14, or 28, (I). Now, as 7 divides both itself and 28, it will divide 35, their sum, (II). It will also divide 2 times 35, or 70, (I); and since it is a common divisor of 70 and 14, it must divide their sum, 84, which is one of the given numbers, (II). It will also divide 2 times 84, or 168, (I); and since it is a common divisor of 168 and 35, it must divide their sum, 203, the larger number, (II). Hence, 7 is a *common divisor* of the given numbers.

Again, tracing the work in the *direct* order, as indicated below, we know that the greatest common divisor, *whatever it may be*, must divide 2 times 84, or 168, (I). Then since it will divide both 168 and 203, it must divide their difference, 35, (III). It will also divide 2 times 35, or 70, (I); and as it will divide both 70 and 84, it must divide their difference, 14, (III). It will also divide 2 times 14 or 28, (I); and as it will divide both 28 and 35, it must divide their difference, 7, (III); *hence, it cannot be greater than* 7.



Thus we have shown,

1. That 7 is a *common divisor* of the given numbers.

2. That their greatest common divisor, whatever it may be, cannot be *greater than* 7. Hence it must be 7.

This work may also be indicated by the ordinary form of division.

OPERATION.

84)203(2
168
35)84(2
70
14)35(2
28
7)14(2
14
0

SOLUTION. — Using the larger number, 203, as dividend, and the smaller, 84, as divisor, we obtain the quotient 2, with a remainder of 35. We divide the former divisor, 84, by this remainder, and obtain a quotient, 2, with a remainder of 14. Dividing 35 by this remainder, we obtain a quotient, 2, with a remainder of 7. Dividing 14 by the last remainder, we obtain a quotient of 2, with no remainder. 7, the last divisor, is the greatest common divisor of the given numbers.

RULE I. — *Divide the greater number by the less. If there is a remainder, divide the preceding divisor by it, and so continue until there is no remainder. The last divisor will be the greatest common divisor.*

II. *If more than two numbers are given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last; the last common divisor found will be the greatest common divisor of all the given numbers.*

1. When more than two numbers are given, it is better to begin with the least two.

2. If at any point in the operation a *prime* number occurs as a remainder, it must be a common divisor, or the given numbers have no common divisor.

2. Find the greatest common divisor of 154 and 210. *Ans.* 14.

3. What is the greatest common divisor of 316 and 664? *Ans.* 4.

4. What is the greatest common divisor of 679 and 1869? *Ans.* 7.

5. Find the greatest common divisor of 688 and 1578. *Ans.* 2.

6. Find the greatest common divisor of 327 and 1308. *Ans.* 327.

7. What is the greatest common divisor of 917 and 1495? *Ans.* 1.

8. What is the greatest common divisor of 1313 and 4108? *Ans.* 13.

9. What is the greatest common divisor of 1649 and 5423? *Ans.* 17.

10. John has 35 pennies, and Charles 50. How may they arrange them in parcels, so that each boy shall have the same number in each parcel? *Ans.* 5 in each parcel.

11. A speculator has 3 fields, the first containing 18, the second 24, and the third 40 acres, which he wishes to divide into the largest possible lots having the same number of acres in each. How many acres will there be in each lot? *Ans.* 2 acres.

12. A farmer had 231 bushels of wheat, and 273 bushels of oats, which he wished to put into the least number of bins containing the same number of bushels, without mixing the two kinds. What number of bushels must each bin hold? *Ans.* 21.

13. A village street is 332 rods long; A owns 124 rods front, B 116 rods, and C 92 rods; they agree to divide their land into equal lots of the largest size that will allow each one to form an exact number of lots. What will be the width of the lots? *Ans.* 4 rods.

14. A forwarding merchant has 2722 bushels of wheat, 1822 bushels of corn, and 1226 bushels of beans, which he wishes to forward, in the fewest bags of equal size that will exactly hold either kind of grain. How many bags will it take? *Ans.* 2885.

15. A has $120, B $240, and C $384; they agree to purchase cows, at the highest price per head that will allow each man to invest all his money. What price must they pay, and how many cows can each man purchase? *Ans.* $24 per head. A 5, B 10, and C 16.

MULTIPLES.

105. A **Multiple** is a number exactly divisible by a given number.

Thus, 20 is a multiple of 4; 50 is a multiple of 10; 360 is a multiple of 90; 10000 is a multiple of 10.

106. A **Common Multiple** is a number exactly divisible by two or more given numbers.

Thus, 20 is a common multiple of 2, 4, 5, and 10; 60 is a common multiple of 2, 3, 5, 6, 10, and 15.

The **Least Common Multiple** is the least number exactly divisible by two or more given numbers.

Thus, 24 is the least common multiple of 3, 4, 6, and 8; 30 is the least common multiple of 2, 3, 5, 6, 10, and 15.

107. From the definition (**106**) it is evident that the product of two or more numbers, or any number of times their product, must be a common multiple of the numbers. Hence,

A common multiple of two or more numbers may be found by taking the product of the given numbers.

108. By a little practice the multiples and least common multiples of the smaller numbers may be readily found by inspection.

Since all even numbers are multiples of 2, the multiples of 2 are the most numerous.

The following table presents the multiples from 1 to 100 of the prime numbers from 1 to 11:

FACTORS.	MULTIPLES.
2	All even numbers to 100.
3	3, 6, 9, 12, etc., to 99.
5	5, 10, 15, 20, etc., to 100.
7	7, 14, 21, 28, etc., to 98.
11	11, 22, 33, 44, etc., to 99.
2, 3	6, 12, 18, 24, etc., to 96.
2, 5	10, 20, 30, 40, etc., to 100.
2, 7	14, 28, 42, 56, etc., to 98.
2, 11	22, 44, 66, 88.
3, 5	15, 30, 45, 60, etc., to 90.
3, 7	21, 42, 63, 84.
3, 11	33, 66, 99.
5, 7	35, 70.
5, 11	55.
7, 11	77.
2, 3, 5	30, 60, 90.
2, 3, 7	42, 84.
2, 3, 11	66.

Find by inspection a multiple of:

1. 3, 5.
2. 2, 7.
3. 5, 11.
4. 2, 9.
5. 7, 11.
6. 2, 3.
7. 2, 3, 11.
8. 2, 3, 7.
9. 3, 5, 7.

Find by inspection the least common multiple of:

10. 3, 5, 6.
11. 4, 8, 12.
12. 3, 6, 9.
13. 2, 6, 10.
14. 2, 5, 6.
15. 2, 3, 4.
16. 2, 3, 5, 6.
17. 2, 3, 4, 5.
18. 3, 4, 5, 7.
19. 4, 5, 7, 8.
20. 2, 3, 5, 7.
21. 3, 5, 7, 11.

The least common multiple of numbers mutually prime is always the least common multiple of their product.

EXAMPLES.

109. To find the least common multiple.

FIRST METHOD.

From the nature of prime numbers we derive the following principles:

I. If a number exactly contains another, it will contain all the prime factors of that number.

II. If a number exactly contains two or more numbers, it will also contain all the prime factors of those numbers.

III. The least number that will exactly contain all the prime factors of two or more numbers, is the least common multiple of those numbers.

1. Find the least common multiple of 30, 42, 66, and 78.

OPERATION.

$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$
$$66 = 2 \times 3 \times 11$$
$$78 = 2 \times 3 \times 13$$

$2 \times 3 \times 13 \times 11 \times 7 \times 5 = 30030$, *Ans.*

SOLUTION.—The number cannot be less than 78, since it must contain 78; hence it must contain the factors of 78, viz.:

$2 \times 3 \times 13$.

We here have all the prime factors of 78, and also all the factors of 66 except the factor 11. Annexing 11 to the series of factors,

$$2 \times 3 \times 13 \times 11,$$

we have all the prime factors of 78 and 66, and also all the factors of 42 except the factor 7. Annexing 7 to the series of factors,

$$2 \times 3 \times 13 \times 11 \times 7,$$

we have all the prime factors of 78, 66, and 42, and also all the factors of 30 except the factor 5. Annexing 5 to the series of factors,

$$2 \times 3 \times 13 \times 11 \times 7 \times 5,$$

we have all the prime factors of each of the given numbers; and hence the product of the series of factors is a common multiple of

the given numbers, (II). And as no factor of this series can be omitted without omitting a factor of one of the given numbers, the product of the series is the least common multiple of the given numbers, (III).

RULE. — I. *Resolve the given numbers into their prime factors.*

II. *Take all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

When a prime factor is repeated in any of the given numbers, it must be used as many times, as a factor of the multiple, as the greatest number of times it appears in any of the given numbers.

Find the least common multiple of:

2. 7, 35, 98. *Ans.* 490.
3. 24, 42, 17. *Ans.* 2856.
4. 4, 9, 6, 8. *Ans.* 72.
5. 8, 15, 77, 385. *Ans.* 9240.
6. 10, 45, 75, 90. *Ans.* 450.
7. 12, 15, 18, 35. *Ans.* 1260.

SECOND METHOD.

110. 1. What is the least common multiple of 4, 6, 9, and 12?

OPERATION.

2	4	6	9	12
2	2	3	9	6
3		3	9	3
3			3	

$2 \times 2 \times 3 \times 3 = 36$, *Ans.*

SOLUTION. — We first write the given numbers in a series, with a vertical line at the left. Since 2 is a factor of some of the given numbers, it must be a factor of the least common multiple sought. Dividing as many of the numbers as are divisible by 2, we write the quotients and undivided number, 9, in a line underneath. We now perceive that some of the numbers in the second line contain the factor 2; hence the least common multiple must contain another 2, and we again divide by 2, omitting to write down any quotient when it is 1. We next divide by 3 for a like reason, and again by 3. By this process we have transferred all the factors of each of the numbers to the left of the vertical line; and their product, 36, must be the least common multiple sought, (**109,** III).

2. What is the least common multiple of 10, 12, 15, and 75?

OPERATION.

2, 5	10	12	15	75
2, 3		6	3	15
5				5

$2 \times 5 \times 2 \times 3 \times 5 = 300$, *Ans.*

SOLUTION. — We readily see that 2 and 5 are among the factors of the given numbers, and must be factors of the least common multiple; hence, we divide every number that is divisible by *either* of these factors or by their *product;* thus, we divide 10 by both 2 and 5; 12 by 2; 15 by 5; and 75 by 5. We next divide the second line in like manner by 2 and 3; and afterwards the third line by 5. By this process we collect the factors of the given numbers into *groups;* and the product of the factors at the left of the vertical line is the least common multiple sought.

3. What is the least common multiple of 6, 15, 35, 42, and 70?

OPERATION.

3, 7	15	42	70
2, 5	5	2	10

$3 \times 7 \times 2 \times 5 = 210$, *Ans.*

SOLUTION. — In this operation we omit the 6 and 35, because they are exactly contained in some of the other given numbers; thus, 6 is contained in 42, and 35 in 70; and whatever will contain 42 and 70 must contain 6 and 35. Hence, we have only to find the least common multiple of the remaining numbers, 15, 42, and 70.

RULE. — I. *Write the numbers in a line, omitting any of the smaller numbers that are factors of the larger, and draw a vertical line at the left.*

II. *Divide by any prime factor, or factors, that may be contained in one or more of the given numbers, and write the quotients and undivided numbers in a line underneath.*

III. *In like manner divide the quotients and undivided numbers, and continue the process till all the factors of the given numbers have been transferred to the left of the vertical line. Then find the product of these factors which will be the least common multiple required.*

4. What is the least common multiple of 12, 15, 42, and 60? *Ans.* 420.

5. What is the least common multiple of 21, 35, and 42? *Ans.* 210.

6. What is the least common multiple of 25, 60, 100, and 125? *Ans.* 1500.

7. What is the least common multiple of 16, 40, 96, and 105? *Ans.* 3360.

8. What is the least common multiple of 4, 16, 20, 48, 60, and 72? *Ans.* 720.

9. What is the least common multiple of 84, 100, 224, and 300? *Ans.* 16800.

10. What is the least common multiple of 270, 189, 297, 243? *Ans.* 187110.

11. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 8, 9? *Ans.* 2520.

12. What is the smallest sum of money for which I could purchase an exact number of books, at $5, or $3, or $4, or $6 each? *Ans.* $60.

13. A farmer has three teams; the first can draw 12 barrels of flour, the second 15 barrels, and the third 18 barrels. What is the smallest number of barrels that will make full loads for any of the teams? *Ans.* 180.

14. What will be the capacity of the smallest cask that will be filled to the brim by using a 2 pt., 3 pt., 5 pt., 6 pt., or 8 pt. measure to fill it? *Ans.* 120 pts.

15. What is the smallest sum of money with which I can purchase cows at $30 each, oxen at $55 each, or horses at $105 each? *Ans.* $2310.

16. A can shear 41 sheep in a day, B 63, and C 54. What is the number of sheep in the smallest flock that would furnish exact days' labor for each of them shearing alone? *Ans.* 15498.

ORDER OF SIGNS.

111. When we wish to indicate that several quantities are to be subjected to the same operation, we inclose them in parentheses ( ), in braces { }, in brackets [ ], or place them under a vinculum ——. These are called the *signs of aggregation.*

1. A quantity in parenthesis must be changed to a single quantity by performing the operations indicated. When there is one parenthesis within another, the inside one should be first removed, and then the next until none remains.

2. Precedence is given to the signs × and ÷ over the signs + and − ; hence the operations of multiplication and division should always be performed before addition and subtraction.

1. Find the value of:

I. $15[3+5\{8+(9+6)\div 5+(12\times 3)+\overline{15-3}\}+10-8]=$

OPERATION.

II. $15[3+5\{8+3+36+12\}+10-8]=$

III. $15[3+295+10-8]=$

IV. $15\times 300=4500$, *Ans.*

SOLUTION. — Removing the inner parentheses ( ) and vinculum by performing the indicated operations, we have II. Removing the braces { }, we have III. Removing the brackets [ ], we have IV ; and multiplying 300 by 15 as indicated, we have the answer, 4500.

RULE. — *Remove all the expressions in parenthesis by performing the operations indicated, beginning with the inner parenthesis. The answer to the last operation indicated will be the value of the expression.*

2. $5\times[13+2(3+4\times 6)+5]$. *Ans.* 360.

3. $25\times(6\times 3)\times 4-(9\times 8+90)$. *Ans.* 1638.

4. $\{200-\overline{8\times 8}+(3\times 9)-8\}\div 5$. *Ans.* 31.

5. $8\times(96-26)\times\overline{5\times 6}-13\times 30(5\times 4)$. *Ans.* 9000.

6. $9\times[3+\overline{16+5}\div 3+\{3+(44\times 5)+\overline{18\div 6}\}+22-15]\times 8$. *Ans.* 17496.

7. $10\times\{16-4+3(2+8-2)+3\times 6(4\div 2)+8\}+10$.

FRACTIONS.

112. If a unit is divided into 2 equal parts, one of the parts is called *one half*.

If a unit is divided into 3 equal parts, one of the parts is called *one third*, two of the parts *two thirds*.

If a unit is divided into 4 equal parts, one of the parts is called *one fourth*, two of the parts *two fourths*, three of the parts *three fourths*.

If a unit is divided into 5 equal parts, one of the parts is called *one fifth*, two of the parts *two fifths*, three of the parts *three fifths*, etc.

The parts are expressed by figures:

One half is written	$\frac{1}{2}$	One fifth is written	$\frac{1}{5}$
One third "	$\frac{1}{3}$	Two fifths "	$\frac{2}{5}$
Two thirds "	$\frac{2}{3}$	One seventh "	$\frac{1}{7}$
One fourth "	$\frac{1}{4}$	Three eighths "	$\frac{3}{8}$
Two fourths "	$\frac{2}{4}$	Five ninths "	$\frac{5}{9}$
Three fourths "	$\frac{3}{4}$	Eight tenths "	$\frac{8}{10}$

The parts into which a unit is divided take their *name*, and their *value*, from the *number* of equal parts into which the unit is divided. If we divide an orange into 2 equal parts, the parts are called *halves;* if into 3 equal parts, *thirds;* if into 4 equal parts, *fourths*, etc. Each *third* is less in value than each *half*, each *fourth* less than each *third*, and the *greater* the *number* of parts, the *less* their *value*.

113. A **Fraction** is one or more of the equal parts of a unit.

114. A **Fractional Unit** is *one* of the equal parts into which a unit is divided.

Thus, if a unit is divided into 5 equal parts, 1 is the fractional unit.

To write a fraction, two integers are required, one to express the number of parts into which the whole number is divided, and the other to express the number of these parts taken. If one dollar is divided into 4 equal parts, the parts are called *fourths*, and three of these parts are called *three fourths* of a dollar. This three fourths may be written.

$\frac{3}{4}$ the number of parts taken.
the number of parts into which the dollar is divided.

115. The **Denominator** is the number below the line.

It denominates or names the parts; and

It shows how many parts are equal to a unit.

116. The **Numerator** is the number above the line.

It numerates or numbers the parts; and

It shows how many parts are taken or expressed by the fraction.

117. The **Terms** of a fraction are the numerator and denominator, taken together.

118. *Fractions indicate division*, the numerator corresponding to the dividend, and the denominator to the divisor.

119. The **Value** of a fraction is the quotient of the numerator divided by the denominator.

120. To analyze a fraction is to designate and describe its numerator and denominator. Thus, $\frac{3}{4}$ is analyzed as follows:

4 is the *denominator*, and shows that the integer is divided into 4 equal parts; it is the divisor.

3 is the *numerator*, and shows that 3 parts are taken; it is the dividend, or integer divided.

3 and 4 are the *terms*, considered as dividend and divisor.

The *value* of the fraction is the quotient of $3 \div 4$, or $\frac{3}{4}$.

EXAMPLES.

Express the following fractions by figures:

1. Seven *eighths*.
2. Three *twenty-fifths*.
3. Nine *one-hundredths*.
4. Sixteen *thirtieths*.
5. Thirty-one *one hundred eighteenths*.
6. Seventy-five *ninety-sixths*.
7. Two hundred fifty-four *four hundred forty-thirds*.
8. Eight *nine hundred twenty-firsts*.
9. One thousand, two hundred thirty-two *seventy-five thousand six hundredths*.
10. Nine hundred six *two hundred forty-three thousand eighty-seconds*.

Read and analyze the following fractions:

11. $\frac{9}{10}$; $\frac{7}{12}$; $\frac{5}{20}$; $\frac{12}{28}$; $\frac{15}{75}$; $\frac{9}{112}$; $\frac{45}{220}$; $\frac{125}{428}$; $\frac{213}{884}$; $\frac{506}{1030}$.
12. $\frac{90}{100}$; $\frac{325}{1000}$; $\frac{450}{1240}$; $\frac{25}{1500}$; $\frac{12}{2000}$; $\frac{726}{3475}$; $\frac{622}{10000}$; $\frac{5240}{12000}$.
13. $\frac{17}{104}$; $\frac{1}{10110}$; $\frac{915}{84621}$; $\frac{880650}{4581429}$; $\frac{123456}{6543210}$; $\frac{567483}{852190}$.

121. Fractions are distinguished as **Proper** and **Improper.**

A *Proper Fraction* is one whose numerator is less than its denominator; its *value* is less than the unit, 1.

Thus, $\frac{7}{12}$, $\frac{5}{16}$, $\frac{9}{10}$, $\frac{12}{84}$ are proper fractions.

An *Improper Fraction* is one whose numerator equals or exceeds its denominator; its *value* is never less than the unit, 1.

Thus, $\frac{7}{7}$, $\frac{3}{3}$, $\frac{15}{4}$, $\frac{35}{8}$, $\frac{50}{10}$, $\frac{180}{90}$ are improper fractions.

122. A **Mixed Number** is a number expressed by an integer and a fraction.

Thus, $4\frac{1}{4}$, $17\frac{18}{25}$, $9\frac{3}{10}$ are mixed numbers.

123. Since fractions indicate division, all changes in the *terms* of a fraction will affect the *value* of that fraction according to the laws of division.

If we modify the language of the General Principles of Division (**86**) by substituting the words *numerator*, *denominator*, and *fraction*, or *value of the fraction*, for the words *dividend*, *divisor*, and *quotient*, respectively, we shall have the following principles:

General Principles of Fractions.

124. Principles. — I. *Multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.*

II. *Multiplying the denominator divides the fraction, and dividing the denominator multiplies the fraction.*

III. *Multiplying or dividing both terms of the fraction by the same number does not alter the value of the fraction.*

These three principles may be embraced in one general law.

General Law. — *A change in the* NUMERATOR *produces a* LIKE *change in the value of the fraction; but a change in the* DENOMINATOR *produces an* OPPOSITE *change in the value of the fraction.*

REDUCTION.

125. Reduction of fractions is the process of changing their form without altering their value.

EXAMPLES.

126. To reduce fractions to their lowest terms.

A fraction is in its *lowest terms* when its numerator and denominator are prime to each other; that is, when both terms have no common divisor.

1. Reduce the fraction $\frac{48}{60}$ to its lowest terms.

FIRST OPERATION.

$\frac{48}{60} = \frac{24}{30} = \frac{12}{15} = \frac{4}{5}$, *Ans.*

SOLUTION. — Dividing both terms of a fraction by the same number does not alter the value of the fraction or quotient, (**124,** III); hence, we divide both terms of $\frac{48}{60}$, by 2, both terms of the result, $\frac{24}{30}$, by 2, and both terms of this result by 3. Since the terms of $\frac{4}{5}$ are prime to each other, the lowest terms of $\frac{48}{60}$ are $\frac{4}{5}$. We have, in effect, canceled all the factors common to the numerator and denominator.

SECOND OPERATION.

$12)\frac{48}{60} = \frac{4}{5}$, *Ans.*

SOLUTION. — We find the greatest common divisor of 48 and 60 (**103**) which is 12, and divide both terms of the fraction by it, thus performing the reduction at a single division.

RULE. — *Cancel or reject all factors common to both numerator and denominator.* Or,

Divide both terms by their greatest common divisor.

2. Reduce $\frac{144}{432}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
3. Reduce $\frac{288}{360}$ to its lowest terms. *Ans.* $\frac{4}{5}$.
4. Reduce $\frac{441}{462}$ to its lowest terms. *Ans.* $\frac{21}{22}$.
5. Reduce $\frac{288}{504}$ to its lowest terms.
6. Reduce $\frac{1134}{21168}$ to its lowest terms.
7. Reduce $\frac{453}{1057}$ to its lowest terms.
8. Reduce $\frac{172}{1118}$ to its lowest terms.
9. Reduce $\frac{3060}{5940}$ to its lowest terms. *Ans.* $\frac{17}{33}$.
10. Reduce $\frac{5643}{5940}$ to its lowest terms. *Ans.* $\frac{19}{20}$.

11. Express in its simplest form the quotient of 189 divided by 273. *Ans.* $\frac{9}{13}$.

12. Express in its simplest form the quotient of 1344 divided by 1536. *Ans.* $\frac{7}{8}$.

127. To reduce an improper fraction to a whole or mixed number.

1. Reduce $\frac{324}{15}$ to a whole or mixed number.

OPERATION.

$\frac{324}{15} = 324 \div 15 = 21\frac{9}{15} = 21\frac{3}{5}$, *Ans.*

SOLUTION. — Since 15 fifteenths equal 1, 324 fifteenths are equal to as many times 1 as 15 is contained times in 324, which is $21\frac{9}{15}$ times. Or, since the numerator is a dividend and the denominator a divisor (**118**), we reduce the fraction to an equivalent whole or mixed number, by dividing the numerator, 324, by the denominator, 15.

RULE. — *Divide the numerator by the denominator.*

1. When the denominator is an exact divisor of the numerator, the result will be a whole number.

2. In all answers containing fractions reduce the fractions to their lowest terms.

2. Change $\frac{13}{7}$ of a week to a mixed number.

3. In $\frac{117}{5}$ of a bushel, how many bushels are there?

4. In $\frac{481}{8}$ of a dollar, how many dollars are there?

5. In $\frac{872}{16}$ of a pound, how many pounds are there?

6. Reduce $\frac{1258}{28}$ to a mixed number.

7. Reduce $\frac{738}{18}$ to a whole number.

8. Change $\frac{1512}{81}$ to a mixed number. *Ans.* $18\frac{3}{9}$.

9. Change $\frac{7321}{17}$ to a mixed number.

10. Change $\frac{2370490}{225}$ to a mixed number. *Ans.* $1053\frac{28}{45}$.

11. Change $\frac{25315200}{360}$ to a whole number. *Ans.* 7032.

128. To reduce a whole number to a fraction having a given denominator.

OPERATION.

$\frac{46}{4}$

$\frac{184}{4}$, *Ans.*

1. Reduce 46 yards to fourths.

SOLUTION. — Since in 1 yard there are 4 fourths, in 46 yards there are 46 times 4 fourths, which are 184 fourths $= \frac{184}{4}$.

RULE. — *Multiply the whole number by the given denominator; take the product for a numerator, under which write the given denominator.*

A whole number is reduced to a fractional form by writing 1 under it for a denominator; thus, $9 = \frac{9}{1}$.

2. How many eighths of a bushel are there in 25 bushels? *Ans.* $\frac{200}{8}$.

3. How many fourths of a gallon are there in 63 gallons? *Ans.* $\frac{252}{4}$.

4. Reduce 140 pounds to sixteenths of a pound.

5. How many tenths of a dollar are there in 56 dollars? *Ans.* $\frac{560}{10}$.

6. Reduce 94 to a fraction whose denominator is 9.

7. Reduce 180 to seventy-fifths.

8. Change 42 to the form of a fraction. *Ans.* $\frac{42}{1}$.

9. Change 247 to the form of a fraction.

10. Change 347 to a fraction with a denominator of 14. *Ans.* $\frac{4858}{14}$.

129. To reduce a mixed number to an improper fraction.

1. In $\$5\frac{3}{8}$, how many eighths of a dollar are there?

OPERATION.

$5\frac{3}{8}$
$\frac{8}{}$
$\frac{43}{8}$, *Ans.*

SOLUTION. — Since in 1 dollar there are 8 eighths, in 5 dollars there are 5 times 8 eighths, or 40 eighths, and 40 eighths + 3 eighths = 43 eighths, or $\frac{43}{8}$.

RULE. — *Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

2. Change $4\frac{1}{2}$ dollars, to half dollars. *Ans.* $\frac{9}{2}$.

3. Change $71\frac{3}{7}$ weeks, to sevenths of a week.

4. In $341\frac{3}{4}$ acres, how many fourths are there?

5. Change $12\frac{7}{12}$ years to twelfths.

6. Change $56\frac{9}{17}$ to an improper fraction. *Ans.* $\frac{961}{17}$.

7. Reduce $21\frac{7}{60}$ to an improper fraction. *Ans.* $\frac{1261}{60}$.
8. Reduce $225\frac{14}{25}$ to an improper fraction. *Ans.* $\frac{5639}{25}$.
9. Change $96\frac{40}{120}$ to an improper fraction.
10. Change $1297\frac{3}{84}$ to eighty-fourths. *Ans.* $\frac{108951}{84}$.
11. What improper fraction will express $400\frac{20}{87}$?

130. To reduce a fraction to a given denominator.

As fractions may be reduced to *lower terms* by division, they may also be changed to *higher terms* by multiplication; and all higher terms must be *multiples* of the lowest terms (**105**).

1. Reduce $\frac{3}{4}$ to a fraction whose denominator is 20.

OPERATION.

$20 \div 4 = 5$

$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$, *Ans.*

SOLUTION. — First we divide 20, the required denominator, by 4, the denominator of the given fraction, to ascertain if it is a multiple of this term, 4. The division shows that it is a multiple, and that 5 is the factor which must be employed to produce this multiple of 4. We therefore multiply both terms of $\frac{3}{4}$ by 5 (**124**), and obtain $\frac{15}{20}$, the desired result.

RULE. — *Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

2. Reduce $\frac{2}{5}$ to a fraction whose denominator is 15. *Ans.* $\frac{6}{15}$.
3. Reduce $\frac{5}{7}$ to a fraction whose denominator is 35.
4. Reduce $\frac{2}{17}$ to a fraction whose denominator is 51. *Ans.* $\frac{6}{51}$.
5. Reduce $\frac{29}{80}$ to a fraction whose denominator is 150.
6. Reduce $\frac{125}{436}$ to a fraction whose denominator is 3488. *Ans.* $\frac{1000}{3488}$.
7. Reduce $\frac{1}{125}$ to a fraction whose denominator is 1000. *Ans.* $\frac{8}{1000}$.

131. To change two or more fractions to a common denominator.

1. Reduce $\frac{3}{4}$ and $\frac{2}{5}$ to a common denominator.

OPERATION.

$$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$$

$$\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$$

SOLUTION. — We multiply the terms of the first fraction by the denominator of the second, and the terms of the second fraction by the denominator of the first (**124**). This reduces each fraction to the same denominator, for each new denominator is the product of the given denominators.

2. Change $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$ to a common denominator.

OPERATION.

$$\frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$$

$$\frac{1}{2} \times \frac{12}{12} = \frac{12}{24}$$

$$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$$

SOLUTION. — We multiply the terms of each fraction by the product of the denominators of the other two (**124**). This reduces each fraction to the same denominator, since each new denominator will be the product of the three given denominators.

RULE. — *Multiply the terms of each fraction by the denominators of all the other fractions.*

Mixed numbers must first be reduced to improper fractions.

3. Change $\frac{3}{7}$ and $\frac{4}{9}$ to fractions having a common denominator. *Ans.* $\frac{27}{63}$, $\frac{28}{63}$.

4. Change $\frac{4}{5}$, $\frac{7}{12}$, and $\frac{5}{6}$ to fractions having a common denominator. *Ans.* $\frac{288}{360}$, $\frac{210}{360}$, $\frac{300}{360}$.

5. Change $\frac{3}{7}$, $\frac{5}{8}$, $\frac{2}{3}$, and $\frac{1}{2}$ to fractions having a common denominator. *Ans.* $\frac{144}{336}$, $\frac{210}{336}$, $\frac{224}{336}$, $\frac{168}{336}$.

6. Change $\frac{9}{16}$, $\frac{1}{8}$, and $\frac{2}{9}$ to fractions having a common denominator. *Ans.* $\frac{243}{432}$, $\frac{144}{432}$, $\frac{96}{432}$.

7. Change $\frac{5}{6}$, $2\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{8}$ to fractions having a common denominator. *Ans.* $\frac{120}{144}$, $\frac{360}{144}$, $\frac{108}{144}$, $\frac{18}{144}$.

8. Change $1\frac{7}{8}$, $\frac{3}{10}$, and 4 to fractions having a common denominator. *Ans.* $\frac{150}{80}$, $\frac{24}{80}$, $\frac{320}{80}$.

9. Change $1\frac{1}{3}$, $\frac{7}{20}$, and 9 to fractions having a common denominator. *Ans.* $\frac{80}{60}$, $\frac{21}{60}$, $\frac{540}{60}$.

132. To change fractions to the least common denominator.

The **Least Common Denominator** of two or more fractions is the least denominator to which they can all be reduced, and is the least common multiple of the lowest denominators.

OPERATION.

2	6	8	12
2		4	6
2		2	3
3			3

$2 \times 2 \times 2 \times 3 = 24$

$\frac{1}{6} = \frac{4}{24}$
$\frac{3}{8} = \frac{9}{24}$
$\frac{5}{12} = \frac{10}{24}$ } *Ans.*

1. Change $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{5}{12}$ to the least common denominator.

SOLUTION. — The least common multiple of the given denominators is 24. This is the least common denominator to which the fractions can be reduced. We multiply the terms of each fraction by such a number as will reduce the fraction to the denominator, 24. Reducing each fraction to this denominator, by **130** we have the answer.

RULE. — I. *Find the least common multiple of the given denominators, for the least common denominator.*

II. *Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.*

2. Change $\frac{2}{25}$, $\frac{3}{10}$, $\frac{47}{50}$, and $\frac{4}{75}$ to their least common denominator. *Ans.* $\frac{12}{150}$, $\frac{45}{150}$, $\frac{141}{150}$, $\frac{8}{150}$.

3. Change $\frac{1}{2}$, $\frac{4}{7}$, $\frac{3}{16}$, $\frac{2}{21}$ to their least common denominator. *Ans.* $\frac{168}{336}$, $\frac{192}{336}$, $\frac{63}{336}$, $\frac{32}{336}$.

4. Change $\frac{3}{9}$, $\frac{1}{21}$, $\frac{3}{4}$, and 6 to their least common denominator. *Ans.* $\frac{56}{252}$, $\frac{12}{252}$, $\frac{189}{252}$, $\frac{1512}{252}$.

5. Change $5\frac{1}{2}$, $2\frac{1}{4}$, and $1\frac{3}{8}$ to their least common denominator. *Ans.* $\frac{44}{8}$, $\frac{18}{8}$, $\frac{11}{8}$.

6. Change $\frac{9}{11}$, $\frac{3}{8}$, $\frac{4}{7}$, and $\frac{1}{4}$ to their least common denominator. *Ans.* $\frac{504}{616}$, $\frac{231}{616}$, $\frac{352}{616}$, $\frac{154}{616}$.

7. Change $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{7}$, $2\frac{5}{6}$, and $\frac{5}{14}$ to their least common denominator. *Ans.* $\frac{126}{168}$, $\frac{21}{168}$, $\frac{48}{168}$, $\frac{476}{168}$, $\frac{60}{168}$.

8. Change $\frac{4}{5}$, $\frac{7}{15}$, $3\frac{2}{3}$, 9, and $\frac{7}{9}$ to equivalent fractions having the least common denominator.

ADDITION.

EXAMPLES.

133. 1. What is the sum of $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$?

OPERATION.

$\frac{1}{8} + \frac{3}{8} + \frac{5}{8} + \frac{7}{8} = \frac{16}{8} = 2$, *Ans.*

SOLUTION. — Since the given fractions have a common denominator, 8, their sum may be found by adding their numerators, 1, 3, 5, and 7, and placing the sum, 16, over the common denominator. We thus obtain $\frac{16}{8} = 2$, the required sum.

2. Add $\frac{7}{10}$, $\frac{3}{10}$, $\frac{1}{10}$, $\frac{5}{10}$, and $\frac{9}{10}$. *Ans.* $2\frac{1}{2}$.

3. Add $\frac{4}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{1}{12}$, $\frac{3}{12}$, and $\frac{11}{12}$. *Ans.* $2\frac{7}{12}$.

4. What is the sum of $\frac{7}{25}$, $\frac{9}{25}$, $\frac{2}{25}$, $\frac{13}{25}$, $\frac{16}{25}$, and $\frac{21}{25}$?

5. What is the sum of $\frac{41}{120}$, $\frac{63}{120}$, $\frac{71}{120}$, $\frac{89}{120}$, and $\frac{109}{120}$?

6. What is the sum of $\frac{13}{225}$, $\frac{76}{225}$, $\frac{140}{225}$, $\frac{181}{225}$, and $\frac{223}{225}$?

Ans. $2\frac{61}{75}$.

7. What is the sum of $\frac{3}{5}$ and $\frac{2}{9}$?

OPERATION.

$\frac{3}{5} + \frac{2}{9} = \frac{27}{45} + \frac{10}{45} = \frac{27+10}{45} = \frac{37}{45}$, *Ans.*

SOLUTION. — In whole numbers we can add like numbers only, or those having the same unit value; so in fractions we can add the numerators when they have a common denominator, but not otherwise. As $\frac{3}{5}$ and $\frac{2}{9}$ have not a common denominator, we must first reduce them to a common denominator, which we find to be 45; $\frac{3}{5} = \frac{27}{45}$ and $\frac{2}{9} = \frac{10}{45}$. Adding the numerators and placing the sum over the common denominator, we find the answer to be $\frac{37}{45}$.

RULE. — I. *When necessary, reduce the fractions to their least common denominator.*

II. *Add the numerators, and place the sum over the common denominator.*

If the amount is an improper fraction, reduce it to a whole or a mixed number.

8. Add $\frac{3}{4}$ to $\frac{2}{3}$. *Ans.* $\frac{35}{36}$.

9. Add $\frac{4}{5}$ to $\frac{11}{14}$. *Ans.* $1\frac{41}{70}$.

10. Add $\frac{3}{4}$, $\frac{1}{8}$, $\frac{2}{7}$, and $\frac{5}{12}$. *Ans.* $1\frac{97}{168}$.

11. Add $\frac{14}{45}$, $\frac{27}{89}$, and $\frac{2}{21}$. *Ans.* $1\frac{4044}{40985}$.

12. Add $\frac{42}{140}$, $\frac{9}{70}$, $\frac{7}{28}$, and $\frac{1}{14}$. *Ans.* $\frac{3}{4}$.
13. Add $\frac{51}{75}$, $\frac{121}{150}$, $\frac{24}{25}$, $\frac{1}{2}$, and $\frac{7}{8}$. *Ans.* $3\frac{17}{25}$.
14. Add $\frac{3}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{9}{10}$. *Ans.* $7\frac{1769}{2520}$.
15. Add $7\frac{1}{2}$, $5\frac{2}{3}$, and $10\frac{3}{4}$.

OPERATION.

$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = 1\frac{11}{12}$
$7 + 5 + 10 = 22$
$23\frac{11}{12}$, *Ans.*

SOLUTION. — The sum of the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ is $1\frac{11}{12}$; the sum of the integers, 7, 5, and 10, is 22; and the sum of both fractions and integers is $23\frac{11}{12}$.

RULE. — *To add mixed numbers, add the fractions and integers separately, and then add their sums.*

If the mixed numbers are small, they may be reduced to improper fractions, and then added after the usual method.

16. What is the sum of $14\frac{4}{5}$, $3\frac{9}{10}$, $1\frac{3}{8}$, and $\frac{19}{20}$?
17. What is the sum of $\frac{7}{8}$, $1\frac{7}{12}$, $10\frac{5}{6}$, and 5? *Ans.* $18\frac{7}{24}$.
18. What is the sum of $17\frac{3}{4}$, $18\frac{5}{12}$, and $26\frac{1}{24}$?
19. What is the sum of $\frac{9}{48}$, $\frac{11}{16}$, $1\frac{1}{8}$, 3, and $\frac{17}{24}$?
20. What is the sum of $125\frac{4}{7}$, $327\frac{5}{12}$, and $25\frac{1}{4}$?
21. What is the sum of $\frac{14}{32}\frac{0}{0}$, $\frac{57}{80}$, $1\frac{1}{10}$, $\frac{19}{20}$, and $\frac{10}{16}\frac{5}{0}$?
22. What is the sum of $3\frac{9}{70}$, $21\frac{2}{35}$, $40\frac{1}{7}$, and $10\frac{1}{10}$?
23. I bought 3 pieces of cloth containing $125\frac{7}{8}$, $96\frac{3}{4}$, and $48\frac{2}{3}$ yards. How many yards did I have?
24. If it takes $5\frac{1}{3}$ yards of cloth to make a coat, $3\frac{1}{8}$ yards for a pair of pantaloons, and $\frac{7}{8}$ of a yard for a vest, how many yards will it take for all? *Ans.* $9\frac{1}{18}$ yards.
25. A farmer divides his farm into 5 fields; the first contains $26\frac{7}{12}$ acres, the second $40\frac{1}{21}$ acres, the third $51\frac{4}{7}$ acres, the fourth $59\frac{3}{4}$ acres, and the fifth $62\frac{3}{5}$ acres. How many acres are there in the farm? *Ans.* $241\frac{13}{21}$.
26. A speculator bought $175\frac{3}{5}$ bushels of wheat for $\$205\frac{1}{3}$, $325\frac{4}{7}$ bushels of barley for $\$296\frac{3}{4}$, $270\frac{13}{21}$ bushels of corn for $\$200\frac{11}{12}$, and $437\frac{7}{15}$ bushels of oats for $\$156\frac{29}{42}$. How many bushels of grain did he buy, and how much did he pay for the whole? *Ans.* $1209\frac{9}{35}$ bushels; $\$859\frac{19}{42}$.

SUBTRACTION.

EXAMPLES.

134. 1. From $\frac{7}{10}$ take $\frac{3}{10}$.

OPERATION.

$\frac{7}{10} - \frac{3}{10} = \frac{4}{10} = \frac{2}{5}$, *Ans.*

SOLUTION. — Since the given fractions have a common denominator, 10, we find the difference by subtracting 3, the less numerator, from 7, the greater, and write the remainder, 4, over the common denominator, 10, thus obtaining $\frac{4}{10} = \frac{2}{5}$, the required difference.

2. From $\frac{8}{9}$ take $\frac{5}{9}$. *Ans.* $\frac{1}{3}$.
3. From $1\frac{4}{12}$ take $1\frac{1}{12}$. *Ans.* $\frac{1}{4}$.
4. From $\frac{20}{27}$ take $\frac{6}{27}$. *Ans.* $\frac{14}{27}$.
5. From $\frac{49}{70}$ take $\frac{36}{70}$. *Ans.* $\frac{13}{70}$.
6. From $\frac{75}{128}$ take $\frac{11}{128}$. *Ans.* $\frac{1}{2}$.
7. From $\frac{182}{348}$ take $\frac{110}{348}$. *Ans.* $\frac{6}{29}$.
8. From $\frac{8}{9}$ take $\frac{5}{6}$.

OPERATION.

$\frac{8}{9} - \frac{5}{6} = \frac{8 \cdot 2}{3 6} - \frac{3 0}{3 6} = \frac{32-30}{36} = \frac{2}{36} = \frac{1}{18}$, *Ans.*

SOLUTION.—As in whole numbers, we can subtract *like* numbers only, or those having the same unit value, so we can subtract fractions only when they have a common denominator. As $\frac{8}{9}$ and $\frac{5}{6}$ have not a common denominator, we first reduce them to a common denominator, 36, and then subtract the less numerator, 30, from the greater, 32, and write the difference, 2, over the common denominator, 36. We thus obtain $\frac{2}{36} = \frac{1}{18}$, the required difference.

RULE. — I. *When necessary, reduce the fractions to a common denominator.*

II. *Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.*

9. From $\frac{1}{2}$ take $\frac{2}{5}$. *Ans.* $\frac{5}{18}$.
10. From $1\frac{5}{4}$ take $\frac{2}{5}$. *Ans.* $\frac{9}{40}$.
11. Subtract $\frac{4}{17}$ from $\frac{3}{8}$. *Ans.* $\frac{19}{136}$.

12. Subtract $\frac{4}{85}$ from $\frac{84}{120}$. *Ans.* $\frac{41}{70}$.

13. Subtract $\frac{50}{72}$ from $\frac{1500}{1728}$. *Ans.* $\frac{25}{144}$.

14. Subtract $\frac{3832}{42272}$ from $\frac{60}{89}$. *Ans.* $\frac{6387}{10678}$.

15. What is the difference between $9\frac{1}{3}$ and $2\frac{3}{4}$.

OPERATION.

$9\frac{1}{3} = 9\frac{4}{12}$

$2\frac{3}{4} = 2\frac{9}{12}$

$6\frac{7}{12}$, *Ans.*

SOLUTION. — We first reduce the fractional parts, $\frac{1}{3}$ and $\frac{3}{4}$, to a common denominator, 12. Since we cannot take $\frac{9}{12}$ from $\frac{4}{12}$, we add $1 = \frac{12}{12}$ to $\frac{4}{12}$, which makes $\frac{16}{12}$, and $\frac{9}{12}$ from $\frac{16}{12}$ leaves $\frac{7}{12}$. We now add 1 to the 2 in the subtrahend (**50**). Three from 9 leaves 6. We thus obtain $6\frac{7}{12}$, the difference required.

RULE. — *To subtract mixed numbers, reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately.* Or,

Reduce the mixed numbers to improper fractions, and subtract the less from the greater by the usual method.

16. From $8\frac{1}{2}$ take $3\frac{7}{9}$. *Ans.* $4\frac{13}{18}$.

17. From $25\frac{5}{6}$ take $9\frac{7}{10}$. *Ans.* $16\frac{2}{15}$.

18. From $4\frac{4}{5}$ take $\frac{14}{15}$.

19. Subtract $1\frac{1}{7}$ from 6.

20. Subtract $120\frac{9}{37}$ from $450\frac{1}{2}$. *Ans.* $330\frac{19}{74}$.

21. Subtract $\frac{43}{125}$ from $3\frac{7}{15}$. *Ans.* $3\frac{146}{375}$.

22. Find the difference between 49 and $75\frac{1}{4}$.

23. Find the difference between $227\frac{5}{9}$ and $196\frac{2}{5}$.

24. From a cask of oil containing $31\frac{1}{2}$ gallons, $17\frac{5}{8}$ gallons were drawn. How many gallons remained? *Ans.* $13\frac{7}{8}$.

25. A farmer, having $450\frac{7}{10}$ acres of land, sold $304\frac{3}{4}$ acres. How many acres had he left? *Ans.* $145\frac{19}{20}$.

26. If flour is bought for $\$6\frac{1}{4}$ per barrel, and sold for $\$7\frac{2}{3}$, what will be the gain per barrel?

27. From the sum of $\frac{5}{7}$ and $3\frac{1}{2}$, take the difference between $4\frac{1}{3}$ and $5\frac{1}{4}$. *Ans.* $3\frac{25}{84}$.

28. What number added to $2\frac{5}{9}$ will make $7\frac{1}{4}$? *Ans.* $4\frac{25}{36}$.

29. What fraction added to $\frac{11}{12}$ will make $\frac{19}{20}$? *Ans.* $\frac{1}{30}$.

MULTIPLICATION.

EXAMPLES.

135. To multiply a fraction by an integer.

1. If 1 yard of cloth costs $\frac{3}{4}$ of a dollar, how much will 5 yards cost?

OPERATION.

$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$, *Ans.*

SOLUTION. — Since 1 yard costs 3 *fourths* of a dollar, 5 yards will cost 5 times 3 *fourths* of a dollar, or 15 *fourths*, equal to $3\frac{3}{4}$ dollars. A fraction is multiplied by multiplying its numerator (**124**).

2. If 1 gallon of molasses costs $\frac{7}{20}$ of a dollar, how much will 5 gallons cost?

OPERATION.

$\frac{7}{20} \times 5 = \frac{7}{4} = 1\frac{3}{4}$, *Ans.*

SOLUTION. — Since 5, the multiplier, is a factor of 20, the denominator of the multiplicand, we perform the multiplication by dividing the denominator, 20, by the multiplier, 5, and we have $\frac{7}{4}$, equal to $1\frac{3}{4}$ dollars. A fraction is multiplied by dividing its denominator (**124**).

RULE. — *To multiply a fraction multiply its numerator, or divide its denominator.*

Always divide the denominator when it is exactly divisible by the multiplier.

3. Multiply $\frac{3}{7}$ by 5. *Ans.* $\frac{15}{7} = 3\frac{1}{7}$.
4. Multiply $\frac{3}{11}$ by 7. *Ans.* $1\frac{10}{11}$.
5. Multiply $\frac{9}{14}$ by 12. *Ans.* $7\frac{5}{7}$.
6. Multiply $\frac{5}{21}$ by 63. *Ans.* 15.
7. Multiply $5\frac{1}{2}$ by 9.

OPERATION.

$5\frac{1}{2}$
9
$4\frac{1}{2}$
45
$49\frac{1}{2}$, *Ans.*

Or,

$5\frac{1}{2} = \frac{11}{2}$.

$\frac{11}{2} \times 9 = \frac{99}{2} = 49\frac{1}{2}$.

SOLUTION. — In multiplying a mixed number, we first multiply the fractional part, and then the integer, and add the two products; or we reduce the mixed number to an improper fraction, and then multiply it.

8. Multiply $7\frac{3}{8}$ by 12. *Ans.* $91\frac{1}{2}$.
9. Multiply $\frac{81}{121}$ by 8. *Ans.* $5\frac{43}{121}$.
10. Multiply $\frac{4}{102}$ by 51. *Ans.* 2.
11. Multiply $15\frac{5}{8}$ by 16. *Ans.* 250.
12. Multiply $\frac{101}{182}$ by 22. *Ans.* $16\frac{5}{8}$.
13. If a man earns $\$8\frac{9}{10}$ a week, how many dollars will he earn in 12 weeks?
14. What will 9 yards of silk cost at $\frac{11}{12}$ of a dollar per yard?
15. What will 27 bushels of barley cost at $\frac{7}{8}$ of a dollar per bushel? *Ans.* $\$23\frac{5}{8}$.

136. To multiply an integer by a fraction.

1. At \$75 an acre, how much will $\frac{3}{5}$ of an acre of land cost?

OPERATION.

$5)\underline{75}$ price of an acre.
$\underline{15}$ cost of $\frac{1}{5}$ of an acre.
$\underline{3}$
Ans. 45 cost of $\frac{3}{5}$ of an acre.

SOLUTION. — One fifth of an acre will cost $\frac{1}{5}$ of \$75, which is \$15; and $\frac{3}{5}$ will cost 3 times as much as $\frac{1}{5}$, or \$45.

Multiplying by a fraction consists in multiplying by the numerator and dividing by the denominator of the multiplier.

2. Multiply 3 by $\frac{4}{9}$. *Ans.* $1\frac{1}{3}$.
3. Multiply 100 by $\frac{9}{14}$. *Ans.* $64\frac{2}{7}$.
4. Multiply 105 by $\frac{17}{21}$. *Ans.* 85.
5. Multiply 19 by $\frac{18}{47}$. *Ans.* $5\frac{17}{47}$.
6. Multiply 24 by $6\frac{5}{8}$.

OPERATION.

24
$6\frac{5}{8}$
$\underline{15}$ $= \frac{5}{8}$ of 24.
$\underline{144}$
159, *Ans.*

Or, $6\frac{5}{8} = \frac{53}{8}$.

$\cancel{24}^{3} \times \frac{53}{\cancel{8}} = 159$.

SOLUTION. — We multiply by the integer and fraction separately, and add the products; or we may reduce the mixed number to an improper fraction, and then multiply by it.

7. Multiply 42 by $9\frac{3}{4}$. *Ans.* $409\frac{1}{2}$.

8. Multiply 80 by $14\frac{9}{16}$. *Ans.* 1165.

9. Multiply 156 by $\frac{27}{39}$. *Ans.* 108.

10. At \$8 a bushel, what will $\frac{5}{6}$ of a bushel of clover seed cost?

11. If a man travels 36 miles a day, how many miles will he travel in $10\frac{3}{8}$ days? *Ans.* 384 miles.

12. If a village lot is worth 450 dollars, what is $\frac{7}{12}$ of it worth? *Ans.* $262\frac{1}{2}$ dollars.

13. At 16 dollars a ton, what is the cost of $2\frac{7}{9}$ tons of hay?

137. To multiply a fraction by a fraction.

1. At $\frac{2}{3}$ of a dollar per bushel, how much will $\frac{3}{4}$ of a bushel of corn cost?

OPERATION.

1st step, $\frac{2}{3} \div 4 = \frac{2}{12}$, cost of $\frac{1}{4}$ of a bushel.

2d step, $\frac{2}{12} \times 3 = \frac{6}{12}$, cost of $\frac{3}{4}$ of a bushel.

Whole work, $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$, *Ans.*

SOLUTION. — Dividing $\frac{2}{3}$ of a dollar by 4, we have $\frac{2}{12}$, the cost of $\frac{1}{4}$ of a bushel. A fraction is divided by multiplying its denominator (**124**). Multiplying the cost of $\frac{1}{4}$ of a bushel by 3, we have $\frac{6}{12}$ of a dollar, the cost of $\frac{3}{4}$ of a bushel. It will readily be seen that we have found the product of the two numerators, 2 and 3, for a new numerator, and of the two denominators, 3 and 4, for a new denominator.

RULE. — I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Find the product of the numerators for a new numerator, and of the denominators for a new denominator.*

Cancel all factors common to numerators and denominators.

2. Multiply $\frac{2}{4}$ by $\frac{4}{6}$. *Ans.* $\frac{1}{3}$.

3. Multiply $\frac{7}{8}$ by $\frac{4}{5}$. *Ans.* $\frac{7}{10}$.

4. Multiply $\frac{11}{24}$ by $\frac{36}{55}$. *Ans.* $\frac{3}{10}$.

5. Multiply $4\frac{1}{5}$ by $\frac{6}{7}$. *Ans.* $3\frac{3}{5}$.

6. What is the product of $\frac{9}{10}$, $\frac{2}{7}$, $\frac{5}{9}$, and $\frac{1}{4}$? *Ans.* $\frac{1}{28}$.

7. What is the product of $1\frac{5}{6}$, $\frac{3}{5}$, 2, and $5\frac{1}{3}$? *Ans.* $11\frac{11}{15}$.

8. What is the product of $\frac{3}{4}$ of $\frac{7}{10}$ times $\frac{5}{6}$ of $\frac{2}{3}$ of $\frac{1}{5}$, by $\frac{4}{7}$ of $1\frac{3}{5}$?

OPERATION.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{7}{\cancel{10}\,\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{7}}{\cancel{8}} \times \frac{\cancel{4}}{\cancel{7}} \times \frac{\cancel{8}}{\cancel{5}} = \frac{7}{30}, \quad \textit{Ans.}$$

SOLUTION. — Canceling like factors in the numerators and denominators, we have left in the numerator 7, and in the denominator $5 \times 6 = 30$, making $\frac{7}{30}$ for the product.

Fractions with the word *of* between them are sometimes called *compound fractions*. The word *of* is simply an equivalent for the sign of multiplication, ×.

9. Multiply $\frac{8}{15}$ of $2\frac{1}{4}$ by $\frac{1}{5}$ of $7\frac{1}{8}$. *Ans.* $1\frac{19}{25}$.

10. Multiply $\frac{3}{7}$ of 16 by $\frac{7}{10}$ of $26\frac{2}{3}$. *Ans.* $85\frac{1}{3}$.

11. What is the sum of the products of 3, $\frac{1}{2}$ of $\frac{4}{7}$, and $\frac{4}{5}$ of $3\frac{1}{2}$? *Ans.* $3\frac{16}{35}$.

12. What is the value of $2\frac{1}{2}$ times $\frac{3}{4}$ of $\frac{4}{5}$ of $1\frac{1}{3}$? *Ans.* 2.

13. What is the value of $\frac{7}{8}$ of $\frac{1}{2}$ of $1\frac{2}{3}$ times $\frac{3}{2}$ of 8?

14. What is the value of $\frac{3}{5}$ of $\frac{1}{3}$ of $\frac{1}{2}$ of $3\frac{2}{3}$ times $\frac{5}{7}$ of 9? *Ans.* $2\frac{5}{14}$.

15. What is the product of $12\frac{1}{2}$ multiplied by $5\frac{1}{2}$ times $6\frac{3}{4}$? *Ans.* $464\frac{1}{16}$.

16. At $\frac{3}{5}$ of a dollar per yard, what will $\frac{2}{3}$ of a yard of cloth cost? *Ans.* $\frac{2}{5}$ of a dollar.

17. If a man owns $\frac{6}{7}$ of a vessel, and sells $\frac{2}{3}$ of his share, what part of the whole vessel will he sell?

18. When oats are worth $\frac{1}{2}$ of a dollar per bushel, what is $\frac{3}{4}$ of a bushel worth?

19. What will $7\frac{3}{4}$ pounds of tea cost, at $\frac{3}{5}$ of a dollar per pound? *Ans.* $\$4\frac{13}{20}$.

20. What will $5\frac{1}{3}$ yards of cloth cost at $\frac{3}{5}$ of a dollar a yard? *Ans.* $\$3\frac{1}{5}$.

21. If a man owns $\frac{2}{3}$ of a plot of ground and sells $\frac{1}{2}$ of his share, what part of the plot does he sell?

22. What is the product of $9\frac{6}{7}$ by $4\frac{2}{3}$?

OPERATION.

$9\frac{6}{7}$
$4\frac{2}{3}$
$39\frac{3}{7}$ product by 4.
$6\frac{4}{7}$ " " $\frac{2}{3}$.
Ans. 46 " " $4\frac{2}{3}$.

Or, $9\frac{6}{7} \times 4\frac{2}{3} = \frac{\overset{23}{69}}{7} \times \frac{\overset{2}{14}}{3} = 46.$

SOLUTION. — Multiplying $9\frac{6}{7}$ by 4, we obtain $39\frac{3}{7}$; and multiplying $9\frac{6}{7}$ by $\frac{2}{3}$, we obtain $6\frac{4}{7}$. Adding these products, we find the sum to be 46. Or, reducing $9\frac{6}{7}$ and $4\frac{2}{3}$ to improper fractions, we multiply one by the other as before. Canceling like factors, the result is 46.

RULE. — *To find the product of mixed numbers, either multiply by the integer and fractional part separately, and then add their products; or, reduce both numbers to improper fractions, and then multiply as in the foregoing rule.*

23. Multiply $12\frac{3}{4}$ by $8\frac{1}{2}$. *Ans.* $108\frac{3}{8}$.

24. What will be the cost of $6\frac{3}{8}$ cords of wood at $\$2\frac{4}{5}$ a cord?

25. What will $\frac{3}{4}$ of $2\frac{1}{2}$ tons of hay cost at $\$11\frac{3}{10}$ a ton? *Ans.* $\$21\frac{3}{16}$.

26. What will $8\frac{5}{8}$ cords of wood cost at $\$2\frac{3}{5}$ per cord? *Ans.* $\$22\frac{17}{40}$.

27. What must be paid for $\frac{4}{5}$ of $6\frac{1}{2}$ tons of coal at $\frac{7}{8}$ of $\$7\frac{1}{4}$ per ton?

28. A man owning $\frac{7}{9}$ of a farm, sold $\frac{1}{3}$ of his share. What part of the whole farm had he left? *Ans.* $\frac{14}{27}$.

29. A man bought a horse for $\$125\frac{3}{4}$, and sold him for $\frac{4}{5}$ of what he cost. What was the loss? *Ans.* $\$25\frac{3}{20}$.

30. A owned $\frac{2}{5}$ of $123\frac{5}{6}$ acres of land, and sold $\frac{3}{5}$ of his share. How many acres did he sell? *Ans.* $49\frac{3}{15}$ acres.

31. If a family consumes $1\frac{1}{4}$ barrels of flour a month, how many barrels will five such families consume in $4\frac{9}{10}$ months?

DIVISION.

EXAMPLES.

138. To divide a fraction by an integer.

1. If a horse eats $\frac{9}{10}$ of a ton of hay in 3 months, what part of a ton will last him 1 month?

OPERATION. $\frac{9}{10} \div 3 = \frac{3}{10}$, *Ans.*

SOLUTION. — In 1 month he will eat $\frac{1}{3}$ of $\frac{9}{10}$ of a ton, or $\frac{3}{10}$ of a ton (**124**, I).

2. If 3 yards of ribbon cost $\frac{5}{6}$ of a dollar, what will 1 yard cost?

OPERATION. $\frac{5}{6} \div 3 = \frac{5}{18}$, *Ans.*

SOLUTION. — Here we cannot exactly divide the numerator by 3; but, since a fraction is divided by multiplying the denominator (**124**, II), we multiply the denominator of the fraction, $\frac{5}{6}$, by 3, and have $\frac{5}{18}$, the required result.

RULE. — *To divide a fraction divide its numerator, or multiply its denominator.*

We divide the numerator when it is exactly divisible by the divisor; otherwise we multiply the denominator.

3. Divide $\frac{6}{7}$ by 2. *Ans.* $\frac{3}{7}$.
4. Divide $\frac{9}{21}$ by 3. *Ans.* $\frac{1}{7}$.
5. Divide $\frac{14}{15}$ by 5. *Ans.* $\frac{14}{75}$.
6. Divide $\frac{75}{125}$ by 25.
7. Divide $\frac{13}{17}$ by 14. *Ans.* $\frac{13}{238}$.
8. If 6 pounds of sugar cost $\frac{3}{8}$ of a dollar, what will 1 pound cost?
9. At \$7 a barrel, what part of a barrel of flour can be bought for $\frac{7}{8}$ of a dollar? *Ans.* $\frac{1}{8}$ of a barrel.
10. If a yard of cloth costs \$5, what part of a yard can be bought for $\frac{6}{7}$ of a dollar? *Ans.* $\frac{6}{35}$ of a yard.
11. If 9 bushels of barley cost \$$7\frac{1}{5}$, what will 1 bushel cost?

OPERATION. $7\frac{1}{5} = \frac{36}{5}$.
$\frac{36}{5} \div 9 = \frac{4}{5}$ of a dollar, *Ans.*

SOLUTION. — We reduce the mixed number to an improper fraction, and divide as before.

12. If 12 barrels of flour cost $\$76\frac{4}{5}$, what will 1 barrel cost?

OPERATION.

$12\overline{)76\frac{4}{5}}$

$\$6\frac{2}{5}$, *Ans.*

SOLUTION. — We first divide as in simple numbers, and have a remainder of $4\frac{4}{5}$. We reduce this remainder to an improper fraction, $\frac{24}{5}$, which we divide (as in Ex. 1), and annex the result, $\frac{2}{5}$, to the partial quotient, 6, and we have $6\frac{2}{5}$, the required result.

13. How many times will $16\frac{3}{4}$ gallons of cider fill a vessel that holds 3 gallons? *Ans.* $5\frac{7}{12}$ times.

14. If 9 men consume $\frac{3}{4}$ of $9\frac{3}{5}$ pounds of meat in a day, how much does each man consume?

Ans. $\frac{4}{5}$ of a pound.

15. A man paid $\$99\frac{2}{25}$ for 4 cows. How much was that apiece? *Ans.* $\$24\frac{24}{25}$.

139. To divide an integer by a fraction.

1. At $\frac{3}{4}$ of a dollar a yard, how many yards of cloth can be bought for 12 dollars?

FIRST OPERATION.

12

4

$3\overline{)48}$

16 yards.

SOLUTION. — Integers cannot be divided by *fourths*, because they are not of the same denomination. Reducing 12 dollars to *fourths* by multiplying, we have 48 *fourths;* and 3 *fourths* is contained in 48 *fourths* 16 times, the required number of yards.

SECOND OPERATION.

$3\overline{)12}$

4

4

16 yards.

SOLUTION. — If the cloth cost $3 a yard, we could buy 4 yards for $12; but since it costs only $\frac{1}{4}$ as much, we can buy 4 times as much for the same money. Therefore, we can buy 4×4 yards $= 16$ yards.

RULE. — *To divide a whole number by a fraction, multiply it by the denominator, and divide by the numerator of the divisor.*

2. Divide 18 by $\frac{3}{8}$. *Ans.* 48.

3. Divide 63 by $\frac{7}{18}$. *Ans.* 117.

4. Divide 42 by $\frac{6}{7}$. *Ans.* 49.

5. Divide 120 by $\frac{7}{12}$. *Ans.* $205\frac{5}{7}$.

6. Divide 316 by $\frac{9}{25}$. *Ans.* $877\frac{7}{9}$.

7. How many bushels of oats, worth $\frac{2}{5}$ of a dollar per bushel, will pay for $\frac{2}{3}$ of a barrel of flour, worth $9 a barrel? *Ans.* 15 bushels.

8. If $\frac{3}{7}$ of an acre of land sells for $21, what will an acre sell for at the same rate? *Ans.* $49.

9. When potatoes are worth $\frac{4}{9}$ of a dollar a bushel, and corn $\frac{5}{8}$ of a dollar a bushel, how many bushels of potatoes are equal in value to 16 bushels of corn?

Ans. $22\frac{1}{2}$ bushels.

10. If a man can chop $2\frac{3}{4}$ cords of wood in a day, in how many days can he chop 22 cords?

OPERATION.

$2\frac{3}{4} = \frac{11}{4}$.

22

4

11)88

8 days, *Ans.*

SOLUTION. — We reduce the mixed number to an improper fraction, and then divide the integer in the same manner as by a proper fraction.

11. Divide 75 by $13\frac{4}{5}$. *Ans.* $5\frac{10}{23}$.

12. Divide 149 by $24\frac{1}{6}$. *Ans.* $6\frac{24}{145}$.

13. A farmer distributed 15 bushels of corn among several persons, giving them $1\frac{2}{3}$ bushels apiece. Among how many persons did he divide it?

14. Divide $\frac{5}{6}$ of 320 by $\frac{5}{8}$ of $9\frac{1}{3}$. *Ans.* $25\frac{5}{7}$.

15. I bought $\frac{1}{8}$ of $7\frac{1}{2}$ cords of wood for $\frac{1}{4}$ of $32. How much did 1 cord cost? *Ans.* $3\frac{1}{5}$.

16. A father divided 183 acres of land equally among his sons, giving them $45\frac{3}{4}$ acres apiece. How many sons had he? *Ans.* 4 sons.

17. A man bought $\frac{3}{4}$ of $2\frac{2}{3}$ yards of cloth for $8. How much would 5 yards of the same cloth cost? *Ans.* $20.

140. To divide a fraction by a fraction.

1. How many pounds of tea can be bought for $\frac{11}{12}$ of a dollar, at $\frac{2}{3}$ of a dollar a pound?

OPERATION.

1st step, $\frac{11}{12} \times 3 = \frac{33}{12}$.

2d step, $\frac{33}{12} \div 2 = \frac{33}{24} = 1\frac{3}{8}$.

Whole work, $\frac{11}{12} \div \frac{2}{3} = \frac{11}{\cancel{12}_{4}} \times \frac{\cancel{3}}{2} = \frac{11}{8} = 1\frac{3}{8}$, *Ans.*

SOLUTION. — One is contained in $\frac{11}{12}$, $\frac{11}{12}$ times, and $\frac{1}{3}$ is contained in $\frac{11}{12}$ 3 times as many times as 1, or 3 times $\frac{11}{12}$, which is $\frac{33}{12}$ times; $\frac{2}{3}$ is contained but $\frac{1}{2}$ as many times as $\frac{1}{3}$, and $\frac{33}{12}$ divided by 2 gives $\frac{33}{24}$, equal to $1\frac{3}{8}$.

In the operation, we have multiplied the dividend by the denominator of the divisor, and divided the result by the numerator of the divisor, which is in accordance with **138** for dividing a fraction. Hence, by inverting the terms of the divisor, the two fractions will stand in such relation to each other that we can find the product of the two numerators for the numerator of the quotient, and of the two denominators for the denominator, as shown in the operation.

RULE. — I. *Reduce integers and mixed numbers to improper fractions.*

II. *Invert the terms of the divisor, and proceed as in multiplication.*

1. The dividend and divisor may be reduced to a common denominator, and the numerator of the dividend be divided by the numerator of the divisor; this will give the same result as the rule.

2. Apply cancellation where practicable.

2. Divide $\frac{7}{8}$ by $\frac{3}{4}$. *Ans.* $1\frac{1}{6}$.

3. Divide $\frac{5}{9}$ by $\frac{1}{6}$. *Ans.* $3\frac{1}{3}$.

4. Divide $\frac{4}{7}$ by $\frac{9}{10}$. *Ans.* $\frac{40}{63}$.

5. Divide $\frac{1}{2}$ by $\frac{7}{18}$. *Ans.* $\frac{18}{14}$.

6. Divide $\frac{2}{8}$ by $\frac{27}{28}$. *Ans.* $\frac{56}{81}$.

7. How many times is $\frac{4}{5}$ contained in $\frac{5}{6}$? *Ans.* $1\frac{1}{24}$.

8. How many times is $\frac{3}{7}$ contained in $1\frac{2}{3}$? *Ans.* $3\frac{8}{9}$.

9. How many times is $\frac{7}{19}$ contained in $\frac{17}{19}$? *Ans.* $2\frac{3}{7}$.

10. How many times is $\frac{5}{16}$ contained in $\frac{13}{20}$?

11. How many times is $\frac{1}{2}$ of $\frac{3}{4}$ contained in $\frac{2}{7}$ of $2\frac{1}{2}$?

12. What is the quotient of $\frac{9}{10}$ of 4, divided by $\frac{5}{6}$ of $3\frac{1}{4}$?

13. What is the quotient of $\frac{1}{8}$ of $\frac{7}{9}$ of 36 divided by $1\frac{7}{8}$ times $\frac{3}{5}$? *Ans.* $3\frac{1}{9}$.

14. What is the value of $\frac{3\frac{1}{2}}{4\frac{3}{8}}$?

OPERATION.

$$\frac{3\frac{1}{2}}{4\frac{3}{8}} = \frac{\frac{7}{2}}{\frac{35}{8}} = \frac{7}{2} \div \frac{35}{8} = \frac{\cancel{7}}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{35}}} = \frac{4}{5}, \text{ Ans.}$$

SOLUTION. — This example is only another form for expressing division of fractions; it is sometimes called a *complex fraction*, and the process of performing the division is called *reducing a complex fraction to a simple one.*

We simply reduce the upper number, or dividend, to an improper fraction, and the lower number, or divisor, to an improper fraction, and then divide as before.

15. What is the value of $\frac{6\frac{2}{9}}{8\frac{2}{3}}$? *Ans.* $\frac{28}{39}$.

16. What is the value of $\frac{11\frac{3}{7}}{\frac{4}{7}}$? *Ans.* 20.

17. What is the value of $\frac{\frac{5}{11}}{4\frac{2}{5}}$? *Ans.* $\frac{25}{242}$.

18. What is the value of $\frac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$? *Ans.* $\frac{1}{8}$.

19. If a horse eats $\frac{3}{8}$ of a bushel of oats in a day, in how many days will he eat $5\frac{1}{4}$ bushels? *Ans.* 14 days.

20. How many times will $4\frac{3}{8}$ gallons of oil fill a vessel that holds $\frac{1}{2}$ of $\frac{5}{6}$ of 1 gallon? *Ans.* $10\frac{1}{2}$ times.

21. If 14 acres of meadow land produce $32\frac{2}{3}$ tons of hay, how many tons will 5 acres produce? *Ans.* $11\frac{2}{3}$ tons.

RELATION OF NUMBERS.

EXAMPLES.

141. To find what part one number is of another.

1. What part of 8 is 7?

OPERATION.

$1 = \frac{1}{8}$ of 8.

$7 \times \frac{1}{8} = \frac{7}{8}$ of 8.

SOLUTION. — Since 1 is $\frac{1}{8}$ of 8, 7 is 7 times $\frac{1}{8}$, or $\frac{7}{8}$ of 8.

What part of:

2. 9 is 8?	5. 96 is 72?	8. 500 is 250?
3. 24 is 15?	6. 120 is 90?	9. 1200 is 400?
4. 48 is 32?	7. 450 is 150?	10. 1800 is 450?

11. What part of 2 is $\frac{1}{3}$?

OPERATION.

$1 = \frac{1}{2}$ of 2.

$\frac{1}{3} = \frac{1}{3}$ of $\frac{1}{2}$ or $\frac{1}{6}$ of 2.

Or,

$2 = \frac{12}{6}$; $\frac{1}{3} = \frac{2}{6}$.

$\frac{2}{6} = \frac{2}{12}$ or $\frac{1}{6}$ of $\frac{12}{6}$.

$\therefore \frac{1}{3} = \frac{1}{6}$ of 2.

SOLUTION. — Since 1 is $\frac{1}{2}$ of 2, $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{1}{2}$ or $\frac{1}{6}$ of 2.

Or, changing the numbers to fractions with a common denominator, $\frac{12}{6}$ and $\frac{2}{6}$, we find what part the smaller numerator is of the greater.

12. What part of $\frac{4}{5}$ is $\frac{2}{3}$?

OPERATION.

$\frac{1}{5} = \frac{1}{4}$ of $\frac{4}{5}$.

1 or $\frac{5}{5} = 5 \times \frac{1}{4}$ or $\frac{5}{4}$ of $\frac{4}{5}$.

$\frac{2}{3} = \frac{2}{3}$ of $\frac{5}{4} = \frac{10}{12}$ or $\frac{5}{6}$ of $\frac{4}{5}$.

Or,

$\frac{4}{5} = \frac{12}{15}$; $\frac{2}{3} = \frac{10}{15}$.

$\frac{10}{15} = \frac{10}{12}$ or $\frac{5}{6}$ of $\frac{12}{15}$.

SOLUTION. — We first find what part $\frac{1}{5}$ is of $\frac{4}{5}$, which is $\frac{1}{4}$; then $\frac{5}{5}$ or $1 = \frac{5}{4}$ of $\frac{4}{5}$, and $\frac{2}{3} = \frac{2}{3}$ of $\frac{5}{4} = \frac{10}{12}$ or $\frac{5}{6}$ of $\frac{4}{5}$.

Or, we may reduce both fractions to the same denominator and find what part the smaller numerator is of the greater.

13. What part of $2\frac{4}{5}$ is $1\frac{2}{3}$?

OPERATION.

$2\frac{4}{5} = \frac{14}{5}$; $1\frac{2}{3} = \frac{5}{3}$.

$\frac{1}{5} = \frac{1}{14}$ of $\frac{14}{5}$.

1 or $\frac{5}{5} = 5 \times \frac{1}{14}$ or $\frac{5}{14}$ of $\frac{14}{5}$.

$\frac{5}{3} = \frac{5}{3}$ of $\frac{5}{14}$ or $\frac{25}{42}$ of $\frac{14}{5}$.

Or,

$2\frac{4}{5} = \frac{42}{15}$; $1\frac{2}{3} = \frac{25}{15}$.

$\frac{25}{15} = \frac{25}{42}$ of $\frac{42}{15}$.

$\therefore$ $1\frac{2}{3} = \frac{25}{42}$ of $2\frac{4}{5}$.

SOLUTION. — We first reduce the mixed numbers to improper fractions. $2\frac{4}{5} = \frac{14}{5}$ and $1\frac{2}{3} = \frac{5}{3}$. The problem now reads: What part of $\frac{14}{5}$ is $\frac{5}{3}$? Proceeding as in Example 12, we find the answer to be $\frac{25}{42}$.

Or, changing both numbers to fractions with a common denominator, we may find what part the smaller numerator is of the greater.

RULE. — *Divide the number of which the part is to be found by the part.*

What part of:

14. $\frac{9}{10}$ is $\frac{3}{5}$? *Ans.* $\frac{2}{3}$. 16. $\frac{3}{4}$ is $\frac{1}{2}$? 18. $2\frac{1}{3}$ is $1\frac{1}{4}$?

15. $\frac{5}{9}$ is $\frac{1}{6}$? *Ans.* $\frac{3}{10}$. 17. $4\frac{1}{4}$ is $\frac{3}{4}$? 19. $7\frac{1}{2}$ is $1\frac{1}{4}$?

142. To find a number when a fractional part of it is given.

1. 180 is $\frac{3}{4}$ of what number?

OPERATION.

$\frac{1}{4}$ the number $= \frac{1}{3}$ of $180 = 60$.

$4 \times 60 = 240$, *Ans.*

Or,

$\frac{1}{4}$ the number $= \frac{1}{3}$ of 180.

$\frac{4}{4}$ the number $= \frac{4}{3}$ of $\overset{60}{\cancel{180}} = 240$, *Ans.*

SOLUTION. — Since 180 is $\frac{3}{4}$ of a certain number, $\frac{1}{4}$ of the number will be $\frac{1}{3}$ of 180, which is 60; and the number will be 4 times 60, which is 240. Hence, 180 is $\frac{3}{4}$ of 240. Or, $\frac{1}{4}$ the number will be $\frac{1}{3}$ of 180, and $\frac{4}{4}$, or the whole number, will be 4 times $\frac{1}{3} = \frac{4}{3}$ of 180, which is 240.

RULE. — *Multiply the number by the fraction with its terms inverted; or, divide the number by the fraction.*

Find the number of which:

2. 75 is $\frac{5}{7}$. 4. $15\frac{3}{4}$ is $\frac{3}{7}$. 6. 392 is $\frac{4}{9}$. 8. 625 is $\frac{5}{6}$.

3. 84 is $\frac{12}{20}$. 5. 180 is $\frac{3}{4}$. 7. $\frac{12}{17}$ is $\frac{4}{5}$. 9. 450 is $\frac{5}{8}$.

PROMISCUOUS EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{1}{4}$ to equivalent fractions whose denominators will be 24. *Ans.* $\frac{12}{24}$, $\frac{20}{24}$, $\frac{9}{24}$, $\frac{6}{24}$.

2. Change $\frac{4}{7}$ to an equivalent fraction having 91 for its denominator. *Ans.* $\frac{52}{91}$.

3. Find the least common denominator of $\frac{3}{4}$, $1\frac{2}{3}$, $\frac{1}{2}$ of $\frac{4}{5}$, 2, $\frac{1}{8}$ of $\frac{1}{4}$ of $1\frac{1}{10}$.

4. Add $4\frac{1}{3}$, $\frac{7}{9}$, $\frac{4}{5}$ of $1\frac{1}{8}$, 3, and $\frac{11}{12}$.

5. Find the difference between $\frac{2}{3}$ of $6\frac{7}{10}$ and $\frac{5}{9}$ of $4\frac{8}{15}$. *Ans.* $1\frac{28}{135}$.

6. The less of two numbers is $4756\frac{4}{9}$, and their difference is $128\frac{3}{4}$. What is the greater number? *Ans.* $4885\frac{7}{36}$.

7. What is the difference between the continued products of 3, $\frac{7}{8}$, $\frac{2}{3}$, $4\frac{2}{5}$, and $3\frac{7}{8}$, $\frac{2}{3}$, 4, $\frac{2}{5}$? *Ans.* $3\frac{17}{80}$.

8. Reduce the fractions $\frac{4}{\frac{1}{5}}$ and $\frac{2\frac{1}{2}}{1\frac{1}{3}}$ to their simplest form.

9. What number multiplied by $\frac{8}{9}$ will produce $1825\frac{7}{8}$?

10. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{4}$ in another, and the remainder, which were 77, in a third pasture. How many sheep had he? *Ans.* 140 sheep.

11. What will $7\frac{3}{4}$ cords of wood cost at $\frac{1}{3}$ of \$ $9\frac{1}{2}$ per cord? *Ans.* \$ $24\frac{13}{24}$.

12. At $\frac{1}{4}$ of a dollar per bushel, how many bushels of apples can be bought for \$ $5\frac{7}{8}$?

13. I paid \$ $1837\frac{5}{8}$ for $7350\frac{1}{2}$ bushels of oats. How much was that per bushel? *Ans.* $\frac{1}{4}$ of a dollar.

14. If $235\frac{1}{2}$ acres of land cost \$ $4725\frac{3}{8}$, what will 628 acres cost? *Ans.* \$ 12601.

15. A man, owning $\frac{2}{5}$ of an iron foundry, sold $\frac{1}{3}$ of his share for \$ $540\frac{3}{4}$. What was the value of the foundry? *Ans.* \$ $4055\frac{5}{8}$.

16. $14\frac{2}{7}$ less $\frac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}}$ is $\frac{2}{3}$ of $\frac{7}{9}$ of what number? *Ans.* 27.

17. A merchant bought $4\frac{3}{4}$ cords of wood at $\$3\frac{1}{4}$ per cord, and paid for it in cloth at $\frac{5}{8}$ of a dollar per yard. How many yards were required to pay for the wood?

18. How many yards of cloth, $\frac{3}{4}$ of a yard wide, will line $20\frac{1}{2}$ yards, $1\frac{1}{4}$ yards wide? *Ans.* $34\frac{1}{6}$ yards.

19. If the dividend is $\frac{7}{9}$, and the quotient $\frac{4}{3\,9}$, what is the divisor?

20. If the sum of two fractions is $\frac{5}{8}$, and one of them is $\frac{9}{2\,0}$, what is the other? *Ans.* $\frac{7}{4\,0}$.

21. If the smaller of two fractions is $\frac{24}{81}$, and their difference is $\frac{7}{9\,3}$, what is the greater? *Ans.* $\frac{7\,9}{9\,3}$.

22. If $3\frac{3}{8}$ pounds of sugar cost 33 cents, what must be paid for $65\frac{1}{2}$ pounds?

23. A certain sum of money is to be divided among 5 persons; A is to have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{1\,0}$, D $\frac{1}{2\,0}$, and E the remainder, which is $\$20$. What is the whole sum to be divided? *Ans.* $\$50$.

24. What number, diminished by the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of itself, leaves a remainder of 34? *Ans.* 40.

25. If 324 bushels of barley can be had for $259\frac{1}{5}$ bushels of corn, how much barley can be had for 2000 bushels of corn? *Ans.* 2500 bushels.

26. If $\frac{3}{8}$ of a farm is valued at $\$1728$, what is the value of the whole?

27. I bought 320 sheep at $\$2\frac{1}{4}$ per head; I afterwards bought 435 at $\$1\frac{7}{8}$ per head; then sold $\frac{2}{5}$ of the whole number at $\$1\frac{3}{4}$ per head, and the remainder at $\$2\frac{1}{8}$. Did I gain or lose, and how much? *Ans.* Lost $\$44\frac{1}{2}$.

28. If 5 is added to both terms of the fraction $\frac{7}{8}$, will its value be increased or diminished? *Ans.* Increased $\frac{5}{1\,0\,4}$.

29. If 5 is added to both terms of the fraction $\frac{3}{7}$, will its value be increased or diminished? *Ans.* Diminished $\frac{5}{8\,4}$.

30. How many times can a bottle holding $\frac{1}{4}$ of $\frac{2}{3}$ of a gallon, be filled from a demijohn containing $\frac{3}{4}$ of $1\frac{2}{3}$ gallons? *Ans.* $7\frac{1}{2}$.

31. I bought $\frac{1}{3}$ of $7\frac{1}{2}$ cords of wood for $\frac{1}{4}$ of $32. What did 1 cord cost?

32. A man bought 728 pounds of candles at $16\frac{2}{3}$ cents a pound. Had they been purchased for $3\frac{7}{8}$ cents less a pound, how many pounds could have been bought for the same money? *Ans.* $953\frac{17}{25}$.

33. What number, divided by $1\frac{3}{8}$, will give a quotient of $9\frac{1}{8}$? *Ans.* $12\frac{35}{64}$.

34. The product of two numbers is 6, and one of them is 1846. What is the other? *Ans.* $\frac{3}{923}$.

35. A stone mason worked $11\frac{2}{3}$ days, and after paying his board and other expenses with $\frac{3}{7}$ of his earnings, he had $20 left. How much did he receive a day?

36. If $\frac{2}{5}$ of 4 tons of coal cost $5\frac{1}{3}$, what will $\frac{3}{4}$ of 2 tons cost? *Ans.* $5.

37. A man gave $6\frac{2}{3}$ pounds of butter, at 12 cents a pound, for $\frac{4}{5}$ of a gallon of oil. What was the oil worth a gallon? *Ans.* 100 cents.

38. In an orchard $\frac{3}{4}$ of the trees are apple trees, $\frac{1}{10}$ peach trees, and the remainder are pear trees, which are 20 more than $\frac{1}{8}$ of the whole. How many trees are there in the orchard? *Ans.* 800.

39. A man, having $271\frac{1}{2}$ acres of land, sold $\frac{1}{3}$ of it, and gave $\frac{3}{8}$ of it to his son. What was the value of the remainder, at $57\frac{1}{5}$ per acre? *Ans.* $4577\frac{3}{80}$.

40. A horse and a wagon cost $270; the horse cost $1\frac{1}{4}$ times as much as the wagon. What was the cost of the wagon?

41. What number taken from $2\frac{1}{2}$ times $12\frac{4}{5}$ will leave $20\frac{3}{4}$? *Ans.* $11\frac{1}{4}$.

42. A merchant bought a cargo of flour for $2173\frac{1}{2}$,

and sold it for $\frac{22}{25}$ of the cost, thereby losing $\frac{3}{4}$ of a dollar per barrel. How many barrels did he purchase?

Ans. 126.

43. A and B can do a piece of work in 14 days; A can do $\frac{3}{4}$ as much as B. In how many days can each do it?

Ans. A in $32\frac{2}{3}$ days; B in $24\frac{1}{2}$ days.

44. How many yards of cloth $\frac{4}{5}$ of a yard wide, are equal to 12 yards $\frac{3}{4}$ of a yard wide? *Ans.* $11\frac{1}{4}$.

45. A, B, and C can do a piece of work in 5 days; B and C can do it in 8 days. In what time can A do it?

46. A man's family expenses are $\$2465\frac{1}{5}$ a year, which is $\frac{6}{7}$ of his income. What does he save?

47. A man put his money into 4 packages; in the first he put $\frac{2}{5}$, in the second $\frac{1}{3}$, in the third $\frac{1}{6}$, and in the fourth the remainder, which was \$ 24 more than $\frac{1}{15}$ of the whole. How much money had he? *Ans.* \$ 720.

48. A man bequeathed to his son \$ 35,000, which was $\frac{5}{7}$ of what he left his wife. How much did he leave his wife?

49. If $\$7\frac{1}{4}$ will buy $3\frac{1}{4}$ cords of wood, how many cords can be bought for $\$10\frac{1}{2}$? *Ans.* $4\frac{4}{5}\frac{1}{8}$ cords.

50. How many times is $\frac{1}{6}$ of $\frac{4}{5}$ of 27 contained in $\frac{7}{8}$ of $\frac{1}{2}$ of $42\frac{2}{3}$?

51. A boy lost $\frac{1}{2}$ of his kite string, and then added 30 feet, when it was just $\frac{4}{5}$ of its original length. What was the length at first? *Ans.* 100 feet.

52. A man bought $\frac{6}{7}$ of a box of candles, and having used $\frac{7}{8}$ of them, sold the remainder for $\frac{16}{25}$ of a dollar. How much would a box cost at the same rate? *Ans.* $\$5\frac{7}{75}$.

53. A post stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and 21 feet above the water. What is its length?

54. A father left his elder son $\frac{3}{7}$ of his estate, his younger son $\frac{4}{7}$ of the remainder, and his daughter the remainder. She received $\$1723\frac{5}{8}$ less than the youngest son. What was the value of the estate? *Ans.* $\$21114\frac{3}{8}\frac{3}{2}$.

DECIMAL FRACTIONS.

143. Decimal Fractions are fractions which have for their denominator 10, 100, 1000, or 1 with any number of ciphers annexed, and are generally written like the orders of integers.

1. The word *decimal* is derived from the Latin *decem*, which signifies *ten*.
2. Decimal fractions are commonly called *decimals*.
3. Since $\frac{1}{10} = \frac{10}{100}$, $\frac{1}{100} = \frac{10}{1000}$, etc., the denominators of decimal fractions increase and decrease in a tenfold ratio, in the same way as simple numbers.

DECIMAL NOTATION AND NUMERATION.

144. *Decimal Fractions* are the decimal divisions of a unit into **tenths, hundredths,** etc., just as common fractions are the common divisions of a unit into any number of equal parts, as **halves, fifths,** etc.

Thus, a unit is divided into ten equal parts, called *tenths;* each of these tenths is divided into ten other equal parts, called *hundredths;* each of these hundredths into ten other equal parts, called *thousandths;* and so on.

Since the denominators of decimal fractions increase and decrease by the scale of ten, the same as in simple numbers, in writing decimals the denominators may be omitted.

In simple numbers, the unit, 1, is the starting point of notation and numeration; and so also is it in decimals. We extend the scale of notation to the

left of units' place in writing integers, and to the right of units' place in writing decimals.

The first place at the left of units is tens, and the first place at the right of units is tenths; the second place at the left is hundreds, and the second place at the right is hundredths; the third place at the left is thousands, and the third place at the right is thousandths; and so on.

The *Decimal Point* is a period (.), which must always be placed before or at the left hand of the decimal.

Thus, $\frac{6}{10}$ is expressed .6
$\frac{54}{100}$ is expressed .54
$\frac{279}{1000}$ is expressed .279

The decimal point is also called the *Separatrix*. This is a correct name for it only when it stands between the integral and decimal parts of the same number.

.5 is 5 tenths, which $= \frac{1}{10}$ of 5 units;
.05 is 5 hundredths, " $= \frac{1}{10}$ of 5 tenths;
.005 is 5 thousandths, " $= \frac{1}{10}$ of 5 hundredths.

Universally, the value of a figure in any decimal place is $\frac{1}{10}$ the value of the same figure in the next left hand place.

The relation of decimals and integers to each other is clearly shown by the following table:

NUMERATION TABLE.

Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
INTEGERS.										DECIMALS.							

By examining this table we see that

Tenths are expressed by one figure.
Hundredths are expressed by two figures.
Thousandths are expressed by three figures.
Ten thousandths are expressed by four figures.

And any order of decimals is expressed by one figure less than the corresponding order of integers.

145. Since the denominator of tenths is 10, of hundredths 100, of thousandths 1000, and so on, a decimal may be expressed by writing the numerator only; but in this case the numerator or decimal must always contain as many decimal places as are equal to the number of ciphers in the denominator; and the denominator of a decimal will always be the unit, 1, with as many ciphers annexed as are equal to the number of figures in the decimal or numerator.

The decimal point must never be omitted.

EXAMPLES.

1. Express in figures thirty-eight hundredths.
2. Write seven tenths.
3. Write three hundred twenty-five thousandths.
4. Write four hundredths. *Ans.* .04.
5. Write sixteen thousandths.
6. Write seventy-four hundred-thousandths. *Ans.* .00074.
7. Write seven hundred forty-five millionths.
8. Write four thousand two hundred thirty-two ten-thousandths.
9. Write five hundred thousand millionths.

10. Read the following decimals:

.05	.681	.9034	.19248
.24	.024	.0005	.001385
.672	.8471	.100248	.1000087

To read a decimal, first numerate from left to right, and the name of the right hand figure is the name of the denominator. Then numerate from right to left as in whole numbers, to read the numerator.

146. A Mixed Decimal Number is a number consisting of integers and decimals; thus, 71.406 consists of the integral part, 71, and the decimal part, .406; it is read the same as $71\frac{406}{1000}$, 71 and 406 thousandths.

1. Write eighteen, and twenty-seven thousandths.
2. Write four hundred, and nineteen ten-millionths.
3. Write fifty-four, and fifty-four millionths.
4. Write eighty-one, and 1 ten-thousandth.
5. Write one hundred, and 67 ten-thousandths.
6. Read the following numbers:

18.027	100.0067	400.0000019
81.0001	54.000054	3.03
75.075	9.2806	40.40404

147. From the foregoing explanations and illustrations we derive the following important principles:

PRINCIPLES OF DECIMAL NOTATION AND NUMERATION.

1. *The value of any decimal figure depends upon its place from the decimal point.*

Thus, .3 is ten times .03.

2. *Prefixing a cipher to a decimal decreases its value the same as dividing it by ten.*

Thus, .03 is $\frac{1}{10}$ the value of .3.

3. *Annexing a cipher to a decimal does not alter its lue, since it does not change the place of the significant ures of the decimal.*

Thus, $\frac{6}{10}$, or .6, is the same as $\frac{60}{100}$, or .60.

4. *Decimals increase from right to left, and decrease m left to right, in a tenfold ratio; and therefore they y be added, subtracted, multiplied, and divided in the ne manner as whole numbers.*

5. *The denominator of a decimal, though never expressed, always the unit* 1, *with as many ciphers annexed as re are figures in the decimal.*

3. *To read decimals requires two numerations; first, m units, to find the name of the denominator, and ond, towards units, to find the value of the numerator.*

RULE FOR DECIMAL NOTATION. — I. *Write the decimal the same way as a whole number, placing ciphers where essary to give each significant figure its true local value.*
II. *Place the decimal point before the first figure.*

RULE FOR DECIMAL NUMERATION. — I. *Numerate m the decimal point, to determine the denominator.*
II. *Numerate towards the decimal point, to determine the nerator.*
III. *Read the decimal as a whole number, giving it the ne or denomination of the right hand figure.*

1. Write 425 millionths.
2. Write six thousand ten-thousandths.
3. Write one thousand eight hundred fifty-nine hun-ed-thousandths.
4. Write 260 thousand 8 billionths.
5. Write 26 thousand and 26 thousandths.
6. Write 1 million and 1 millionth.

7. Read the following decimals:

.6321	.748243	.2962999
.5400027	.60000000	.00000006

8. Write five hundred two, and one thousand millionths.

9. Write thirty-one, and two ten-millionths.

10. Write eleven thousand, and eleven hundr thousandths.

11. Write nine million, and nine billionths.

12. Write one hundred two tenths. *Ans.* 10.

13. Write one hundred twenty-four thousand thr hundred fifteenth thousandths.

14. Write seven hundred thousandths.

15. Write seven hundred-thousandths.

16. Read the following numbers:

12.36	9.052	62.9999
142.847	32.004	1858.4583
1.02	4.0005	27.00045
305.005	14.342	3489.9843
4000.004	634.0634	2.000002
16.0902	5.00005	80000.567890
568.070	500.50000	3.0802050
.9095	33.33333	18752.5439082

17. Write three thousand two hundred five, and fi hundred six hundred-thousandths.

18. Write five twenty-five and five thousand forty-fi ten-thousandths.

19. Write seven ninety-seven, and six thousand thr forty-nine hundred-thousandths.

20. Write three million six hundred thousand, a nine ten-millionths.

21. Write two, and four million one twenty-thr *thousand* four ninety-two ten-millionths.

REDUCTION.

EXAMPLES.

148. To reduce decimals to a common denominator.

1. Reduce .5, .375, .3.25401, and 46.13 to their least common decimal denominator.

OPERATION.

.50000
.37500
3.25401
46.13000

SOLUTION. — The third number contains five decimal places, and hence 100000 must be a common denominator. As annexing ciphers to decimals does not alter their value (**147**, 3), we give to each number five decimal places by annexing ciphers, and thus reduce the given decimals to a common denominator.

RULE. — *Give to all the numbers the same number of decimal places, by annexing ciphers if necessary.*

1. If the numbers are reduced to the denominator of that one of the given numbers having the greatest number of decimal places, they will have their least common decimal denominator.

2. A whole number may readily be reduced to decimals by placing the decimal point after units, and annexing ciphers; one cipher reducing it to *tenths*, two ciphers to *hundredths*, three ciphers to *thousandths*, and so on.

2. Reduce .17, 24.6, .0003, 84, and 721.8000271 to their least common denominator.

3. Reduce 7 tenths, 24 thousandths, 187 millionths, 5 hundred millionths, and 10845 hundredths to their least common denominator.

4. Reduce to their least common denominator the following decimals: 1000.001, 841.78, 2.6004, 90.000009, and 6000.035.

5. Reduce the following decimals to their least common denominator: 5.05, .006, 200.5632, 5000.9, .352186, .799, and .89432.

6. Reduce .63, 57.56, .297, and .123456789 to their least common denominator.

149. To reduce a decimal to a common fraction.

1. Reduce .75 to its equivalent common fraction.

OPERATION.

$.75 = \frac{75}{100} = \frac{3}{4}$.

SOLUTION. — We omit the decimal point, supply the proper denominator to the decimal, and then reduce the common fraction thus formed to its lowest terms.

RULE. — *Omit the decimal point, and supply the proper denominator.*

2. Reduce .125 to a common fraction. *Ans.* $\frac{1}{8}$.
3. Reduce .16 to a common fraction. *Ans.* $\frac{4}{25}$.
4. Reduce .655 to a common fraction. *Ans.* $\frac{131}{200}$.
5. Reduce .9375 to a common fraction. *Ans.* $\frac{15}{16}$.
6. Reduce .0008 to a common fraction. *Ans.* $\frac{1}{1250}$.

150. To reduce a common fraction to a decimal.

1. Reduce $\frac{21}{40}$ to thousandths.

OPERATION.

$$\begin{array}{r} 40)21.000 \\ \hline .525 \end{array}$$

SOLUTION. — $\frac{21}{40}$ is $\frac{1}{40}$ of 21. 21 = 21 thousand thousandths, or 21.000; $\frac{1}{40}$ of 21.000 = 21.000 ÷ 40 = .525. ∴ $\frac{21}{40}$ = .525.

2. Reduce $\frac{1}{16}$ to its equivalent decimal.

OPERATION.

$$\begin{array}{r} 16)1.0000 \\ \hline .0625, \textit{Ans.} \end{array}$$

SOLUTION. — Dividing as in the former example, we obtain a quotient of 3 figures, 625. But since we annexed 4 ciphers, there must be 4 places in the required decimal; hence we prefix 1 cipher.

RULE. — I. *Annex ciphers to the numerator, and divide by the denominator.*

II. *Point off as many decimal places in the result as are equal to the number of ciphers annexed.*

Common fractions in their lowest terms can be reduced to exact or perfect decimals when their denominators contain only the prime factors 2 and 5, and not otherwise. A perfect decimal is called a *finite* decimal. When a decimal is not exact, the remainder is usually indicated by a common fraction or by the sign +.

The following table of common fractions with their decimal equivalents should be carefully committed to memory:

$\frac{1}{2} = .50.$	$\frac{4}{5} = .80.$	$\frac{6}{7} = .85\frac{5}{7}.$	$\frac{5}{9} = .55\frac{5}{9}.$
$\frac{1}{3} = .33\frac{1}{3}.$	$\frac{1}{6} = .16\frac{2}{3}.$	$\frac{1}{8} = .12\frac{1}{2}.$	$\frac{7}{9} = .77\frac{7}{9}.$
$\frac{2}{3} = .66\frac{2}{3}.$	$\frac{5}{6} = .83\frac{1}{3}.$	$\frac{3}{8} = .37\frac{1}{2}.$	$\frac{8}{9} = .88\frac{8}{9}.$
$\frac{1}{4} = .25.$	$\frac{1}{7} = .14\frac{2}{7}.$	$\frac{5}{8} = .62\frac{1}{2}.$	$\frac{1}{11} = .09\frac{1}{11}.$
$\frac{3}{4} = .75.$	$\frac{2}{7} = .28\frac{4}{7}.$	$\frac{7}{8} = .87\frac{1}{2}.$	$\frac{1}{12} = .08\frac{1}{3}.$
$\frac{1}{5} = .20.$	$\frac{3}{7} = .42\frac{6}{7}.$	$\frac{1}{9} = .11\frac{1}{9}.$	$\frac{1}{16} = .06\frac{1}{4}.$
$\frac{2}{5} = .40.$	$\frac{4}{7} = .57\frac{1}{7}.$	$\frac{2}{9} = .22\frac{2}{9}.$	$\frac{1}{20} = .05.$
$\frac{3}{5} = .60.$	$\frac{5}{7} = .71\frac{3}{7}.$	$\frac{4}{9} = .44\frac{4}{9}.$	$\frac{1}{25} = .04.$

3. Reduce $\frac{1}{3}$ to a decimal.

OPERATION.

$$3\overline{)1.0000}$$
$$.3333\frac{1}{3}$$

SOLUTION. — Proceeding as before, we find that each successive figure in the quotient is 3 with a remainder of $\frac{1}{3}$, which, reduced to the next lower denomination, gives another quotient of 3 with a remainder of $\frac{1}{3}$. Hence we cannot reduce $\frac{1}{3}$ to an exact decimal, and the answer must be an *infinite number* of 3's with a remainder.

4. Reduce $\frac{19}{37}$ to a decimal.

OPERATION.

$$37\overline{)19.000000}$$
$$.513513\frac{19}{37}$$

Or,

$$.513513513513+$$

SOLUTION. — Proceeding as before and performing the division to millionths, we obtain for a quotient $.513513\frac{19}{37}$. This is not an exact decimal, but since the remainder $\frac{19}{37}$ is the same as the fraction originally reduced, we know that the quotient figures of the lower denominations will be 513 repeated. Hence we may add as many of these figures as we desire, adding the sign +, after the last.

5. Reduce $\frac{1}{6}$ to millionths.

OPERATION.

$$6\overline{)1.000}$$
$$.166666+$$

Or, $.1\dot{6}$

SOLUTION. — Proceeding as before, we have for a quotient .166 with a remainder, and we readily see that this remainder reduced to any number of lower denominations will repeat the quotient figure 6. Hence we add as many 6's as are necessary to reduce the number to the required denomination, and place the sign +, after the result.

151. Decimals like those in Examples 3, 4, 5, in which certain figures or sets of figures constantly repeat, are called repeating decimals, or **circulating decimals.**

The figure, or set of figures, repeated is called the **repetend.** A repetend need be written but once, and when it consists of a single figure a point is placed over it. When it consists of more than one figure, points are placed over the first and last figures.

Thus, .666 is written $.\dot{6}$; .297297 is written $.\dot{2}9\dot{7}$.

When the decimal commences with a repetend, as .3 in Example 3 and .513 in Example 4, it is called a pure circulating decimal; but when it is preceded by one or more decimal figures which do not repeat, as by .1 in Example 5, it is called a mixed circulating decimal.

Every pure circulating decimal is equal to a fraction whose numerator is the repetend and whose denominator consists of as many 9's as there are places in the repetend.

Thus $.3+ = \frac{3}{9}$, or $\frac{1}{3}$; $.3333+ = \frac{3333}{9999}$; $.5555+ = \frac{5555}{9999}$, etc.

6. Reduce $\frac{2}{25}$ to a decimal.
7. Reduce $\frac{3}{64}$ to a decimal.
8. Reduce $\frac{3}{80}$ to a decimal.
9. Reduce $\frac{3}{125}$ to a decimal.
10. Reduce $\frac{3}{800}$ to a decimal.
11. Reduce $\frac{7}{27}$ to a pure circulating decimal.

Ans. $.\dot{2}5\dot{9}$.

12. Reduce $\frac{5}{6}$ to a mixed circulating decimal.

Ans. $.8\dot{3}$.

13. Reduce $.\dot{7}$ to a common fraction. *Ans.* $\frac{7}{9}$.
14. Reduce $.\dot{2}1\dot{6}$ to a common fraction. *Ans.* $\frac{216}{999} = \frac{8}{37}$.
15. Change $.2\dot{9}\dot{7}$ to a common fraction. *Ans.* $\frac{11}{37}$.
16. Change $.\dot{1}42857\dot{1}$ to a common fraction. *Ans.* $\frac{1}{7}$.
17. Reduce $\frac{1}{9}$ to a decimal. *Ans.* $.\dot{1}$.
18. Reduce $\frac{2}{9}$ to a decimal; $\frac{3}{9}$; $\frac{4}{9}$.
19. Reduce $\frac{1}{99}$ to a decimal; $\frac{5}{99}$. *Ans.* $.\dot{0}\dot{1}$; $.\dot{0}\dot{5}$.
20. Reduce $\frac{1}{999}$ to a decimal. *Ans.* $.\dot{0}0\dot{1}$.
21. Change $\frac{7}{9}$ to a pure circulating decimal. *Ans.* $.\dot{7}$.
22. Change $\frac{12}{37}$ to a pure circulating decimal. *Ans.* $.\dot{3}2\dot{4}$.
23. Change $\frac{47}{150}$ to a mixed circulating decimal. *Ans.* $.31\dot{3}$.
24. Reduce $\frac{97}{160}$ to a decimal.
25. Reduce $\frac{13}{256}$ to a decimal.
26. Reduce $\frac{18}{52}$ to a decimal.
27. Reduce $.3\dot{2}\dot{4}$ to a common fraction.
28. Reduce $.\dot{0}11\dot{7}$ to a common fraction.
29. Reduce $\frac{226}{1180}$ to a decimal.

Tell by inspection the fractional equivalents of:

30. $.66\frac{2}{3}$; $.57\frac{1}{7}$; $.09\frac{1}{11}$; $.16\frac{2}{3}$.
31. $.40$; $.42\frac{8}{7}$; $.33\frac{1}{3}$; $.80$.
32. $.11\frac{1}{9}$; $.08\frac{1}{3}$; $.55\frac{5}{9}$; $.04$.
33. $.87\frac{1}{2}$; $.50$; $.44\frac{4}{9}$; $.06\frac{1}{4}$.
34. $.83\frac{1}{3}$; $.71\frac{3}{7}$; $.62\frac{1}{2}$; $.28\frac{4}{7}$.
35. $.77\frac{7}{9}$; $.85\frac{5}{7}$; $.05$; $.37\frac{1}{2}$.
36. $.14\frac{2}{7}$; $.22\frac{2}{9}$; $.75$; $.12\frac{1}{2}$.
37. $.60$; $.20$; $.88\frac{8}{9}$; $.25$.

Give instantly the decimal equivalents of:

38. $\frac{1}{2}$; $\frac{4}{5}$; $\frac{6}{7}$; $\frac{5}{9}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{1}{8}$; $\frac{7}{9}$.
39. $\frac{2}{3}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{8}{9}$; $\frac{1}{4}$; $\frac{1}{7}$; $\frac{5}{8}$; $\frac{1}{11}$.
40. $\frac{3}{4}$; $\frac{2}{7}$; $\frac{7}{8}$; $\frac{1}{12}$; $\frac{1}{5}$; $\frac{3}{7}$; $\frac{1}{9}$; $\frac{1}{16}$.
41. $\frac{2}{5}$; $\frac{4}{7}$; $\frac{2}{9}$; $\frac{1}{20}$; $\frac{3}{5}$; $\frac{5}{7}$; $\frac{4}{9}$; $\frac{1}{25}$.

ADDITION.

EXAMPLES.

152. 1. What is the sum of 3.703, 621.57, .672, and 20.0074?

OPERATION.

3.703
621.57
.672
20.0074
645.9524

SOLUTION. — We write the numbers so that figures of like orders of units stand in the same column; that is, units under units, tenths under tenths; hundredths under hundredths, etc. This brings the decimal points directly under one another. Commencing at the right hand we add each column separately, carry as in whole numbers, and in the result place a decimal point between units and tenths.

RULE. — I. *Write the numbers so that the decimal points stand directly under each other.*

II. *Add as in whole numbers, and place the decimal point, in the result, directly under the points in the numbers added.*

2. Add .199, 2.7569, .25, .654. Sum, 3.8599

3. Add 4.015, 6.75, 27.38203, 375.01, 2.5. Amount, 415.65703

4. Add 1152.01, 14.11018, 152348.21, 9.000083.

Ans. 153523.330263.

5. Add 37.03, .521, .9, 1000, 4000.0004.

Ans. 5038.4514.

6. What is the sum of twenty-six, and twenty-six hundredths; seven tenths; six, and eighty-three thousandths; four, and four thousandths? *Ans.* 37.047.

7. What is the sum of thirty-six, and fifteen thousandths; three hundred, and six hundred five ten-

thousandths; five, and three millionths; sixty, and eighty-seven ten-millionths? *Ans.* 401.0755117.

8. What is the sum of fifty-four, and thirty-four hundredths; one, and nine ten-thousandths; three, and two hundred seven millionths; twenty-three thousandths; eight, and nine tenths; four, and one hundred thirty-five thousandths? *Ans.* 71.399107.

9. How many yards are there in three pieces of cloth, the first piece containing 18.375 yards, the second piece 41.625 yards, and the third piece 35.5 yards?

10. A's farm contains 61.843 acres, B's contains 143.75 acres, C's 218.4375 acres, and D's 21.9 acres. How many acres are there in the four farms?

11. My farm consists of 7 fields, containing $12\frac{3}{4}$ acres, $18\frac{2}{5}$ acres, 9 acres, $24\frac{1}{8}$ acres, $4\frac{13}{16}$ acres, $8\frac{9}{10}$ acres, and $15\frac{13}{20}$ acres respectively. How many acres are there in my farm? *Ans.* 93.6375.

Reduce the common fractions to decimals before adding.

12. A grocer has $2\frac{1}{2}$ barrels of A sugar, $5\frac{3}{4}$ barrels of B sugar, $3\frac{5}{8}$ barrels of C sugar, 3.0642 barrels of crushed sugar, and 8.925 barrels of pulverized sugar. How many barrels of sugar has he? *Ans.* 23.8642.

13. A tailor made 3 suits of clothes; for the first suit he used $2\frac{1}{8}$ yards of broadcloth, $3\frac{1}{16}$ yards of cassimere, and $\frac{7}{8}$ yard of satin; for the second suit 2.25 yards of broadcloth, 2.875 yards of cassimere, and 1 yard of satin; and for the third suit $5\frac{1}{16}$ yards of broadcloth, and $1\frac{1}{8}$ yards of satin. How many yards of each kind of goods did he use? How many yards of all?

Ans. to last, 18.375 yards.

14. A dressmaker bought in the course of a week $3\frac{3}{8}$ yards cloth, $5\frac{5}{8}$ yards, $9\frac{7}{8}$ yards, $4\frac{1}{6}$ yards, and $7\frac{5}{8}$ yards. How much did she buy altogether during the week?

Ans. 30.87½ yards.

SUBTRACTION.

EXAMPLES.

153. 1. From 91.73 take 2.18.

OPERATION.

91.73
2.18
Ans. 89.55

2. From 2.9185 take 1.42.

OPERATION.

2.9185
1.42
Ans. 1.4985

3. From 124.65 take 95.58746.

OPERATION.

124.65
95.58746
Ans. 29.06254

SOLUTION. — In each of these three examples, we write the subtrahend under the minuend, placing units under units, tenths under tenths, etc. Commencing at the right hand, we subtract as in whole numbers, and in the remainders we place the decimal points directly under those in the numbers above. In the second example, the number of decimal places in the minuend is greater than the number in the subtrahend, and in the third example the number is less. In each case we reduce both minuend and subtrahend to the same number of decimal places, by supposing ciphers to be annexed, before performing the subtraction.

RULE. — I. *Write the numbers so that the decimal points stand directly under one another.*

II. *Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

4. Find the difference between 714 and .916.
5. How much greater is 2 than .298? *Ans.* 1.702.
6. From 21.004 take 75 hundredths. *Ans.* 20.254.
7. From 10.0302 take two ten-thousandths.
8. From 900 take .009. *Ans.* 899.991.
9. From two thousand take two thousandths.
10. From one take one millionth. *Ans.* .999999.

MULTIPLICATION.

EXAMPLES.

154. -1. What is the product of .35 multiplied by .5?

OPERATION.

.35
.5
.175, *Ans.*

SOLUTION. — We perform the multiplication in the same manner as in whole numbers. To determine how many places to point off, we may reduce the decimals to common fractions; thus, $.35 = \frac{35}{100}$ and $.5 = \frac{5}{10}$. Performing the multiplication, we have $\frac{35}{100} \times \frac{5}{10} = \frac{175}{1000}$, and this product, expressed decimally, is .175. Here we see that the product contains as many decimal places as are contained in both multiplicand and multiplier.

RULE. — *Multiply as in whole numbers, and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply a decimal by 10, 100, 1000, etc., remove the point as many places to the right as there are ciphers on the right of the multiplier.

2. Multiply 1.245 by .27. *Ans.* .33615.

3. Multiply 79.347 by 23.15. *Ans.* 1836.88305.

4. Multiply 350 by .7853.

5. Multiply one tenth by one tenth. *Ans.* .01.

6. Multiply 25 by twenty-five hundredths. *Ans.* 6.25.

7. Multiply .132 by .241. *Ans.* .031812.

8. Multiply 24.35 by 10.

9. Multiply .006 by 1000. *Ans.* 6.

10. Multiply .23 by .009. *Ans.* .00207.

11. Multiply sixty-four thousandths by thirteen millionths. *Ans.* .000000832.

12. Multiply eighty-seven ten-thousandths by three hundred fifty-two hundred-thousandths.

13. Multiply one million by one millionth. *Ans.* 1.

14. Multiply sixteen thousand by sixteen ten-thousandths. *Ans.* 25.6.

DIVISION.

EXAMPLES.

155. 1. What is the quotient of .175 divided by .5?

OPERATION.

.5)$\overline{.175}$

.35, *Ans.*

SOLUTION. — We perform the division in the same manner as in whole numbers. To determine how many places to point off, we may reduce the decimals to common fractions; thus, $.175 = \frac{175}{1000}$, and $.5 = \frac{5}{10}$. Performing the division, we have

$$\frac{175}{1000} \div \frac{5}{10} = \frac{\overset{35}{\cancel{175}}}{\underset{\cancel{\emptyset}}{100}} \times \frac{\cancel{10}}{\cancel{5}} = \frac{35}{100};$$

and this quotient, expressed decimally, is .35. Here we see that the dividend contains as many decimal places as are contained in both divisor and quotient.

RULE. — *Divide as in whole numbers, and from the right hand of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient is less than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by prefixing ciphers.

2. If there is a remainder after dividing the dividend, annex ciphers, and continue the division; the ciphers annexed are decimals of the dividend.

3. The dividend must always contain at least as many decimal places as the divisor, before commencing the division.

4. In most business transactions, the division is considered sufficiently exact when the quotient is carried to 4 decimal places, unless great accuracy is required.

5. To divide by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers on the right hand of the divisor.

Divide:

2. .675 by .15. *Ans.* 4.5.
3. .288 by 3.6. *Ans.* .08.
4. 81.6 by 2.5.
5. 2.3421 by 21.1.
6. 2.3421 by .211.
7. 8.297496 by .153.
8. 12 by .7854.
9. 15.34 by 2.7.
10. 785.4 by 1000.
11. .1 by .7.
12. 45.30 by .015.
13. .003753 by 625.5.
14. 9 by 450.
15. 2.39015 by .007.
16. 365 by 100.
17. 1000 by .001.

PROMISCUOUS EXAMPLES.

1. Add six hundred, and twenty-five thousandths; four tenths; seven, and sixty-two ten-thousandths; three, and fifty-eight millionths; ninety-two, and seven hundredths. *Ans.* 702.501258.

2. What is the sum of $81.003 + 5000.4 + 5.0008 + 73.87563 + 1000 + 25 + 3.000548 + .0315$?

3. From eighty-seven take eighty-seven thousandths.

4. What is the difference between nine million and nine millionths? *Ans.* 8999999.999991.

5. Multiply .365 by .15. *Ans.* .05475.

6. Multiply three thousandths by four hundredths.

7. If one acre produces 42.57 bushels of corn, how many bushels will 18.73 acres produce? *Ans.* 797.3361.

8. Divide .125 by 8000. *Ans.* .000015625.

9. Divide .7744 by .1936.

10. Divide 27.1 by 100000. *Ans.* .000271.

11. If 6.35 acres produce 70.6755 bushels of wheat, what does one acre produce? *Ans.* 11.13 bushels.

12. Reduce .625 to a common fraction. *Ans.* $\frac{5}{8}$.

13. Express 26.875 by an integer and a common fraction. *Ans.* $26\frac{7}{8}$.

14. Reduce $\frac{2}{125}$ to a decimal fraction. *Ans.* .016.

15. Reduce $\frac{8\frac{3}{4}}{17\frac{1}{2}}$ to a decimal fraction. *Ans.* .5.

16. How many times will .5 of 1.75 be contained in .25 of $17\frac{1}{2}$? *Ans.* 5.

17. What will be the cost of $3\frac{5}{8}$ bales of cloth, each bale containing 36.75 yards, at $\$.85$ per yard?

18. Traveling at the rate of $4\frac{3}{5}$ miles an hour, how many hours will a man require to travel 56.925 miles?

19. Change $\frac{9}{22}$ to a mixed circulating decimal.

20. Change $\frac{5}{21}$ to a pure circulating decimal.

DECIMAL CURRENCY.

156. Coin is money stamped, and has a given value established by law.

157. Currency is coin, bank bills, treasury notes, etc., in circulation as a medium of trade.

A *Decimal Currency* is a currency whose denominations increase and decrease in a tenfold ratio.

The currency of the United States is decimal currency, and is sometimes called *Federal Money;* it was adopted by Congress in 1786.

NOTATION AND NUMERATION.

158. The **Coin** of the United States consists of gold, silver, nickel, and bronze.

The Gold Coins are the double-eagle, eagle, half-eagle, quarter-eagle.

The Silver Coins are the dollar, half-dollar, quarter-dollar, and the ten-cent pieces.

The Nickel Coin is the five-cent piece.

The Bronze Coin is the one-cent piece.

TABLE.

10 mills (*m.*)	make	1 cent	c.
10 cents	"	1 dime	d.
10 dimes	"	1 dollar	$.
10 dollars	"	1 eagle	E.

1. The mill is a denomination used only in computations; it is not a coin.
2. The character $ is supposed to be a contraction of U. S. (United States), the S being placed upon the U.
3. All gold and silver coins must consist of 9 parts pure metal and 1 part alloy.

159. The gold dollar, weighing 25.8, is the *unit* of U. S. money; dimes, cents, and mills are fractions of a dollar, and are separated from the dollar by the decimal point.

Thus, $2.125 = two dollars one dime two cents five mills.

Dimes are not read as *dimes*, but the two places of dimes and cents are appropriated to cents; thus, 1 dollar 3 dimes 2 cents, or $1.32, is read one dollar thirty-two cents; hence,

When the number of cents is less than 10, we *write a cipher before it in the place of dimes.*

The half-cent is frequently written as 5 mills; thus, 24½ cents, written $.245.

Business men frequently write *cents* as common fractions of a dollar; thus, three dollars thirteen cents are written 3\frac{13}{100}$, and read, three and thirteen hundredths dollars. In business transactions, when the final result of a computation contains 5 mills or *more*, these mills are regarded as one cent, and when it contains *less* than 5 mills, they are rejected.

160. By examining the *table*, we see that the *dime* is a *tenth* part of the unit, or dollar; the *cent* a tenth part of the dime, or a *hundredth* part of the dollar; and the *mill* a tenth part of the cent, etc.

Hence we see that the denominations of decimal currency increase and decrease in the same way as decimal fractions, and are expressed according to the same decimal system of notation. They are therefore added, subtracted, multiplied, and divided in the same manner as decimals.

EXAMPLES.

Write:

1. Four dollars five cents.
2. Two dollars nine cents.
3. Ten dollars ten cents.
4. Eight dollars seven mills.
5. Sixty-four cents.
6. Three cents two mills.
7. Read $5.272; $100.025; $17.005.
8. Read $16.205; $215.081; $1000.011; $4.002.

REDUCTION.

161. By examining the table of Decimal Currency, we see that 10 mills make one cent, and 100 cents, or 1000 mills, make one dollar; hence,

RULE. — I. *To change dollars to cents, multiply by* 100; *that is, annex two ciphers.*

II. *To change dollars to mills, annex three ciphers.*

III. *To change cents to mills, annex one cipher.*

EXAMPLES.

1. Change $792 to cents. *Ans.* 79200 cents.
2. Change $36 to cents.
3. Reduce $5248 to cents.
4. Change $6.25 to cents *Ans.* 625 cents.

To change dollars and cents to cents, or dollars, cents, and mills to mills, remove the decimal point and the sign, $.

5. Change $63.045 to mills. *Ans.* 63045 mills.
6. Change 16 cents to mills.
7. Reduce $3.008 to mills.
8. Change 89 cents to mills.

162. Conversely,

RULE. — I. *To change cents to dollars, divide by* 100; *that is, point off two figures from the right.*

II. *To change mills to dollars, point off three figures.*

III. *To change mills to cents, point off one figure.*

1. Change 875 cents to dollars. *Ans.* $8.75.
2. Change 1504 cents to dollars.
3. In 13875 cents how many dollars are there?
4. In 16525 mills how many dollars are there?
5. Reduce 524 mills to cents.
6. Reduce 6524 mills to dollars.
7. Reduce $77.09 to cents.

ADDITION.

EXAMPLES.

163. 1. A man bought a cow for $21.50, a horse for $125.37½, a harness for $46.75, and a carriage for $210. How much did he pay for all?

OPERATION.

$21.50	
125.375	
46.75	
210.00	
Ans. $403.625	

SOLUTION. — Writing dollars under dollars, cents under cents, etc., so that the decimal points stand under one another, we add and point off as in addition of decimals.

RULE. — I. *Write dollars under dollars, cents under cents, etc.*

II. *Add as in simple numbers, and place the point in the amount as in addition of decimals.*

2. What is the sum of 50 dollars 7 cents, 1000 dollars 75 cents, 60 dollars 3 mills, 18 cents 4 mills, 1 dollar 1 cent, and 25 dollars 45 cents 8 mills? *Ans.* $1137.475.

3. Add 364 dollars 54 cents 1 mill, 486 dollars 6 cents, 93 dollars 9 mills, 1742 dollars 80 cents, 3 dollars 27 cents 6 mills. *Ans.* $2689.686.

4. Add 92 cents, 10 cents 4 mills, 35 cents 7 mills, 18 cents 6 mills, 44 cents 4 mills, 12½ cents, and 99 cents.

5. A lady bought a dress for 9 dollars 17 cents, trimmings for 87½ cents, a paper of pins for 6¼ cents, some tape for 4 cents, some thread for 8 cents, and a comb for 11 cents. What did she pay for all? *Ans.* $10.3375.

6. A farmer receives $89.74 for wheat, $13.03 for corn, $6.37½ for potatoes, and $19.62½ for oats. What does he receive for the whole? *Ans.* $128.77.

7. I bought a ton of coal for $6.08, a barrel of sugar for $26.625, a box of tea for $16, and a barrel of flour for $7.40. What was the cost of all?

SUBTRACTION.

EXAMPLES.

164. 1. A man, having $327.50, paid out $186.75 for a horse. How much had he left?

OPERATION.

$327.50
186.75
Ans. $140.75

SOLUTION. — Writing the less number under the greater, dollars under dollars, cents under cents, etc., we subtract and point off in the result as in subtraction of decimals.

RULE. — I. *Write the subtrahend under the minuend, dollars under dollars, cents under cents.*

II. *Subtract as in simple numbers, and place the point in the remainder, as in subtraction of decimals.*

2. From 365 dollars 5 mills take 267 dollars 1 cent 8 mills. *Ans.* $97.987.

3. From 50 dollars take 50 cents. *Ans.* $49.50.

4. From 100 dollars take 1 mill. *Ans.* $99.999.

5. From 1000 dollars take 3 cents 7 mills.

6. A man bought a farm for $1575.24, and sold it for $1834.16. What did he gain? *Ans.* $258.92.

7. I sold a horse for 145 dollars 27 cents, which is 37 dollars 69 cents more than he cost me. What did he cost me?

8. A merchant bought flour for $5.62½ a barrel, and sold it for $6.84 a barrel. What did he gain on a barrel?

9. A man, having $14725, gave $3560 for a store, and $7015.87½ for goods. How much money had he left?

10. A lady bought a silk dress for $13¾, a bonnet for $5¼, a pair of gaiters for $1⅛, and a fan for $⅞; she gave the shopkeeper a twenty dollar bill and a five dollar bill. How much change should he return to her? *Ans.* $3.75.

Reduce the fractions of a dollar to cents and mills.

MULTIPLICATION.

EXAMPLES.

165. 1. If a barrel of flour costs $6.375, what will 85 barrels cost?

OPERATION.

$6.375
85
31875
51000
Ans. $541.875

SOLUTION. — We multiply as in simple numbers, always regarding the multiplier as an *abstract* number, and point off from the right hand of the result, as in multiplication of decimals.

RULE. — *Multiply as in simple numbers, and place the point in the product, as in multiplication of decimals.*

2. If a cord of wood is worth $4.275, what will 300 cords be worth? *Ans.* $1282.50.

3. What will 175 barrels of apples cost, at $2.45 per barrel? *Ans.* $428.75.

4. What will 800 barrels of salt cost, at $1.28 per barrel?

5. A grocer bought 372 pounds of cheese at $.15 a pound, 434 pounds of coffee at $.12½ a pound, and 16 bushels of potatoes at $.33 a bushel. What did the whole cost?

6. A boy, being sent to purchase groceries, bought 3 pounds of tea at 56 cents a pound, 15 pounds of rice at 7 cents a pound, 27 pounds of sugar at 8 cents a pound; he gave the grocer 5 dollars. How much change ought he to receive?

7. A farmer sold 125 bushels of oats at $.37½ a bushel, and received in payment 75 pounds of sugar at $.09 a pound, 12 pounds of tea at $.60 a pound, and the remainder in cash. How much cash did he receive?

Ans. $32.92½.

DIVISION.

EXAMPLES.

166. 1. If 125 barrels of flour cost $850, how much will 1 barrel cost?

OPERATION.

125) $850.00 ($6.80, *Ans.*
750
1000
1000
0

SOLUTION. — We divide as in simple numbers, and as there is a remainder after dividing the dollars, we reduce the dividend to cents, by annexing two ciphers, and continue the division.

RULE. — *Divide as in simple numbers, and place the point in the quotient, as in division of decimals.*

1. In business transactions it is never necessary to carry the division further than to mills in the quotient.

2. If the dividend will not contain the divisor an exact number of times, ciphers may be annexed, and the division continued as in division of decimals. In this case it is always safe to reduce the dividend to mills, or to 3 more decimal places than the divisor contains, before commencing the division.

2. If 33 gallons of oil cost $41.25, what is the cost per gallon? *Ans.* $1.25.

3. If 27 yards of broadcloth cost $94.50, what will 1 yard cost?

4. If 64 gallons of olives cost $136, what will 1 gallon cost? *Ans.* $2.125.

5. At 12 cents apiece, how many pineapples can be bought for $1.32? *Ans.* 11.

6. If 1 pound of tea costs 54 cents, how many pounds can be bought for $405?

7. If a man earns $1800 in a year, how much does he earn in a month?

8. If 100 acres of land cost $2847.50, what will 1 acre cost? *Ans.* $28.475.

9. What will 1 pound of beef cost, if 894 pounds cost $80.46? *Ans.* $.09

ADDITIONAL APPLICATIONS.

167. An **Aliquot Part** of a number is such a part as will exactly divide that number.

Thus, 3, 5, and $7\frac{1}{2}$ are aliquot parts of 15.

An *aliquot part* may be a whole or a mixed number, while a *factor* must be a whole number.

ALIQUOT PARTS OF ONE DOLLAR.

50 cents $= \frac{1}{2}$ of \$1.	$12\frac{1}{2}$ cents $= \frac{1}{8}$ of \$1.
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of \$1.	$11\frac{1}{9}$ cents $= \frac{1}{9}$ of \$1.
25 cents $= \frac{1}{4}$ of \$1.	10 cents $= \frac{1}{10}$ of \$1.
20 cents $= \frac{1}{5}$ of \$1.	$8\frac{1}{3}$ cents $= \frac{1}{12}$ of \$1.
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of \$1.	$6\frac{1}{4}$ cents $= \frac{1}{16}$ of \$1.
$14\frac{2}{7}$ cents $= \frac{1}{7}$ of \$1.	5 cents $= \frac{1}{20}$ of \$1.

EXAMPLES.

168. To find the cost of any number or quantity, when the price of a unit is an aliquot part of one dollar.

1. What will be the cost of 3784 yards of flannel, at 25 cents a yard?

OPERATION.

4)3784

Ans. \$946

SOLUTION. — Since the price is $\frac{1}{4}$ of a dollar a yard, the whole cost will be $\frac{1}{4}$ as many dollars as there are yards; or, $\frac{1}{4}$ of \$3784 = \$946.

RULE. — *Take such a fractional part of the given number as the price is part of one dollar.*

2. What will 963 bushels of oats cost at $33\frac{1}{3}$ cents per bushel? *Ans.* \$321.

3. What is the cost of 478 yards of delaine, at 50 cents per yard?

4. What is the cost of 4266 yards of sheeting, at $8\frac{1}{3}$ cents a yard? *Ans.* \$355.50.

5. What is the cost of 1250 bushels of apples at $12\frac{1}{2}$ cents per bushel? *Ans.* \$156.25.

6. What will 3126 spools of thread cost, at $6\frac{1}{4}$ cents per spool? *Ans.* $195.375.

7. At $16\frac{2}{3}$ cents per dozen, what do 1935 dozen eggs cost? *Ans.* $322.50.

8. What is the cost of 56480 yards of calico, at $12\frac{1}{2}$ cents per yard?

169. The price of one and the quantity being given, to find the cost.

1. How much will 9 barrels of flour cost, at $6.25 per barrel?

OPERATION.

$6.25
9
Ans. $56.25

SOLUTION. — Since one barrel costs $6.25, 9 barrels will cost 9 times $6.25; $6.25 × 9 = $56.25.

RULE. — *Multiply the price of one by the quantity.*

2. If a pound of beef costs 9 cents, what will 864 pounds cost? *Ans.* $77.76.

3. What do 87 acres of government land cost at $1.25 per acre?

4. What will be the cost of 400 barrels of salt, at $1.45 per barrel? *Ans.* $580.

5. What do 16 chests of tea cost, each chest containing 52 pounds, at 44 cents per pound?

170. The cost and the quantity being given, to find the price of one.

1. If 30 bushels of corn cost $20.70, what will 1 bushel cost?

OPERATION.

30)$20.70
$.69

SOLUTION. — One bushel will cost $\frac{1}{30}$ of $20.70, which is $.69.

RULE. — *Divide the cost by the quantity.*

2. If 25 acres of land cost $175, what will 1 acre cost?

3. If 48 yards of broadcloth cost $200, what will 1 yard cost? *Ans.* $4.16⅔.

4. If 96 tons of hay cost $1200, what will 1 ton cost?

5. If 10 dictionaries cost $56.25, what will 1 cost? *Ans.* $5.62½.

6. A man bought 18 pounds of tea for $11.70. What was the price per pound? *Ans.* $.65.

7. If 53 pounds of butter cost $10.07, what will 1 pound cost?

8. A merchant bought 800 barrels of salt for $1016. What did it cost him per barrel?

9. If 343 sheep cost $874.65, what will 1 sheep cost? *Ans.* $2.55.

10. If board for a family is $684.37½ for 1 year, how much is it per day? *Ans.* $1.87½.

11. If 69 dozen eggs cost $17.25, what will 1 dozen cost? *Ans.* $.25.

171. The price of one and the cost of a quantity being given, to find the quantity.

1. At $6 a barrel for flour, how many barrels can be bought for $840?

OPERATION.

6)840

Ans. 140 barrels.

SOLUTION. — Since $6 will buy 1 barrel of flour, $840 will buy as many barrels as $6 is contained times in $840; 840 ÷ 6 = 140; $840 will buy 140 barrels.

RULE. — *Divide the cost of the quantity by the price of one.*

2. How many dozen eggs can be bought for $5.55, if one dozen costs $.15? *Ans.* 37 dozen.

3. At $12 a ton, how many tons of hay can be bought for $216? *Ans.* 18 tons.

4. How many bushels of wheat can be bought for $2178.75, if 1 bushel costs $1.25? *Ans.* 1743 bushels.

5. A dairyman expends $643.50 in buying cows at $19½ apiece. How many cows does he buy?

Ans. 33 cows.

6. At $.45 per gallon, how many gallons of molasses can be bought for $52.65?

7. A drover bought horses at $264 a pair. How many horses did he buy for $6336?

8. At $65 a ton, how many tons of railroad iron can be bought for $117715? *Ans.* 1811 tons.

172. To find the cost of articles sold by the 100, or 1000.

1. What do 475 feet of timber cost at $5.24 per 100 feet?

OPERATION.

$5.24
475
2620
3668
2096
100)$2489.00
Ans. $24.89

SOLUTION. — If the price were $5.24 per *foot*, the cost of 475 feet would be 475 × $5.24 = $2489. But since $5.24 is the price of 100 feet, $2489 is 100 times the true value. Therefore, to obtain the true value, we divide $2489 by 100, which we may do by cutting off two figures from the right, and the result is $24.89.

RULE. — I. *Reduce the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand.*

II. *Multiply the price by the quantity, and point off in the result as in multiplication of decimals.*

The letter C is used to indicate hundreds, and M to indicate thousands.

2. What will 42650 bricks cost, at $4.50 per M?

Ans. $191.925.

3. What is the freight on 2489 pounds from Boston to New York, at $.85 per 100 pounds? *Ans.* $21.156.

4. What will 7842 feet of pine boards cost, at $17.25 per M? *Ans.* $135.274.

5. What will 2348 pineapples cost, at $12½ per 1000?

6. A broom maker bought 1728 broom-handles, at $3 per 1000. What did they cost him?

7. What is the cost of 2400 feet of boards, at $7 per M; 865 feet of scantling, at $5.40 per M; and 1256 feet of lath, at $.80 per C? *Ans.* $31.519.

8. What will be the cost of 1476 pounds of beef, at $4.37½ per hundred pounds?

173. To find the cost of articles sold by the ton of 2000 pounds.

1. How much will 2376 pounds of hay cost, at $9.50 per ton?

OPERATION.

2)$9.50
$4.75
2.376
$11.28600

SOLUTION. — Since 1 ton, or 2000 pounds, costs $9.50, 1000 pounds, or ½ ton, will cost ½ of $9.50, which is $4.75. One pound will cost $\frac{1}{1000}$, or .001, of $4.75, and 2376 pounds will cost $\frac{2376}{1000}$, or 2.376 times $4.75, which is $11.286.

RULE. — I. *Divide the price of* 1 *ton by* 2, *and the quotient will be the price of* 1000 *pounds.*

II. *Multiply this quotient by the given number of pounds expressed as thousandths, as in* **172.**

2. At $7 a ton, what will 1495 pounds of hay cost?
Ans. $5.2325.

3. At $8.75 a ton, what will 325 pounds of hay cost?

4. What is the cost of 3142 pounds of plaster, at $3.84 per ton? *Ans.* $6.032.

5. What is the cost of 1848 pounds of coal, at $5.60 per ton?

6. A man bought 125 sacks of guano, each sack containing 148 pounds, at $18 a ton. What was the cost?

BILLS.

174. A **Bill,** in business transactions, is a written statement of articles bought or sold, together with the prices of each, and the whole cost.

Find the cost of the several articles, and the amount or footing of the following bills:

1.

New York, June 20, 1894.

Mr. John Rice,

Bought of Baldwin & Sherwood.

7	yd.	Broadcloth,	@	$3.60
9	"	Satinet,	"	1.12½
12	"	Vesting,	"	.90
24	"	Cassimere,	"	1.37½
32	"	Flannel,	"	.65
				$99.925

Received Payment,

Baldwin & Sherwood.

2.

Boston, Jan. 1, 1895.

Daniel Chapman & Co.,

Bought of Palmer & Brother.

67	pairs	Calf Boots,	@	$3.75
108	"	Thick "	"	2.62
75	"	Gaiters,	"	1.12
27	"	Buskins,	"	.86
35	"	Slippers,	"	.70
50	"	Rubbers,	"	1.04
				$717.93

Received Payment,

Palmer & Brother,

By Geo. Baker.

3.

CHARLESTON, Sept. 6, 1892.

G. B. GRANNIS,

Bought of STEWART & HAMMOND.

325 lb.	A. Sugar,	@	$	.05
148 "	B. "	"		.04½
286 "	Rice,	"		.05
95 "	O. J. Coffee	"		.28
50 boxes	Oranges,	"		2.75
75 "	Lemons,	"		3.62½

Received Payment, by note at 4 *mo.*,

STEWART & HAMMOND.

4.

ST. LOUIS, Oct. 15, 1896.

MESSRS. OSBORN & EATON,

Bought of ROBERT H. CARTER & Co.

20000 feet	Pine Boards,	@	$	15.00 per M.
7500 "	Plank,	"		9.50 "
10750 "	Scantling,	"		6.25 "
3960 "	Timber,	"		2.62½ "

Received Payment,

ROBERT H. CARTER & Co.

5.

CINCINNATI, May 3, 1893.

MR. J. C. SMITH,

Bought of SILAS JOHNSON.

26 lb.	Coffee Sugar,	@	$	.04½
5 "	Y. H. Tea,	"		.62½
26 "	Mackerel,	"		.06¼
4 gal.	Molasses,	"		.42
46 yds.	Sheeting,	"		.09
30 "	Bleached Shirting,	"		.11
			$	15.04

Charged in ⁄c.

PROMISCUOUS EXAMPLES.

1. What will 62.75 tons of potash cost, at $124.35 per ton? *Ans.* $7802.9625.

2. What will 15 pounds of butter cost, at $.17 a pound? *Ans.* $2.55.

3. A cargo of corn, containing 2250 bushels, was sold for $1406.25. What did it sell for per bushel? *Ans.* $\$\frac{5}{8}$.

4. If 12 yards of cloth cost $48.96, what will one yard cost?

5. A merchant bought 16 bales of cotton cloth, each bale containing 13 pieces, and each piece 26 yards, at $.07 per yard. What did the whole cost? *Ans.* $378.56.

6. At $.12$\frac{1}{2}$ a yard, how many yards of calico can I get for 27 pounds of butter at $.33$\frac{1}{3}$ a pound?

7. A man bought a horse for $105, a yoke of oxen for $125, 4 cows at $35 apiece, and sold them all for $400. What was gained or lost in the transaction?

8. I bought 28 tons of hay at $19 a ton, and sold it at $15 a ton. How much did I lose? *Ans.* $112.

9. If a man spends 4\frac{3}{5}$ a day, in how many days will he spend 34\frac{1}{2}$? *Ans.* 7.5 days.

10. At $.31$\frac{1}{4}$ per bushel, how many bushels of potatoes can be bought for $9? *Ans.* 28.8 bushels.

11. If a man's income is $2000 a year, and his expenses are $3.50 a day, what will he save at the end of a year, or 365 days?

12. A merchant deposits in a bank, at one time, $687.25, and at another, $943.64. If he draws out $875.29, what will remain in the bank?

13. I bought 288 barrels of flour for $1728, and sold one half the quantity for the same price I gave for it, and the other half for $8 per barrel. What did I receive for the whole? *Ans.* $2016.

14. What will eight hundred seventy-five thousandths of a cord of wood cost, at $3.75 per cord? *Ans.* $3.281+.

15. A drover bought cattle at $46.56 per head, and sold them at $65.42 per head, thereby gaining $3526.82. How many cattle did he buy? *Ans.* 187.

16. If 36.48 yards of cloth cost $54.72, what will 14.25 yards cost? *Ans.* $21.375.

17. A house cost $3548, which was 4 times as much as the furniture cost. What did the furniture cost?

Ans. $887.

18. How many bushels of onions, at $.82 per bushel, can be bought for $112.34?

19. If 46 tons of iron cost $3461.50, what will 5 tons cost?

20. A man left his widow one third of his property, worth $24000, and the remainder was to be divided equally among 5 children. What was the portion of each child? *Ans.* $3200.

21. A man purchased one lot, containing 160 acres of land, at $1.25 per acre; and another lot, containing 80 acres, at $5 per acre; he sold them both at $2.50 per acre. What did he gain or lose in the transaction?

22. A druggist bought 54 gallons of oil for $72.90, and lost 6 gallons of it by leakage. He sold the remainder at $1.70 per gallon. What did he gain?

Ans. $8.70.

23. A miller bought 122½ bushels of wheat of one man, and 75¼ bushels of another, at $.93¾ per bushel. He sold 60 bushels at a profit of $12.50. If he sells the remainder at $.81¼ per bushel, what will be his entire gain or loss? *Ans.* $4.718 + loss.

24. How many pounds of butter, at $.16 per pound, must be given for 39 yards of sheeting, at $.08 a yard?

Ans. 19½ pounds.

25. A laborer receives $1.40 per day, and spends $.75 for his support. How much does he save in a week?

26. What is the cost of 23487 feet of hemlock boards, at $4.50 per 1000 feet? *Ans.* $105.6915.

27. A man has an income of $1200 a year. How much must he spend per day to use it all?

28. I bought 28 firkins of butter, each containing 56 pounds, at $.17 per pound. What was the whole cost?

29. What do 4868 bricks cost, at $4.75 per M?

30. A farmer sold 27 bushels of potatoes, at $.33⅓ per bushel; 28 bushels of oats, at $.25 per bushel; and 19 bushels of corn, at $.50 per bushel. What did he receive for the whole? *Ans.* $25.50.

31. A man bought an overcoat for $36¾, and a coat for $28½, and gave in payment one fifty, and two ten dollar bills. How much change should he receive?

32. If 4¼ barrels of flour cost $32.3, what will 7½ barrels cost? *Ans.* $51.

33. If .875 of a ton of coal costs $5.635, what will 9¼ tons cost? *Ans.* $59.57.

34. For the first three years of business, a trader gained $1200.25 a year; for the next three, he gained $1800.62 a year, and for the next two he lost $950.87 a year. Supposing his capital at the beginning of trade to have been $5000, what was he worth at the end of the eighth year? *Ans.* $12100.87.

35. What will be the cost of 18640 feet of timber, at $4.50 per 100? *Ans.* $838.80.

36. A gardener sold, from his garden, 120 bunches of onions at $.12½ a bunch, 18 bushels of potatoes at $.62½ per bushel, 47 heads of cabbage at $.07 a head, 6 dozen cucumbers at $.18 a dozen; he expended $1.50 in spading, $1.27 for fertilizers, $1.87 for seeds, $2.30 in planting and hoeing. What were the profits of his garden?

37. If a man has a capital of $ 3495 and loses .25 of it, how much has he left? *Ans.* $ 2621.25.

38. I paid a man $ 90 for the use of $ 1500 for 1 year. How much did I pay him for the use of each dollar?

39. A man bought a carriage for $ 150. He paid $ 15 for repairs and then sold it at a gain of .15 of the total cost. For how much did he sell it? *Ans.* $ 189.75.

40. If at a closing out sale I can get $ 8 worth of goods for every $ 4, how much can I get for $ 75?

41. I loaned a man a sum of money. He repaid me $ 24, which was .75 of the sum loaned him. How much does he still owe me? *Ans.* $ 8.

42. A merchant sells goods amounting to $ 5346. If he reserves as commission .02 of the amount sold, what is his commission? *Ans.* $ 106.92.

43. A merchant sold goods for $ 5346 and received $ 106.92 commission. How much commission did he receive for each dollar's worth of goods he sold?

44. A man has a capital of $ 50000; he invests .6 of it in business, .01 of it in stocks, and the remainder in bonds and mortgages. How much does he invest in bonds and mortgages? *Ans.* $ 19500.

45. I paid a man $ 6 for the use of $ 100 for 1 year. At the same rate, how much would I pay him for the use of the same sum for 5 years? *Ans.* $ 30.

46. At the same rate, what would I pay him for the use of $ 500 for 1 year? For 5 years?

Ans. $ 30. $ 150.

47. If I import $ 10000 worth of goods on which the duty is .12½ of the value of the goods, how much duty must I pay? *Ans.* $ 1250.

48. A and B engaged in trade, each with $ 5000. A gained .12½ of his capital and B .27 of his. How much more did B gain than A? *Ans.* $ 725.

SCALES.

175. A scale is a series of units, increasing or decreasing by fixed multipliers or divisors.

In the ordinary decimal notation this multiplier is 10. The decimal notation will be better understood when pupils have learned something of scales in general and of the possibility of writing numbers in any scale.

Scales may be uniform or varying.

In the uniform scale the multiplier or divisor is the same. In the varying scale it varies.

The Arabic system of notation and all the tables of the metric system are based on a uniform scale, the decimal. Most of the other tables in Denominate Numbers are based on varying scales.

The number of units which make one of the next higher order is the root or **radix** of the scale.

Thus, in the decimal scale 10 units make 1 ten; 10 tens, 1 hundred, etc.; therefore 10 is the radix.

Other uniform scales besides the decimal are: the binary (radix 2), ternary (3), quaternary (4), quinary (5), senary (6), septenary (7), octary (8), nonary (9), undenary (11), duodecimal (12), which derive their names from their radix.

A scale contains as many digits as there are units in its radix; and every scale must have the digit, 0.

DECIMAL.	**10 digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.**
NONARY.	**9 digits, 1, 2, 3, 4, 5, 6, 7, 8, 0.**

The octary scale has eight digits, 1 to 7 and 0; the septenary seven, 1 to 6 and 0; the senary six, 1 to 5 and 0; the quinary five, 1 to 4 and 0; the quaternary four, 1 to 3 and 0; the ternary three, 1, 2, 0; the binary two, 1, 0; the undenary eleven, 1 to 9, and one more (which may be expressed by any character, as *a*), and 0: and the duodecimal twelve, 1 to 9, *a*, *b*, 0.

By combining the digits in any scale, we obtain the notation of that scale.

In the decimal scale when we pass 9, we must use two digits for the next number, which is 10. In like manner in the quaternary scale, when we pass 3, we must use two figures; therefore 4 in the decimal scale is written 10 in the quaternary scale. In the quinary scale, when we pass 4, we must use two figures; therefore 5 in the decimal scale is written 10 in the quinary. In the decimal scale, when we pass 99, we have to use three digits for the next number, which is 100; but the highest number we can write in the quinary scale in two figures is 44, which corresponds to 24 in the decimal scale. Therefore 25 in the decimal scale would be written 100 in the quinary; and for the same reason 125 would be written 1000; 625, 10000, etc.

In the ternary scale, 3 would be written 10, 9 would be written 100, 27 would be written 1000, 81, 10000, etc. In the senary scale, 6 would be written 10, 36 would be 100, 216 would be 1000, 1296 would be 10000, etc.

The following table shows the notation in the various scales with the corresponding value of the numbers, in the decimal scale:

Decimal.	1	2	3	4	5	6	7	8	9	10	11	12	13
Binary	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101
Ternary	1	2	10	11	12	20	21	22	100	101	102	110	111
Quaternary	1	2	3	10	11	12	13	20	21	22	23	30	31
Quinary	1	2	3	4	10	11	12	13	14	20	21	22	23
Senary	1	2	3	4	5	10	11	12	13	14	15	20	21
Septenary	1	2	3	4	5	6	10	11	12	13	14	15	16
Octary	1	2	3	4	5	6	7	10	11	12	13	14	15
Nonary	1	2	3	4	5	6	7	8	10	11	12	13	14
Undenary	1	2	3	4	5	6	7	8	9	*a*	10	11	12
Duodecimal	1	2	3	4	5	6	7	8	9	*a*	*b*	10	11

EXAMPLES.

1. Write in the duodecimal scale the numbers corresponding to those from 14 to 36 in the decimal scale.

2. Write in the undenary, septenary, and nonary scales the numbers from 13 to 44 in the decimal scale.

3. Write the numbers from 1 to 100 in the following scales:

Ternary.	Senary.	Binary.
Quaternary.	Quinary.	Octary.

4. Express in the decimal scale the number 10000, as used in the ternary and quatenary scales.

5. Express in the decimal scale the number 1000000, as used in the senary scale.

6. Express in the decimal scale the numbers 100, 1000, 10000, 100000, as used in the quinary scale.

175a. Since varying scales do not increase by one constant multiplier, in working examples in such scales we must bear in mind how many units of each order make one of the next higher.

Thus in the scale, 24 grains (gr.) make 1 pennyweight (pwt.); 20 pwt., 1 ounce (oz.); 12 oz., 1 pound (lb.); 24 units of the first order make 1 of the second; 20 of the second, 1 of the third, and 12 of the third, 1 of the fourth. Hence, to reduce grains to pounds, we must multiply successively by 24, 20, and 12; to reduce pounds to grains, we divide successively by 12, 20, and 24.

Reduce:

1. 1 lb. to gr.	3. 480 gr. to oz.	5. 1 lb. to oz.
2. 24 gr. to pwt.	4. 1 pwt. to gr.	6. 1 lb. to pwt.

Numbers written in a uniform scale, as the decimal, are called **simple numbers.** Those written in the varying scales are **compound numbers.**

Numbers may be added, subtracted, multiplied, and divided by the same rules as apply to decimal notation, modified to adapt them to the particular scale in which the numbers are written.

DENOMINATE NUMBERS.

176. A Denominate Number is a number composed of one or more units of any denomination, and may be *simple* or *compound.*

A simple denominate number is a number composed of but one denomination.

Thus, 7 miles; 6 hours; 5 tons.

A compound denominate number is a number composed of two or more denominations of the same nature.

Thus, 10 lb. 6 oz.; 6 yr. 3 mo.

177. Reduction is the process of changing a number from one denomination to another without altering its value.

A number is changed to a *lower* denomination when it is changed to a denomination of *less* unit value.

Thus, $1 = 10 dimes = 100 cents.

A number is changed to a *higher* denomination when it is changed to a denomination of *greater* unit value.

Thus, 100 cents = 10 dimes = $1.

Most of the tables in Denominate Numbers except those in the Metric System are based on varying scales.

The only other tables that are not based on a varying scale are the table of counting 12 things make 1 dozen, 12 dozen 1 gross, 12 gross 1 great gross, etc., based on the duodecimal scale; the table of United States Currency, and some of the tables of Foreign Currency, based on the decimal scale.

CURRENCY.*

I. CANADA MONEY.

178. The currency of the Dominion of Canada is decimal, and the table and denominations are the same as those of United States money.

1. The currency of the whole Dominion of Canada was made uniform July 1, 1871. Before the adoption of the decimal system, pounds, shillings, and pence were used.

2. The *silver coins* are the 50 cent piece, 25 cent piece, 10 cent piece, and 5 cent piece. The *bronze coin* is the cent.

3. The *gold coin* used in Canada is the British *Sovereign*, worth $4.86⅔, and the *Half-Sovereign*.

FOREIGN CURRENCY.

II. ENGLISH MONEY.

179. English Currency is the currency of Great Britain. The unit is the pound sterling.

TABLE.

			U. S. Value.
4 Farthings (far.)	make 1 Penny	*d.*	$0.0202+
12 Pence	" 1 Shilling	*s.*	.2433+
20 Shillings	" 1 Pound or Sov.	£., or sov. .	$4.8665

1. Farthings are generally expressed as fractions of a penny; thus 1 far., sometimes called 1 quarter (qr.), = ¼*d.*; 3 far. = ¾*d.*

2. The *gold coins* are the sovereign (= £1), and the half-sovereign.

3. The *silver coins* are the crown (= 5*s.*), the half-crown (= 2*s.* 6*d.*), the florin (= 2*s.*), the shilling, and the sixpenny, fourpenny, and threepenny pieces.

4. The *copper coins* are the penny, halfpenny, and farthing.

5. The guinea (= 21*s.*) and the half-guinea (= 10*s.* 6*d.* sterling) are old gold coins, and are no longer coined.

III. FRENCH MONEY.

180. The currency of France is decimal currency. The unit is the franc (about 20 cents U. S. money) of which the other denominations are divisors.

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* For table of U. S. Currency, see § 158.

TABLE.

10 millimes make 1 centime.
100 centimes " 1 franc.

COINS. — 1. The *gold coins* are the 40, 20, 10, and 5 franc pieces.
2. The *silver coins* are the 5, 2, and 1 franc, the 50 and 20 centime pieces.
3. The *bronze coins* are the 10, 5, 2, and 1 centime pieces.

IV. GERMAN MONEY.

181. The currency of Germany is decimal money. The unit is the mark of 100 pfennige, about 24 cents.

COINS. — 1. The *gold coins* are the 5, 10, and 20 mark pieces.
2. The *silver coins* are the 20 pfennig, 1 and 2 mark pieces.
3. The *nickel coins* are the 5 and 10 pfennig pieces.

EXAMPLES.

182. To change a compound denominate number to a lower denomination.

1. Reduce £21 18*s.* 10*d.* 2 far. to farthings.

OPERATION.

£21 18*s.* 10*d.*
20
438*s.*
12
5266*d.*
4
21066 far., *Ans.*

SOLUTION. — In £1 there are 20*s.*; in £21 there are 21 × 20*s.* = 420*s.* + the given number 18*s.* = 438*s.* in £21 18*s.* In 1*s.* there are 12*d.*; in 438*s.* there are 438 × 12*d.* = 5256*d.* + 10*d.* = 5266*d.* In 1*d.* there are 4 far.; in 5266*d.* there are 5266 × 4 far. = 21064 far. + 2 far. = 21066 far., *Ans.*

RULE. — I. MULTIPLY *the highest denomination of the given number by that number of the scale which will reduce it to the next lower denomination, adding to the product the given number, if any, of that lower denomination.*

II. *Proceed in the same manner with the results obtained in each lower denomination, until the reduction is brought to the denomination required.*

183. To change a compound denominate number to a higher denomination.

1. Reduce 21066 farthings to pounds.

OPERATION.

4 | 21066 far.
12 | 5266*d.* + 2 far.
2|0 | 43|8*s.* + 10*d.*
£21 + 18*s.*

Ans. £21 18*s.* 10*d.* 2 far.

SOLUTION. — We divide 21066 far. by 4, since there are $\frac{1}{4}$ as many pence as farthings, and we find that 21066 far. = 5266*d.* + a remainder of 2 far. Next we divide 5266*d.* by 12, and we find that 5266*d.* = 438*s.* + 10*d.* Lastly, we divide 438*s.* by 20, and we find that 438*s.* = £21 + 18*s.* The last quotient with the several remainders annexed in the order of the succeeding denominations, gives the answer £21 18*s.* 10*d.* 2 far.

RULE. — I. DIVIDE *the given number by that number of the scale which will reduce it to the next higher denomination.*

II. *Divide the quotient by the next higher number in the scale; and so proceed to the highest denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.*

Reduction to lower denominations and reduction to higher denominations mutually prove each other.

2. In 14194 farthings how many pounds are there?
3. In £14 15*s.* 8*d.* 2 far. how many farthings are there?
4. In 15359 farthings how many pounds are there?
5. In 46 sov. 12*s.* 2*d.* how many pence are there?
6. In 11186 pence how many sovereigns are there?
7. In 40 francs how many centimes are there?
8. In 20 mark how many pfennige are there?
9. In 1000 centimes how many francs are there?
10. In 2000 pfennige how many mark are there?
11. In 1624 pence how many pounds are there?
12. How many centimes are there in 22 francs?

MEASURES OF WEIGHT.

184. Weight is the measure of gravity, and varies according to the quantity of matter a body contains.

The scales of weight in common use in the United States are the *Troy* and *Avoirdupois.*

STANDARD OF WEIGHT.

185. In the year 1834 the United States government adopted a uniform standard of weights and measures, for the use of the custom-houses, and the other branches of business connected with the general government.

The *United States standard unit of weight* is the Troy pound of the mint, which is the same as the imperial standard pound of Great Britain, and is equal to 252.458 grains, of which the standard Troy pound contains 5760.

The *United States Avoirdupois pound* is determined from the standard Troy pound, and contains 7000 Troy grains.

TROY WEIGHT.

186. Troy Weight is used in weighing gold, silver, and jewels and in philosophical experiments.

TABLE.

24 Grains (gr.)	make	1 Pennyweight	pwt.
20 Pennyweights	"	1 Ounce	oz.
12 Ounces	"	1 Pound	lb.

In weighing diamonds, pearls, and other jewels, the unit commonly employed is the carat, equal to 4 carat grains or 3.168 troy grains. The term carat is also used to express the number of parts in 24 that are pure gold. Thus gold that is 12 carats fine is $\frac{12}{24}$ pure gold and $\frac{12}{24}$ alloy.

EXAMPLES.

1. How many grains in 14 lb. 10 oz. 18 pwt. 22 gr.?

OPERATION.

14 lb. 10 oz. 18 pwt. 22 gr.
12
178 oz.
20
3578 pwt.
24
85894 gr. *Ans.*

2. How many pounds in 85894 gr.?

OPERATION.

24 | 85894 gr.
20 | 3578 pwt. + 22 gr.
12 | 178 oz. + 18 pwt.
14 lb. + 10 oz.

Ans. 14 lb. 10 oz. 18 pwt. 22 gr.

3. Reduce 5 lb. 7 oz. 12 pwt. 9 gr. to grains.

4. Reduce 32457 grains to pounds.

5. Reduce 41760 grains to pounds. *Ans.* 7 lb. 3 oz.

6. A miner had 14 lb. 10 oz. 18 pwt. of gold dust. What was it worth at $.75 a pwt.? *Ans.* $2683.50.

7. How many spoons, each weighing 2 oz. 15 pwt., can be made from 5 lb. 6 oz. of silver? *Ans.* 24.

8. A goldsmith manufactured 1 lb. 1 pwt. 16 gr. of gold into rings, each weighing 4 pwt. 20 gr.; he sold the rings for $1.25 apiece. What did he receive for them? *Ans.* $62.50.

AVOIRDUPOIS WEIGHT.

187. Avoirdupois Weight is used for all the ordinary purposes of weighing.

TABLE.

16 Ounces	make	1 Pound	lb.
100 Pounds	"	1 Hundred-weight . .	cwt.
20 Cwt., or 2000 lb.	"	1 Ton	T.

The *long* or *gross ton*, hundred-weight, and quarter were formerly in common use; but they are now seldom used except in estimating English goods at the U. S. custom-houses, and in freighting and wholesaling coal at the mines.

LONG TON TABLE.

16 Ounces	make	1 Pound	lb.
28 Pounds	"	1 Quarter	qr.
4 Quarters	"	1 Hundred-weight . .	cwt.
20 Cwt. = 2240 lb.	"	1 Ton	T.

The following denominations are also in use:

100 Pounds of	Grain or Flour	make	1	Cental.
100 "	Dry Fish	"	1	Quintal.
100 "	Nails	"	1	Keg.
196 "	Flour	"	1	Barrel.
200 "	Pork or Beef	"	1	Barrel.
280 "	Salt at N. Y. S. Works	"	1	Barrel.
56 "	Salt at N. Y. S. Works	"	1	Bushel.
240 "	Lime	"	1	Cask.
32 "	Oats	"	1	Bushel.
56 "	Corn	"	1	Bushel.
60 "	Wheat	"	1	Bushel.

APOTHECARIES' MEASURES.

188. Apothecaries' Weight is used by apothecaries and physicians in weighing medicines for prescriptions, but medicines are bought and sold by avoirdupois weight.

TABLE.

20 Grains (gr.)	= 1 Scruple . . .	sc., or ℈
3 Scruples	= 1 Dram	dr., or ʒ
8 Drams	= 1 Ounce . . .	oz., or ℥
12 Ounces	= 1 Pound . . .	lb., or ℔

Apothecaries' Liquid Measure is used in compounding and measuring liquid medicines.

TABLE.

60 Drops (gtt.) or minims (♏)	= 1 Fluid drachm . .	ƒʒ.
8 Fluid drachms	= 1 Fluid ounce . . .	ƒ℥.
16 Fluid ounces	= 1 Pint	*O.*
8 Pints	= 1 Gallon	*Cong.*

189. The **Metric System** of weights and measures originated in France in 1795, and its use is legalized in the United States. It is in general use by scientific men throughout the world.

It is a decimal system in which the various denominations are decimal multiples and submultiples of a unit. The following comparison of our ordinary decimal notation and the metric notation shows them to be the same:

Ten Thousands.	Thousands.	Hundreds.	Tens.	UNITS.	tenths.	hundredths.	thousandths.
1	1	1	1	1.	1	1	1
Kilos	Myrias	Hektos	Dekas	UNITS	decis	centis	millis

The units of the different measures are: long measure, *meter;* land or square measure, *are* and *square meter;* wood or cubic measure, *stere* and *cubic meter;* weight, *gram;* capacity, *liter.* By adding the prefixes given in the table of notation to these words all the metric tables are formed.

1. The fundamental unit of the metric system is the *meter*, from which all the other units have been derived.

2. The principal point of superiority of the metric tables is their decimal scale. By means of the decimal point several denominations may be written together as one number, as in ordinary notation, and a change to higher or lower denominations is effected by simply moving the decimal point to the left or right.

3. One disadvantage in the use of the metric system lies in the fact that with us the other denominations are in such common use that it would require a careful study and comparison of the two systems, on the part of the people at large, to make the use of the metric system practicable. Again, some of the units while well adapted to the social and industrial conditions of France and other European countries, would not be convenient for use here. For instance, the are, the unit of land measure, is $\frac{1}{40}$ of an acre. While this suits the peasant farms of France very well it would be absurdly small for a farm in the United States.

4. The metric system is now used in France, Netherlands, Spain, Italy, Greece, Austria, Germany, Norway, Sweden, Switzerland, Portugal, Mexico, Brazil, Venezuela, Argentine Republic, Haiti, and other states; and to some extent in Great Britain, United States, India, Canada, and Chile.

METRIC WEIGHT.

190. The **gram** is the unit of weight; it is legal at 15.432 Troy grains. It is equal to the weight of a cube of distilled water, the edge of which is $\frac{1}{100}$ of a meter.

TABLE.

10 Milligrams (mg.)	=1 Centigram =	.15432 gr. Troy.
10 Centigrams (cg.)	=1 Decigram =	1.54324 " "
10 Decigrams (dg.)	=1 **Gram** =	15.43248 " "
10 Grams (G.)	=1 Dekagram =	.35273 oz. Avoir.
10 Dekagrams (Dg.)	=1 Hektogram =	3.52739 " "
10 Hektograms (Hg.)	=1 Kilogram =	2.20462 lb.. "
10 Kilograms (Kg.)	=1 Myriagram =	22.04621 " "
10 Myriagrams (Mg.)	=1 Quintal =	220.46212 " "
10 Quintals, or 1000 kilograms	=1 Tonneau =	2204.62125 " "

The weights commonly used are the gram, kilogram, and tonneau. The gram is used in mixing medicines, in weighing the precious metals, and in all cases where great exactness is required. The kilogram — or, as it is commonly called, the "kilo" — is the usual weight for groceries and coarse articles generally; it is very nearly 2¼ pounds Av. The tonneau is used for weighing hay and other heavy articles; it is about 204 lb. more than our ton.

COMPARATIVE TABLE OF WEIGHTS.

	Troy.	Avoirdupois.	Apoth.
1 pound =	5760 grains	= 7000 grains	= 5760 gr.
1 ounce =	480 "	= 437.5 "	= 48 O.
	175 pounds	= 144 pounds.	

EXAMPLES.

1. In 25 T. 15 cwt. 70 lb. how many pounds?

OPERATION.

25 T. 15 cwt. 70 lb.
20
515 cwt.
100
51570 lb., *Ans.*

2. In 51570 pounds how many tons?

OPERATION.

1|00|515|70 lb.
2|0|51|5 cwt. + 70 lb.
25 T. + 15 cwt.

Ans. 25 T. 15 cwt. 70 lb.

3. Reduce 3 T. 14 cwt. 74 lb. 12 oz. to ounces.

4. Reduce 119596 ounces to tons.

5. A tobacconist bought 3 T. 15 cwt. 20 lb. of tobacco, at 22 cents a pound. What did it cost him? *Ans.* $1654.40.

6. What will 115 kilograms of hay cost, at $10 per ton?

7. I paid $360 for 2 tons of cheese, and retailed it for 30 cents a kilogram. What was my whole gain?

8. If a person buys 10 T. 6 cwt. 3 qr. 14 lb. of English iron, by the long ton weight, at 6 cents a pound, and sells the same at $130 per short ton, what will he gain? *Ans.* $115.85.

9. Change 20 lb. 8 oz. 12 pwt. troy weight to avoirdupois weight. *Ans.* $17\frac{41}{875}$ lb.

10. I bought by avoirdupois weight 20 lb. of opium, at 40 cents an ounce, and sold the same by troy weight at 50 cents an ounce. What did I gain? *Ans.* $17.83\frac{1}{3}$.

11. From 25.8 g. take 326 cg. How much remains?

12. A dealer bought 15 tons of coal, and sold at one time 5.32 T., at another 3.045 T., and at another 4.125 T. How much had he left?

MEASURES OF EXTENSION.

191. Extension has one or more of the dimensions — length, breadth, and thickness.

A **Line** has only one dimension — *length*.

A **Surface** or **Area** has two dimensions — *length* and *breadth*.

A **Solid** or **Body** has three dimensions — *length*, *breadth*, and *thickness*.

STANDARD OF EXTENSION.

192. The *United States standard unit of measures of extension*, whether linear, superficial, or solid, is the yard of 3 feet, or 36 inches, and is the same as the imperial standard yard of Great Britain.

1. The old British Bird's standard yard of 1760 was found inadequate, and a new standard was constructed in 1816. This was destroyed by fire in the burning of the houses of Parliament in 1834, and a new standard, reproduced by reference to all the former standards, was constructed in 1844. This was known as the British imperial yard.

2. The British imperial yard is defined by Act of Parliament as the distance between the centers of two cylindrical holes in a certain bar of bronze when the metal has a temperature of 62° F.

LINEAR MEASURE.

193. Linear Measure, also called Long Measure, is used in measuring lengths and distances.

TABLE.

12	Inches (in.)	make	1 Foot	ft.
3	Feet	"	1 Yard	yd.
5½	Yards, or 16½ ft.,	"	1 Rod	rd.
320	Rods	"	1 Statute Mile	mi.

The following denominations are also in use:

3 Barleycorns	make	1 Inch	Used by shoemakers.
4 Inches	"	1 Hand	Used to measure the height of horses at the shoulder.
9 Inches	"	1 Span.	
21.888 inches	"	1 Sacred Cubit.	
6 Feet	"	1 Fathom	Used to measure depths at sea.
3 "	"	1 Pace	Used in pacing distances.
5 Paces	"	1 Rod	
8 Furlongs	"	1 Mile.	
1.152⅔ Statute Mi.	"	1 Geog. mile.	Used to measure distances at sea.
3 Geographic "	"	1 League = 3.458 st. mi.	
60 " "	"	1 Degree	of latitude on a meridian or of longitude on the equator.
69.16 Statute "	"		
360 Degrees	"	the Circumference of the Earth.	

1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into *halves*, *fourths*, *eighths*, and *sixteenths*. The old table of cloth measure is practically obsolete.

2. The geographic mile is $\frac{1}{60}$ of $\frac{1}{360}$ or $\frac{1}{21600}$ of the circumference of the earth. It is a small fraction more than 1.15 statute miles. It is also called a nautical mile.

3. The length of a degree of latitude varies, being 68.72 miles at the equator, 68.9 to 69.05 miles in middle latitudes, and 69.30 to 69.34 miles in the polar regions. The mean or average length is as stated in the table. A degree of longitude is greatest at the equator, where it is 69.16 miles, and it gradually decreases toward the poles, where it is 0.

METRIC LONG MEASURE.

194. The **Meter** is the unit of length. It was intended to be $\frac{1}{10000000}$ part of the distance from the equator to either pole, but it does not exactly correspond with that length.

TABLE.

	1 Millimeter	= .03937079	in.
10 Millimeters (mm.)	= 1 Centimeter	= .3937079	"
10 Centimeters (cm.)	= 1 Decimeter	= 3.937079	"
10 Decimeters (dm.)	= 1 **Meter**	= 39.37079	"
10 Meters (m.)	= 1 Dekameter	= 32.808992	ft.
10 Dekameters (Dm.)	= 1 Hectometer	= 19.927817	rd.
10 Hectometers (Hm.)	= 1 Kilometer	= .62137	mi.
10 Kilometers (Km.)	= 1 Myriameter	= 6.213824	"

The measures chiefly used are the meter (39.37 inches) and kilometer. The meter, like the yard, is used in measuring cloth and short distances; the kilometer is used in measuring long distances, and is .62137 of a common mile.

EXAMPLES.

1. In 2 mi. 192 rd. 2 yd. how many inches?

OPERATION.

2 mi. 192 rd. 2 yd.
320
832 rd.
$5\frac{1}{2}$
416
4162
4578 yd.
3
13734 ft.
12
164808 in., *Ans.*

2. In 164808 inches how many miles?

OPERATION.

1	164808 in.
3	13734 ft.
$5\frac{1}{2}$	4578 yd.
2	2
11	9156
320	832 rd. + $\frac{1}{2}$ yd. = 2 yd.

2 mi. + 192 rd.,

Ans. 2 mi. 192 rd. 2 yd.

3. The diameter of the earth being 7912 miles, how many inches is it? *Ans.* 501304320 in.

4. In 168474 feet how many miles are there?

5. In 31 mi. 290 rd. 3 yd. how many feet are there?

6. If the greatest depth of the Atlantic telegraph cable from Newfoundland to Ireland is 2500 fathoms, how many miles is it? *Ans.* 2 mi. 269 rd. $1\frac{1}{2}$ ft.

7. From a piece of cloth containing 45.75 m., a tailor cut 5 suits of clothes, each suit containing 7.5 m. How much remained?

8. Find the sum of 12.65 m., 3.678 Dm., 2106.32 Hm., and 570.36 dm.

9. A pond of water measures 4 fathoms 3 feet 8 inches in depth. How many inches deep is it? *Ans.* 332.

SURVEYOR'S LONG MEASURE.

195. A **Gunter's Chain,** used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links. The unit is the chain.

TABLE.

7.92	Inches (in.)	make	1 Link	l.
25	Links	"	1 Rod	rd.
4	Rods, or 66 feet,	"	1 Chain	ch.
80	Chains	"	1 Mile	mi.

1. Rods are seldom used in chain measure, distances being taken in chains and hundredths. In measuring city lots a steel tape 50 feet long is generally used, and the measure is expressed in feet and tenths of a foot.

2. An engineer's chain, used by civil engineers, is 100 feet long, and consists of 100 links.

EXAMPLES.

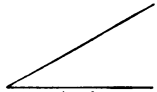
1. In 3 mi. 51 ch. 73 l. how many links are there?

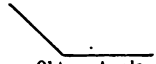
2. Reduce 29173 l. to miles.

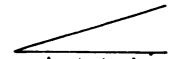
3. A certain field, inclosed by a board fence, is 17 ch. 31 l. long and 12 ch. 87 l. wide. How many feet long is the fence which incloses it? *Ans.* 3983.76 ft.

SQUARE MEASURE.

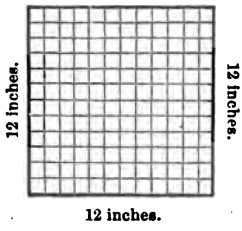
196. A' **Square** is a figure bounded by four equal sides, and having four equal angles.


Angle


Obtuse Angle


Acute Angle

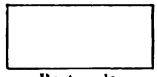
The angles of a square are termed **right angles**; a greater angle than a right angle is termed an **obtuse angle**; a less angle than a right angle is termed an **acute angle**.

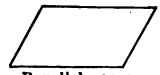


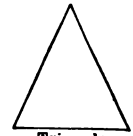
1 square foot is a figure having four sides of 1 ft. or 12 in. each, as shown in the diagram. Its contents are $12 \times 12 = 144$ square inches. Hence,

The contents or area of a square, or of any other figure having a uniform length and a uniform breadth, is found by multiplying the length by the breadth.

Thus, a square foot is 12 in. long and 12 in. wide, and the contents are $12 \times 12 = 144$ square inches.


Rectangle


Parallelogram


Triangle

A **rectangle** is a figure having four right angles and its opposite sides equal.

A board 20 in. long and 10 in. wide is a rectangle, containing $20 \times 10 = 200$ sq. inches.

A **parallelogram** is any four sided figure with its opposite sides equal and parallel.

A **triangle** is a figure bounded by three straight lines. The dotted lines show the height or *altitude* of the figures.

The contents of a parallelogram is found by multiplying the base by the altitude. The contents of a triangle is one half the product of the base and altitude.

197. Square Measure is used in computing areas or surfaces; as of land, boards, painting, plastering, paving, etc.

TABLE.

144	Square Inches (sq. in.)	make 1	Square Foot	sq. ft.
9	Square Feet	" 1	Square Yard	sq. yd.
30¼	Square Yards	" 1	Square Rod	sq. rd.
160	Square Rods	" 1	Acre	A.
640	Acres	" 1	Square Mile	sq. mi.

Artificers estimate their work as follows:

By the square foot; glazing and stone-cutting. By the square yard; painting, plastering, paving, ceiling, and paper-hanging. By the square of 100 feet; flooring, partitioning, roofing, slating, and tiling. Brick-work is generally estimated by the 1000 bricks; sometimes in *cubic feet.*

1. In estimating the painting of moldings, cornices, etc., the measuring-line is carried into all the moldings and cornices.

2. A brick wall which is a brick and a half thick is said to be of the standard thickness. Five courses in the height are called a foot.

The terms *perch* and *pole* (P.) are sometimes used for sq. rod.

METRIC SQUARE MEASURE.

198. The **Square Meter** is the unit for measuring ordinary surfaces of small extent.

TABLE.

1 Centare (ca.)	= 1 Sq. Meter	=	1.196034 sq. yd.
100 Centares "	= 1 **Are**	=	119.6034 sq. yd.
100 Sq. Meters			
100 Ares (A.)	= 1 Hektare (Ha.)	=	2.47114 acres.
10,000 Sq. Meters			
100 Hectares (Ha.)	= 1 Sq. Kilometer	=	.3861 sq. mi.

1. The *Are* is the *unit* of land measure, and is a square whose side is 10 meters, equal to a *square dekameter*, or 119.6 square yards. The square kilometer is used in estimating the areas of countries, seas, etc.

2. Since in this table 100 units of each denomination make 1 of the next higher, in reducing to higher or lower denominations we must move the decimal point *two places* to the right or left for each denomination.

EXAMPLES.

1. In 10 A. 65 sq. rd. 16 sq. yd. 4 sq. ft. 136 sq. in. how many square inches are there?

OPERATION.

10 A. 65 sq. rd. 16 sq. yd. 4 sq. ft. 136 sq. in.
160
1665 sq. rd.
$30\frac{1}{4}$
$416\frac{1}{4}$
49966
$50382\frac{1}{4}$ sq. yd.
9
$453444\frac{1}{4}$ sq. ft.
144
$36 = \frac{1}{4}$ sq. ft.
1813912 with 136 sq. in.
1813776
453444
65296108 sq. in., *Ans.*

SOLUTION. — Since there are 160 sq. rd. in an acre, in 10 A. there are 10 times 160 sq. rd. = 1600 sq. rd. + 65 sq. rd. = 1665 sq. rd. Since there are $30\frac{1}{4}$ sq. yd. in one sq. rd., in $30\frac{1}{4}$ sq. rd. there are 1665 times $30\frac{1}{4}$ sq. yd. = $50366\frac{1}{4}$ sq. yd. + 16 sq. yd. = $50382\frac{1}{4}$ sq. yd. Since there are 9 sq. ft. in one sq. yd., in $50382\frac{1}{4}$ sq. yd. there are $50382\frac{1}{4}$ times 9 sq. ft. = $453440\frac{1}{4}$ sq. ft. + 4 sq. ft. = $453444\frac{1}{4}$ sq. ft. Since there are 144 sq. in. in 1 sq. ft., in $453444\frac{1}{4}$ sq. ft. there are $453444\frac{1}{4}$ times 144 sq. in. = 65295972 sq. in. + 136 sq. in. = 65296108 sq. in., *Ans.*

2. In 65296108 sq. in. how many acres are there?

OPERATION.

144	65296108	sq. in.
9	453445	sq. ft. + 28 sq. in.
$30\frac{1}{4}$	50382	sq. yd. + 7 sq. ft.
4	4	
121	201528	fourths sq. yd.
160	1665	sq. rd. + $\frac{6}{4}^3 = 15\frac{3}{4}$ sq. yd.

10 A. + 65 sq. rd.

Ans. 10 A. 65 sq. rd. 15¾ sq. yd. 7 sq. ft. 28 sq. in.

Or { 10 A. 65 sq. rd. 15 sq. yd. 7 sq. ft. 28 sq. in.
{ 6 sq. ft. 108 sq. in.

Or 10 A. 65 sq. rd. 16 sq. yd. 4 sq. ft. 136 sq. in.

SOLUTION. — Dividing by the numbers in the ascending scale, and arranging the remainders according to their order in a line below, we find the square yards a mixed number, 15¾. But ¾ of a sq. yd. = ¾ of 9 sq. ft. = 6¾ sq. ft.; and ¾ of a sq. ft. = ¾ of 144 sq. in. = 108 sq. in. Therefore ¾ sq. yd. = 6 sq. ft. 108 sq. in.; and adding 108 sq. in. to 28 sq. in. we have 136 sq. in., and adding 6 sq. ft. to 7 sq. ft. we have 13 sq. ft. = 1 sq. yd. 4 sq. ft., and writing the 4 sq. ft. in the result, and adding 1 sq. yd. to 15 sq. yd. we have for the reduced result, 10 A. 65 sq. rd. 16 sq. yd. 4 sq. ft. 136 sq. in.

3. Reduce 87 A. 118 sq. rd. 7 sq. yd. 1 sq. ft. 100 sq. in. to square inches. *Ans.* 550355068 sq. in.

4. Reduce 550355068 square inches to acres.

5. A field 100 rods long and 30 rods wide contains how many acres? *Ans.* 18 A. 120 sq. rd.

6. How many rods of fence will inclose a farm a mile square? *Ans.* 1280 rods.

7. How much additional fence will divide it into four equal square fields? *Ans.* 640 rd.

8. How many acres of land in Boston, at $1 a square foot, will $100000 purchase?

Ans. 2 A. 47 sq. rd. 9 sq. yd. 3¼ sq. ft.

9. How many yards of carpeting, 1 yd. wide, will be required to carpet a room 18½ ft. long and 16 ft. wide?

Ans. 32⅔ yd.

10. What would be the cost of plastering a room 18 ft. long, 16½ ft. wide, and 9 ft. high, at 22 cents a sq. yd.?

Ans. $22.44.

11. What will be the expense of slating a roof 40 feet long and each of the two sides 20 feet wide, at $10 per square? *Ans.* $160.

12. How many square inches are there in a board 3 feet long and 20 inches wide? *Ans.* 720.

13. A man bought a farm 198 rods long and 150 rods wide, and agreed to pay $32 an acre for it. How much did the farm cost him? *Ans.* $5940.

14. A certain rectangular piece of land measures 1000 links by 100. How many acres does it contain?

Ans. 1 A.

15. How many square yards are there in a triangle whose base is 148 ft. and altitude 45 feet? *Ans.* 370 sq. yd.

16. What is the area of a triangle whose base is 100 ft. and whose altitude is 60 ft.?

17. How many acres of land are there in a parallelogram, the length of which is 240 rods and the breadth (corresponding to altitudes in the figure, see **196**) 84 rods?

18. What is the value of 4 acres of land at $2 per sq. meter?

19. In 9.375 hectares, how many sq. meters are there?

20. How many hectares are there in 25.635 sq. kilometers?

SURVEYOR'S SQUARE MEASURE.

199. This measure is used by surveyors in computing the *area* or contents of land.

TABLE.

625 Square Links (sq. l.	make	1 Pole	P.
16 Poles	"	1 Square Chain .	sq. ch.
10 Square Chains	"	1 Acre	A.
640 Acres	"	1 Square Mile . .	sq. mi.
36 Square Miles (6 miles square)	"	1 Township	Tp.

1. A square mile of land is also called a *section*.

2. Engineers commonly use a chain, or measuring tape, 100 feet long, each foot divided into *tenths*.

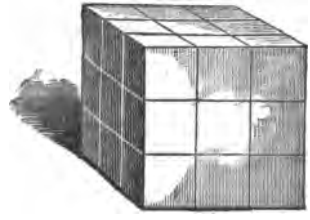
3. The contents of land are commonly estimated in square miles, acres, and hundredths; the denomination, *rood*, is no longer used.

EXAMPLES.

1. How many poles are there in a township of land?
2. Reduce 3686400 P. to sq. mi.
3. In 94 A. 7 sq. ch. 12 P. 118 sq. l. how many square links are there?

CUBIC MEASURE.

200. A **Cube** is a solid, or body, having six equal square sides, or faces. If each side of a cube is 1 yard or 3 feet, 1 foot in thickness of this cube will contain $3 \times 3 \times 1 = 9$ cubic feet, and the whole cube will contain $3 \times 3 \times 3 = 27$ cubic feet.



A solid, or body, may have the three dimensions all alike or all different. A body 4 ft. long, 3 ft. wide, and 2 ft. thick contains $4 \times 3 \times 2 = 24$ cubic or solid feet. Hence,

The cubic or solid contents of a body is the product of the length by the breadth by the thickness.

201. Cubic Measure, also called Solid Measure, is used in estimating the contents of solids, or bodies; as timber, wood, stone, etc.

TABLE.

1728 Cubic Inches (cu. in.)	make	1	Cubic Foot	cu. ft.
27 Cubic Feet	"	1	Cubic Yard	cu. yd.
16 Cubic Feet	"	1	Cord Foot	cd. ft.
8 Cord Feet, or 128 Cubic Feet	"	1	Cord of Wood	Cd.
24¾ Cubic Feet	"	1	Perch of Stone or Masonry . . .	Pch.

1. A cubic yard of earth is called a *load*.

2. Railroad and transportation companies estimate light freight by the *space* it occupies in cubic feet, and heavy freight by *weight*.

3. A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, contains 1 cord; and a cord foot is 1 foot in length of such a pile.

4. A *perch* of stone or of masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

5. Joiners, bricklayers, and masons make no allowance for windows, doors, etc. Bricklayers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of hou es, cellars, etc., but estimate their work by the girt; that is, the entire length of the wall on the outside.

6. Engineers, in making estimates for excavations and embankments, take the dimensions with a line or measure divided into feet and decimals of a foot. The estimates are made in feet and decimals, and the results are reduced to cubic yards.

METRIC CUBIC MEASURE.

202. The **cubic meter** is the unit of solid measure.

TABLE.

1000 Cu. Millimeters (cu. mm.) = 1 Cu. Centimeter = .061 + cu. in.
1000 Cu. Centimeters (cu. cm.) = 1 Cu. Decimeter = 61.026 cu. in.
1000 Cu. Decimeters (cu. dm.) = 1 { **Cu. Meter** or **Stere** } = 35.316+ cu. ft.

1. In measuring wood the cubic meter is called a stere, and is equal to .2759 of a cord.

2. Since in cubic measure 1000 of each denomination make one of the next higher, in reducing to higher or lower denominations we must move the decimal point *three places* to the left or right for each denomination.

EXAMPLES.

1. In 125 cu. ft. 840 cu. in. how many cu. in. are there? *Ans.* 216840 cu. in.

2. Reduce 5224 cubic feet to cords. *Ans.* $40\frac{13}{16}$ cords.

3. In a solid 3 ft. 2 in. long, 2 ft. 2 in. wide, and 1 ft. 8 in. thick, how many cubic inches are there?

Ans. 19760 cu. in.

4. How many small cubes, 1 in. on each edge, can be sawed from a cube 6 ft. on each edge, allowing no waste for sawing? *Ans.* 373248.

5. In a pile of wood 60 ft. long, 20 ft. wide, and 15 ft. high, how many cords are there? *Ans.* $140\frac{5}{8}$ cords.

6. How many cubic ft. are there in a load of wood 10 ft. long, $3\frac{1}{4}$ ft. wide, and $3\frac{1}{2}$ ft. high? *Ans.* $113\frac{3}{4}$ cu. ft.

7. If a load of wood is 12 ft. long and 3 ft. wide, how high must it be to make a cord? *Ans.* $3\frac{5}{9}$ ft. high.

8. The gray limestone of Central New York weighs 175 pounds a cubic foot. What is the weight of 1 solid yard? *Ans.* 2 T. 7 cwt. 25 lb.

9. A cellar wall, 32 ft. by 24 ft., is 6 ft. high and $1\frac{1}{2}$ ft. thick. What did it cost at \$1.25 a perch?

Ans. \$50.909 +.

10. My sleeping-room is 10 ft. long, 9 ft. wide, and 8 ft. high. If I breathe 10 cu. ft. of air in one minute, in how long a time will I breathe as much air as the room contains? *Ans.* 72 min.

11. In a school-room 30 ft. long, 20 ft. wide, and 10 ft. high, with 50 persons breathing each 10 cu. ft. of air in one minute, in how long a time will they breathe as much as the room contains? *Ans.* 12 min.

12. How many steres are there in a pile of wood 20 m. long, 1.25 m. wide, and 4 m. high?

13. How many cords of wood are there in 10 steres?

MEASURES OF CAPACITY.

STANDARD OF CAPACITY.

203. The *United States standard unit of liquid measure* is the old English wine gallon, or 231 cubic inches, which is equal to 8.33888 pounds avoirdupois of distilled water at its maximum density: that is, at the temperature of 39.830 F., the barometer being at 30 inches.

The *standard unit of dry measure* is the British Winchester bushel, which is $18\frac{1}{2}$ inches in diameter, and 8 inches deep, and contains 2150.42 cu. in. A gallon dry measure contains 268.8 cu. in.

LIQUID MEASURE.

204. Liquid Measure, also called Wine Measure, is used in measuring liquids; as liquors, molasses, water, etc.

TABLE.

4	Gills (gi.)	make 1 Pint	pt.
2	Pints	" 1 Quart	qt.
4	Quarts	" 1 Gallon	gal.
31½	Gallons	" 1 Barrel	bbl.
2	Barrels, or 63 gal.	" 1 Hogshead	hhd.

The following denominations are also in use:

36	Gallons	make 1 Barrel	of beer.
54	" or 1½ Barrels	" 1 Hogshead	" "
42	"	" 1 Tierce.	
2	Hogsheads, or 120 gallons	" 1 Pipe or Butt.	
2	Pipes, or 4 Hogsheads	" 1 Tun.	

1. The denominations, barrel and hogshead, are used in estimating the capacity of cisterns, reservoirs, vats, etc.

2. The tierce, hogshead, pipe, butt, and tun are the names of casks, and do not express any fixed or definite measures. They are usually gauged, and have their capacities in gallons marked on them.

3. Ale or beer measure, formerly used in measuring beer, ale, and milk, is almost entirely discarded.

DRY MEASURE.

205. Dry Measure is used in measuring articles not liquid; as grain, fruit, salt, roots, ashes, etc.

TABLE.

2	Pints (pt.)	make 1 Quart.	qt.
8	Quarts	" 1 Peck	pk.
4	Pecks	" 1 Bushel	bu.

1. In England, 8 bu. of 70 lb. each are called a *quarter*, used in measuring grain. The weight of the English quarter is ¼ of a long ton.

2. In measuring grain, seeds, berries, etc., measure must be even full or stricken.

3. In measuring coal, corn in ear, large fruits, vegetables, etc., the measure should be heaped at least six inches. A bushel heaped measure contains 2747.7167 cu. in., or 597.2967 more than a bushel stricken measure. 5 pk. stricken measure equal about a heaped bushel.

4. The liquid (wine) and dry measures of the same denomination are of different capacities. The relative size of each may be seen by the following table:

COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cu. in. in one gallon.	Cu. in. in one quart.	Cu. in. in one pint.	Cu. in. in one gill.
Liquid measure,	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry measure, ($\frac{1}{2}$ pk.)	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

The beer gallon of 282 inches is retained in use only by custom.

METRIC CAPACITY.

206. The **Liter** is the unit of Capacity; it is legal at 1.0567 qt. liquid measure, and .9081 qt. dry measure.

It is a vessel whose volume is equal to a cube whose edge is $\frac{1}{10}$ of a meter.

TABLE.

		Dry.	Liquid.
10 Milliliters, ml. =	1 Centiliter	= .6102 cu. in.	= .338 fluid oz.
10 Centiliters, cl. =	1 Deciliter	= 6.1022 cu. in.	= .845 gill.
10 Deciliters, dl. =	1 **Liter,**	= .908 qt.	= 1.0567 qt.
10 Liters, l. =	1 Dekaliter	= 9.08 qt.	= 2.6417 gal.
10 Dekaliters, Dl. =	1 Hektoliter	= 2.8375 bu.	= 26.417 gal.

The measures commonly used are the liter and hektoliter. The liter is very nearly a quart; it is used in measuring milk, wine, etc., in moderate quantities. The hektoliter is about 2 bu. 3$\frac{1}{2}$ pk.; it is used in measuring grain, fruit, roots, etc., in large quantities.

EQUIVALENTS.

231 cu. in. = 1 gal.
2150.4 cu. in. = 1 bu.
4 bu. = 5 cu. ft. (nearly).
4 heaped bu. = 5 stricken bu.
7$\frac{1}{2}$ gal. = 1 cu. ft. (nearly).
7 cu. ft. corn in ear = 3 bu. shelled corn.
*48 lb. = 1 bu. barley.
*56 lb. = 1 bu. corn.
*32 lb. = 1 bu. oats.
*60 lb. = 1 bu. potatoes.
*56 lb. = 1 bu. rye.
*60 lb. = 1 bu. wheat.

*The number of lb. in a bu. varies in different states. The above are the equivalents in most states.

EXAMPLES.

1. In 2 hhd. 1 bar. 30 gal. 2 qt. 1 pt. 3 gi. how many gills?

OPERATION.

2 hhd. 1 bar. 30 gal. 2 qt. [1 pt. 3 gi.
2
5 bbl.
31½
2½
185
187½ gal.
4
752 qt.
2
1505 pt.
4
6023 gi., *Ans.*

2. In 6023 gi. how many hogsheads?

OPERATION.

4	6023 gi.
2	1505 pt. + 3 gi.
4	752 qt. + 1 pt.
31½	188 gal.
2	2
63	376 [gal.
2	5 bbl. + ½ gal. = 30½

2 hhd. + 1 bar.

Ans. 2 hhd. 1 bar. 30½ gal. 1 pt. 3 gi.

But ½ gal. = 2 qt., making the *Ans.* 2 hhd. 1 bar. 30 gal. 2 qt. 1 pt. 3 gi.

3. Reduce 3 hogsheads to gills.

4. Reduce 6048 gills to hogsheads.

5. In 13 hhd. 15 gal. 1 qt. how many pints are there?

6. In 6674 pints how many hogsheads are there?

7. What will be the cost of a hogshead of alcohol, at 6 cents a gill? *Ans.* $120.96.

8. A grocer bought 10 barrels of cider, at $2 a barrel; after converting it into vinegar, he retailed it all at 5 cents a quart. What was his whole gain?

Ans. $43.

9. At 6 cents a pint, how much molasses can be bought for $3.84? *Ans.* 8 gal.

10. How many demijohns, that will contain 2 gal. 2 qt. 1 pt. each, can be filled from a hogshead? *Ans.* 24.

11. What is the price of a deciliter of cordial, if a liter costs $5?

12. How many hektoliters are there in 500 liters?

13. In 3199 pt. how many bushels are there?

14. Reduce 1 bu. 1 pk. 1 qt. 1 pt. to pints.

15. Reduce 83 pints to bushels.

16. An innkeeper bought a load of 50 bushels of oats at 65 cents a bushel, and retailed them at 25 cents a peck. How much did he make on the load? *Ans.* $17.50.

17. At $4 a hektoliter, what is the value of 4 liters of apples?

18. A fruit dealer bought a bushel of strawberries, dry measure, and sold them by liquid measure. How many quarts did he gain? *Ans.* $5\frac{13}{55}$ quarts.

19. A grocer bought 40 quarts of milk by beer measure, and sold it by liquid measure. How many quarts did he gain? *Ans.* $8\frac{44}{77}$ quarts.

20. A bushel, or 32 quarts, dry measure, contains how many more cubic inches than 32 quarts wine measure?

Ans. $302\frac{2}{5}$ cu. in.

21. If a man buys 10 bushels of chestnuts at $5 per bushel, dry measure, and sells the same at 22 cents per quart, liquid measure, how much is his gain?

Ans. $31.92.

207. Units of the *Common* System may be readily changed to *units* of the *Metric* System by the aid of the following table:

TABLE.

1 Inch	= 2.54 Centimeters.	1 Cu. inch	= 16.39 Cu. Centimeters.
1 Foot	= 30.48 Centimeters.	1 Cu. foot	= 28320 Cu. Centimeters.
1 Yard	= .9144 Meter.	1 Cu. yard	= .7646 Cu. Meter.
1 Rod	= 5.029 Meters.	1 Cord	= 3.625 Steres.
1 Mile	= 1.6093 Kilometers.	1 Fl. ounce	= 2.958 Centiliters.
1 Sq. inch	= 6.4528 Sq. Centimeters.	1 Gallon	= 3.786 Liters.
1 Sq. foot	= 929 Sq. Centimeters.	1 Bushel	= .35242 Hectoliter.
1 Sq. yard	= .8361 Sq. Meter.	1 Troy gr.	= 64.8 Milligrams.
1 Sq. rod	= 25.29 Centares.	1 Troy lb.	= .373 Kilo.
1 Acre	= 40.47 Ares.	1 Av. lb.	= .4536 Kilo.
1 Sq. mile	= 259 Hectares = 2.59 Sq. Km.	1 Ton	= .907 Tonneau.

MISCELLANEOUS TABLES.

TIME.

208. Time is used in measuring periods of duration; as years, days, minutes, etc.

TABLE.

60	Seconds (sec.)	make	1 Minute	min.
60	Minutes	"	1 Hour	h.
24	Hours	"	1 Day	da.
7	Days	"	1 Week	wk.
365	Days	"	1 Common Year . . .	yr.
366	Days	"	1 Leap Year	yr.
12	Calendar Months	"	1 Year	yr.
100	Years	"	1 Century	C.

The calendar year is divided as follows:

Season.	Names.	Abbreviations.	No. of days.
Winter,	1st month, January,	Jan.	31
	2d " February,	Feb.	28 or 29
Spring,	3d " March,	Mar.	31
	4th " April,	Apr.	30
	5th " May,	May	31
Summer,	6th " June,	Jun.	30
	7th " July,	July	31
	8th " August,	Aug.	31
Autumn,	9th " September,	Sept.	30
	10th " October,	Oct.	31
	11th " November,	Nov.	30
Winter,	12th " December,	Dec.	31
			365 or 366

1. The period of time required for the earth to revolve about the sun, or the *true year*, is 365 da. 5 h. 48 min. 49.7 sec., or nearly 365¼ days. This exceeds the common year by 5 h. 48 min. 49.7 sec. (not quite a quarter of a day). Instead of reckoning this ¼ day each year, we consider the year as 365 days, and add the ¼, or 1 day, at the end of every fourth, bissextile, or leap year, to the month of February. But since the excess of the true year over the common year is not *quite* a quarter of a day, and the difference would amount to about a day in 400 years, we discard one leap year in every 400 years; that is, *centennial years exactly divisible by 400, and other years exactly divisible by 4, are considered as leap years. Centennial years divisible by 4, but not by 400, are not leap years.* Thus, the year 1900 will not be a leap year, but the year 1600 was.

2. Julius Cæsar, the Roman Emperor, decreed that the year should consist of 365 days 6 hours; that the 6 hours should be disregarded for 3 successive years, and an entire day be added to every fourth year. This day was inserted in the calendar between the 24th and 25th days of February, and is called the *intercalary* day. As the Romans counted the days backward from the first day of the following month, the 24th of February was called by them *sexto calendas Martii*, the sixth before the calends of March. The intercalary day which followed this was called *bis-sexto calendas Martii*; hence the name *bissextile*.

3. In 1582 the error in the calendar as established by Julius Cæsar had increased to 10 days; that is, too much time had been reckoned as a year, until the civil year was 10 days behind the solar year. To correct this error, Pope Gregory decreed that 10 entire days should be stricken from the calendar, and that the day following the 3d day of October, 1582, should be the 14th. This brought the vernal equinox at March 21 — the date on which it occurred in the year 325, at the time of the Council of Nice.

4. The year as established by Julius Cæsar is sometimes called the *Julian Year*; and the period of time in which it was in force, namely, from 46 years B.C. to 1582, is called the *Julian Period*.

5. The year as established by Pope Gregory is called the *Gregorian Year*, and the calendar now used is the *Gregorian Calendar*.

6. Most Catholic countries adopted the Gregorian Calendar soon after it was established. Great Britain, however, continued to use the Julian Calendar until 1752. At this time the civil year was 11 days behind the solar year. To correct this error, the British Government decreed that 11 days should be stricken from the calendar, and that the day following the 2d day of September, 1752, should be the 14th.

7. Time before the adoption of the Gregorian Calendar is called *Old Style* (O. S.), and since, *New Style* (N. S.). In Old Style the year commenced March 25, and in New Style it commences January 1.

8. Russia still reckons time by Old Style, or the Julian Calendar; hence her dates are now 12 days behind ours.

9. The centuries are numbered from the commencement of the Christian era; the months from the commencement of the year; the days from the commencement of the month, and the hours from the commencement of the day (12 o'clock, midnight). Thus, May 23, 1893, 9 o'clock A.M., is the 9th hour of the 23d day of the 5th month of the 93d year of the 19th century.

10. In most business transactions 30 days are called 1 month. For many purposes 4 weeks constitute a month.

11. The number of days in each calendar month may be easily remembered by committing the following lines to memory:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight; and one day more
We add to it one year in four."

EXAMPLES.

1. Reduce 365 da. 5 h. 48 min. 46 sec. to seconds.
2. Reduce 31556926 seconds to days.
3. In 3114061 seconds how many weeks are there?

4. In 5 wk. 1 da. 1 h. 1 min. 1 sec., how many seconds are there?

5. How many times does a clock pendulum, beating seconds, vibrate in one day? *Ans.* 86400.

6. If a man takes 1 step a yard long in a second, in how long a time will he walk 10 miles?

Ans. 4 h. 53 min. 20 sec.

7. In a lunar month of 29 da. 12 h. 44 min. 3 sec., how many seconds are there? *Ans.* 2551443.

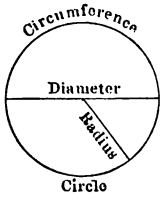
8. How much time will a person gain in 40 years, by rising 45 minutes earlier every day?

Ans. 456 da. 13 h. 30 min.

CIRCULAR OR ANGULAR MEASURE.

209. Circular or **Angular Measure**, or Circular motion, is used principally in surveying, navigation, astronomy, and geography, to measure arcs of angles or circles, for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.

A **Circle** is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within called the *center*.



That part of the circumference which is included within the lines which form the angle, is the **measure** of the **angle.**

The **Radius** of a circle is a line extending from its center to any point in the circumference. It is one half the diameter.

The **Diameter** of a circle is a line passing through its center, and terminated at both ends by the cir-

cumference. The circumference of every circle is 3.1416+ times the diameter. Hence we have the following rules:

RULE. — *To find the circumference of a circle, multiply the diameter by* 3.1416.

To find the diameter of a circle, multiply the circumference by .3183.

The rules for finding the area of a circle are derived from principles of geometry which cannot be here explained.

I. *To find the area of a circle when both diameter and circumference are given, multiply the diameter by the circumference, and divide the product by* 4.

II. *When the diameter is given, multiply the square of the diameter by* .7854.

III. *When the circumference is given, multiply the square of the circumference by* .07958.

The square of a number is the product of that number multiplied by itself.

Every circle, great or small, is divisible into the same number of equal parts, as quarters, called *quadrants*, twelfths, called *signs*, 360ths, called *degrees*, etc. Consequently the parts of different circles, although having the same names, are of different lengths. The unit is the degree.

TABLE.

60 Seconds (″)	make	1 Minute	
60 Minutes	"	1 Degree	°
30 Degrees	"	1 Sign	S.
12 Signs, or 360°,	"	1 Circle	C.

1. Minutes of the earth's circumference are called geographic or nautical miles. The length of a degree of longitude on the earth's surface at the equator is 69.16 miles.

2. The denomination, *sign* (30°), is confined exclusively to astronomy.

3. Degrees are not strictly divisions of a circle, but of the space about a point in any plane.

4. 90° make a quadrant, or right angle, 60° a sextant, or ⅙ of a circle.

EXAMPLES.

1. Reduce 10 S. 10° 10′ 10″ to seconds.

2. Reduce 1116610″ to signs.

3. How many degrees are there in 11400 geographic or nautical miles? *Ans.* 190°.

4. If one degree of the earth's circumference is 69$\frac{1}{6}$ statute miles, how many statute miles are there in 11400 geographic miles, or 190 degrees? *Ans.* 13148.

5. How many minutes, or nautical miles, are there in the circumference of the earth? *Ans.* 21600′ or mi.

6. A ship during 4 days' storm at sea changed her longitude 397 geographic miles. How many degrees and minutes did she change? *Ans.* 6° 37′.

7. The diameter of a circle is 113, and the circumference 355. What is the area? *Ans.* 10028.75.

8. What is the area of a circular island whose diameter is 1 mile?

9. A man has a circular garden requiring 84 rods of fencing to inclose it. How much land is there in the garden? *Ans.* 3 A. 81.5+ P.

210. COUNTING AND PAPER.

COUNTING.

12 Units or things	make	1	Dozen	doz.
12 Dozen	"	1	Gross	gro.
12 Gross	"	1	Great gross	G. gro.
20 Units	"	1	Score	sc.

PAPER.

24 Sheets	make	1	Quire	qre.
20 Quires	"	1	Ream	rm.
2 Reams	"	1	Bundle	bdl.
5 Bundles	"	1	Bale	B.

The terms *folio, quarto, octavo, duodecimo*, etc., indicate the number of leaves into which a sheet of paper is folded. When a sheet is folded into 2, 4, 8, 12, 16, 18, or 24 forms, it is called, respectively, a folio, quarto (4to), octavo (8vo), duodecimo (12mo), 16mo, 18mo, or 24mo.

EXAMPLES.

1. If in Birmingham, England, 150 million pens are manufactured annually, how many great gross will they make per year? *Ans.* 86805 great gross 6 gross 8 dozen.

2. In 100000 sheets of paper, how many bales are there? *Ans.* 20 bales 4 bundles 6 quires 16 sheets.

3. What is the age of a man 4 score and 10 years old?

4. How many printed pages, 2 pages to each leaf, will there be in an octavo book, having 8 fully printed sheets? *Ans.* 128 pages.

5. How large a book will ten 32mo sheets make, if every page is printed? *Ans.* 640 pages.

PROMISCUOUS EXAMPLES IN REDUCTION.

1. How many suits of clothes, each containing $6\frac{3}{4}$ yd. can be cut from $330\frac{3}{4}$ yards of cloth? *Ans.* 49.

2. A man bought a gold chain, weighing 1 oz. 15 pwt., at seven dimes a pennyweight. What did it cost?

Ans. $24.50.

3. If a human heart beats 72 times a minute, how many times will it beat in a day?

4. A man bought 1 T. 11 cwt. 12 lb. of hay, at $1\frac{1}{4}$ cents a pound. What did it cost? *Ans.* $38.90.

5. What will be the cost of a load of oats weighing 1456 pounds, at $37\frac{1}{2}$ cents per bushel? *Ans.* $17.0625.

6. If one bushel of wheat will make 45 pounds of flour, how many barrels will 1000 bushels make?

Ans. 229 bbl. 116 lb.

7. A load of wheat weighing 2430 pounds is worth how much, at $1.20 a bushel? *Ans.* $48.60.

8. I paid $12.50 for a barrel of beef. How much was that per pound? *Ans.* $6\frac{1}{4}$ cents.

9. How many steres of wood are there in a pile 4 meters long, 5 meters wide, and 6 meters high?

10. In 10 mi. 7 ch. 4 rd. 20 l., how many links are there? *Ans.* 80820 links.

11. What is the value of a city lot, 25 ft. wide and 100 ft. long, if every square inch is worth 1 cent? *Ans.* $3600.

12. How many cords of wood can be piled in a shed 50 ft. long, 25 ft. wide, and 10 ft. high?

Ans. 97 Cd. 5 cd. ft. 4 cu. ft.

13. Reduce 25 hektoliters to liters. *Ans.* 2500 l.

14. A cistern 10 ft. square and 10 ft. deep, will hold how many hogsheads of water? *Ans.* 118 hhd. $46\frac{42}{77}$ gal.

15. A bin 8 ft. long, 5 ft. wide, and $4\frac{1}{2}$ ft. high, will hold how many bushels of grain? *Ans.* $144\frac{9}{14}$ bu.

16. How many grains (troy) are there in 10 grams?

17. How many seconds less are there in every autumn than in either spring or summer? *Ans.* 86400 sec.

18. If a person could travel at the rate of a second of distance in a second of time, how much time would he require to travel round the earth? *Ans.* 15 days.

19. A printer calls for 4 reams 10 quires and 10 sheets of paper to print a book. How many sheets does he call for? *Ans.* 2170 sheets.

20. How many bushels are there in 5 hektoliters?

Ans. 14.18+ bu.

21. How many times will a wheel, 16 ft. 6 in. in circumference, turn round in running 42 miles?

22. How many days, working 10 hours a day, will it require for a person to count $10000, at the rate of 1 cent each second? *Ans.* 27 da. 7 h. 46 min. 40 sec.

23. A town, 6 miles long and $4\frac{1}{2}$ miles wide, is equal to how many farms of 80 acres each? *Ans.* 2[illegible]6.

24. At $21.75 per rod, what will be the cost of grading 10 mi. 176 rd. of road? *Ans.* $73[illegible]8.

REDUCTION OF DENOMINATE FRACTIONS.

EXAMPLES.

211. To reduce a denominate fraction from a greater to a less unit.

1. Reduce $\frac{1}{80}$ of a bushel to the fraction of a pint.

OPERATION.

bu. pt.

$\frac{1}{80} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{4}{5}$ *Ans.*

Or,

5	
8Ø	1
	4
	8
	2
5	4 = $\frac{4}{5}$ pt., *Ans.*

SOLUTION. — To reduce bushels to pints, we must multiply by 4, 8, and 2, the numbers in the scale. Since the given number is a fraction of a bushel, we indicate the process as in multiplication of fractions, and after canceling, we obtain $\frac{4}{5}$.

RULE. — *Multiply the fraction of the higher denomination by the numbers in the scale successively, between the given and the required denominations.*

1. Cancellation may be applied wherever practicable.
2. For reduction in the metric system, we reduce the fraction to a decimal and then proceed as in whole numbers by moving the decimal point.

2. Reduce $\frac{1}{1000}$ of a £ to the fraction of a penny.

Ans. $\frac{6}{25}$ *d.*

3. Reduce $\frac{1}{14400}$ of a week to the fraction of a minute. *Ans.* $\frac{7}{10}$ min.

4. What part of a gill is $\frac{1}{40320}$ of a hogshead? *Ans.* $\frac{1}{2}$ gi.

5. What fraction of a grain is $\frac{1}{960}$ of an ounce troy?

Ans. $\frac{1}{2}$ gr.

6. Reduce $\frac{1}{1000000}$ of a mile to the fraction of an inch. *Ans.* $\frac{198}{3125}$ in.

7. Reduce $\frac{2}{9}$ of $\frac{1}{6}$ of 2 pounds to the fraction of an ounce troy. *Ans.* $\frac{8}{9}$ oz.

8. Reduce $\frac{1}{680}$ of a hogshead to the fraction of a pint. *Ans.* $\frac{63}{85}$ pt.

212. To reduce a denominate fraction from a less to a greater unit.

1. Reduce $\frac{4}{5}$ of a pint to the fraction of a bushel.

OPERATION.

$\frac{\cancel{4}}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{\cancel{4}} = \frac{1}{80}$, *Ans.*

Or,

5	$\cancel{4}$
2	
8	
$\cancel{4}$	
80	1

$= \frac{1}{80}$ bu., *Ans.*

SOLUTION. — To reduce pints to bushels, we must divide by 2, 8, and 4, the numbers of the scale. Since the given number of pints is a fraction, we indicate the process, as in division of fractions, and canceling, we obtain $\frac{1}{80}$.

RULE. — *Divide the fraction of the lower denomination by the numbers in the scale, successively, between the given and the required denomination.*

The operation may frequently be shortened by cancellation.

2. What part of a rod is $\frac{1}{8}$ of a foot? *Ans.* $\frac{1}{132}$ rd.

3. What part of a pound is $\frac{3}{5}$ of a dram?

Ans. $\frac{1}{160}$ lb.

4. Reduce $\frac{1}{2}$ of a cent to the fraction of an eagle.

Ans. $\frac{1}{2000}$ E.

5. A hand is $\frac{1}{3}$ of a foot. What fraction of a mile is that?

Ans. $\frac{1}{15840}$ mi.

6. Reduce $\frac{3}{7}$ of 2 pwt. to the fraction of a pound.

Ans. $\frac{1}{2240}$ lb.

7. How much less is $\frac{3}{5}$ of a pint than $\frac{1}{2}$ of a hogshead?

Ans. $\frac{412}{840}$ hhd.

8. $\frac{3}{5}$ of an inch is what fraction of a mile?

Ans. $\frac{1}{105600}$ mi.

9. $\frac{3}{9}$ of an ounce troy is $\frac{2}{9}$ of what fraction of 2 pounds?

10. $\frac{3}{9}$ of an ounce is $\frac{1}{6}$ of what fraction of 2 pounds troy?

213. To reduce a denominate fraction to integers of lower denominations.

1. What is the value of $\frac{5}{8}$ of a hogshead?

OPERATION.

$\frac{5}{8}$ hhd. $\times 63 = \frac{315}{8}$ gal. $= 39\frac{3}{8}$ gal.

$\frac{3}{8}$ gal. $\times 4 = \frac{12}{8}$ qt. $= 1\frac{4}{8}$ qt.; $\frac{4}{8}$ qt. $\times 2 = \frac{8}{8}$ pt. $= 1$ pt.

Ans. 39 gal. 1 qt. 1 pt.

SOLUTION. — $\frac{5}{8}$ hhd. $= \frac{5}{8}$ of 63 gal., or $39\frac{3}{8}$ gal.; and $\frac{3}{8}$ gal. $= \frac{3}{8}$ of 4 qt., or $1\frac{4}{8}$ qt.; and $\frac{4}{8}$ qt. $= \frac{4}{8}$ of 2 pt., or 1 pt.

RULE. — I. *Multiply the fraction by that number in the scale which will reduce it to the next lower denomination, and if the result is an improper fraction, reduce it to a whole or mixed number.*

II. *Proceed with the fractional part, if any, as before, until reduced to the denominations required.*

III. *The units of the several denominations, arranged in their order, will be the required result.*

2. Reduce $\frac{4}{7}$ of a month to lower denominations.

Ans. 17 da. 3 h. 25 min. $42\frac{6}{7}$ sec.

3. What is the value of $\frac{3}{7}$ of a £?

Ans. 8*s.* 6*d.* $3\frac{3}{7}$ far.

4. What is the value of $\frac{2}{5}$ of a bushel?

5. Reduce $\frac{6}{7}$ of 15 cwt. to its equivalent value.

6. Reduce $\frac{2}{3}$ of $\frac{4}{9}$ of a pound avoirdupois to integers.

Ans. $4\frac{24}{27}$ oz.

7. What is the value of $\frac{5}{6}$ of an acre? *Ans.* $133\frac{1}{3}$ P.

8. Reduce $\frac{15}{26}$ of a day to its value in integers.

Ans. 16 h. 36 min. $55\frac{5}{13}$ sec.

9. What is the value of $\frac{3}{5}$ of a pound troy?

10. What is the value of $\frac{7}{9}$ of $5\frac{1}{2}$ tons?

Ans. 4 T. 5 cwt. $55\frac{5}{9}$ lb.

11. What is the value of $\frac{3}{8}$ of $3\frac{2}{3}$ acres?

Ans. 1 A. 60 P.

214. To reduce a compound denominate number to a fraction of a higher denomination.

1. What part of a week is 5 da. 14 h. 24 min.?

OPERATION.

5 da. 14 h. 24 min. = 8064 min.
1 wk. = 10080 min.
$\frac{8064}{10080} = \frac{4}{5}$ wk., *Ans.*

SOLUTION. — To find what part one compound number is of another, both must be of the same denomination. In 5 da. 14 h. 24 min. there are 8064 minutes, and in one week there are 10080 minutes. Since 1 minute is $\frac{1}{10080}$ of a week, 8064 minutes is $\frac{8064}{10080} = \frac{4}{5}$ of a week.

RULE. — *Reduce the given number to its lowest denomination for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction.*

If the given number contains a fraction, the denominator of this fraction must be regarded as the lowest denomination.

2. What part of a mi. is 266 rd. 3 yd. 2 ft.?

Ans. $\frac{5}{6}$ mi.

3. What fraction of a £ is 13*s.* 7*d.* 3 far.?

4. Reduce 10 oz. 10 pwt. 10 gr. to the fraction of a pound troy. *Ans.* $\frac{505}{5760}$ lb.

5. Reduce 2 cd. ft. 8 cu. ft. to the fraction of a cord.

Ans. $\frac{5}{16}$ Cd.

6. Reduce 1 bbl. 1 gal. 1 qt. 1 pt. 1 gi. to the fraction of a hogshead. *Ans.* $\frac{117}{224}$ hhd.

7. What part of 2 rods is 4 yards 1½ feet? *Ans.* $\frac{9}{22}$.

8. Reduce 1⅗ pecks to the fraction of a bushel.

Ans. $\frac{2}{5}$ bu.

9. What part of 9 feet square is 9 square feet?

10. From a piece of cloth containing 8¾ yd. a tailor cut 2¾ yd.; what part of the whole piece did he take? *Ans.* $\frac{3}{7}$.

11. What fraction of a week is 5 da. 4 h. 30 min.?

Ans. $\frac{83}{112}$.

215. To reduce a denominate decimal to integers of lower denominations.

1. Reduce .78125 of a pound troy to integers of lower denominations.

OPERATION.

.78125 lb.
12
9.37500 oz.
20
7.50000 pwt.
24
12.0000 gr.

9 oz. 7 pwt. 12 gr., *Ans.*

SOLUTION. — We multiply by 12 to reduce the given number from pounds to ounces, and the result is 9 oz. and the decimal .375 of an oz. Multiplying this decimal by 20 to reduce it to pennyweights, we get 7 pwt. and .5 of a pwt. This last decimal we multiply by 24, to reduce it to grains, and the result is 12 gr. The answer is 9 oz. 7 pwt. 12 gr.

RULE. — I. *Multiply the given decimal by that number in the scale which will reduce it to the next lower denomination, and point off as in multiplication of decimals.*

II. *Proceed with the decimal part of the product in the same manner until reduced to the required denominations. The integers at the left will be the answer required.*

2. Change .217° to integers of lower denominations.

Ans. 13′ 1.2″.

3. Change .659 of a week to integers of lower denominations. *Ans.* 4 da. 14 h. 42 min. 43.2 sec.

4. Reduce .578125 of a bushel to integers of lower denominations. *Ans.* 2 pk. 2 qt. 1 pt.

5. Reduce .125 bbl. to integers of lower denominations. *Ans.* 3 gal. 3 qt. 1 pt. 2 gi.

6. Reduce £.628125 to integers of lower denominations.

7. Change .22 of a hogshead of molasses to integers of lower denominations. *Ans.* 13 gal. 3 qt. 3.52 gi.

8. Change .67 of a league to integers of lower denominations. *Ans.* 2 mi. 101 rd. 6 ft. 6.24+ in.

9. Change .42857 of a month to integers of lower denominations. *Ans.* 12 da. 20 h. 34 min. $13\frac{1}{5}$ sec.

10. Reduce 5.88125 acres to integers of lower denominations. *Ans.* 5 A. 141 P.

11. Reduce .0055 T. to pounds. *Ans.* 11 lb.

12. Reduce .034375 of a bundle of paper to its value in lower denominations. *Ans.* 1 quire 9 sheets.

13. Reduce .35 lb. apoth. to integers of lower denominations.

14. Reduce 4.75 lb. troy to integers of lower denominations.

15. Reduce 2.55 lb. avoir. to integers of lower denominations.

16. Reduce .625 of a fathom to integers of lower denominations.

216. To reduce a compound denominate number to a decimal of a higher denomination.

1. Reduce 3 pk. 2 qt. to the decimal of a bushel.

OPERATION.

8 | 2.00 qt.
4 | 3.2500 pk.
.8125 bu., *Ans.*

Or, 3 pk. 2 qt. = 26 qt.
1 bu. = 32 qt.
$\frac{26}{32}$ = .8125 bu., *Ans.*

SOLUTION. — Since 8 quarts make 1 peck, and 4 pecks 1 bushel, there will be $\frac{1}{8}$ as many pecks as quarts and $\frac{1}{4}$ as many bushels as pecks.

Or we may reduce 3 pk. 2 qt. to the fraction of a bushel (as in **214**), and we have $\frac{26}{32}$ of a bushel, which, reduced to a decimal, equals .8125.

RULE. — *Divide the lowest denomination given by that number in the scale which will reduce it to the next higher, and annex the quotient as a decimal to that higher. Proceed in the same manner until the whole is reduced to the denomination required.* Or,

Reduce the given number to a fraction of the required denomination, and reduce this fraction to a decimal.

2. Reduce 3 qt. 1 pt. 1 gi. to the decimal of a gallon.
Ans. .90625 gal.

3. Reduce 10 oz. 13 pwt. 9 gr. to the decimal of a pound troy. *Ans.* .8890625 lb.

4. Reduce 1.2 pints to the decimal of a hogshead.
Ans. .00238 + hhd.

5. What part of a bushel is 3 pk. 1.12 qt.?
Ans. .785 bu.

6. What part of an acre is 132.56 P.?

7. Reduce 17 yd. 1 ft. 6 in. to the decimal of a mile.
Ans. .00994318 + mi.

8. Reduce .32 of a pint to the decimal of a bushel.
Ans. .005 bu.

9. Reduce $4\frac{7}{8}$ feet to the decimal of a fathom.
Ans. .8125 fathom.

10. Reduce 150 sheets of paper to the decimal of a ream. *Ans.* .3125 rm.

11. Reduce 47.04 lb. of flour to the decimal of a barrel.

12. Reduce .33 of a foot to the decimal of a mile.

13. Reduce 5 h. 36 min. $57\frac{6}{10}$ sec. to the decimal of a day.

14. What decimal of a league is 2 mi. 3 rd. 1 yd. $3\frac{3}{5}$ in.?

15. Change 18*s.* 5*d.* $2\frac{2}{13}$ far. to the decimal of a £.

16. Reduce 11 oz. 10 pwt. 9 gr. to the decimal of a lb. troy.

17. Reduce 126 A. 4 sq. ch. 12 P. to the decimal of a Tp.

18. Reduce 5 meters 10 dekameters to the decimal of a hektometer.

19. Reduce 1 dekagram to the decimal of a lb. avoir.

20. What decimal of a lb. troy is 2 oz. 14 pwt.?
Ans. .225 lb.

ADDITION.

EXAMPLES.

217. 1. A banker sold at one time £10 4*s.* 6*d.* 3 far. of English money; at another time, £2 9*s.* 3*d.* 1 far.; at another, 11*s.* 2 far.; and at another, £25 10*s.* 11*d.* 3 far. How much did he sell in all?

OPERATION.

£	*s.*	*d.*	far.
10	4	6	3
2	9	3	1
	11	0	2
25	10	11	3
Ans. 38	15	10	1

SOLUTION. — Arranging the numbers in columns, placing units of the *same* denomination under each other, we first add the units in the right hand column, or lowest denomination, and find the amount to be 9 farthings, which is equal to 2*d.* 1 far. We write the 1 far. under the column of farthings, and add the 2*d.* to the column of pence. The amount of the second column is 22*d.*, which is equal to 1*s.* 10*d.* Writing the 10*d.* under the column of pence, we add the 1*s.* to the next column. Adding this column the amount is 35*s.* = £1 15*s.* Placing the 15*s.* under the column of shillings, we add the £1 to the column of pounds. Adding the last column, the amount is £38.

RULE. — I. *Write the numbers so that those of the same unit value stand in the same column.*

II. *Beginning at the right hand, add each denomination as in simple numbers, carrying to each succeeding denomination* 1 *for as many units as it takes of the denomination added, to make one of the next higher denomination.*

It is customary, in business, to reduce the lower denominations to a common or decimal fraction, and to perform the operation with only one denomination.

2.

T.	cwt.	lb.
2	7	50
4	8	20
6	10	0
13	5	70

T.
$2\frac{3}{8}$
$4\frac{41}{100}$
$6\frac{1}{2}$
$13\frac{57}{200}$

T.
2.375
2.41
6.5
13.285

3.

£	s.	d.
48	13	8
51	6	4
67	11	3
76	18	10
244	10	1

4.

T.	cwt.	lb.	oz.
4	7	18	4
	15	98	15
3	9	10	6
1	0	15	0
9	12	42	9

5. Find the sum of 197 sq. yd. 4 sq. ft. 104½ sq. in., 122 sq. yd. 2 sq. ft. 27¾ sq. in., 5 sq. yd. 8 sq. ft. 2⅜ sq. in., and 237 sq. yd. 7 sq. ft. 128½ sq. in.

Ans. 563 sq. yd. 4 sq. ft. 118.825 sq. in.

When common fractions occur, they should be reduced to a common denominator, to decimals, or to integers of a lower denomination, and added according to the usual method.

6. If a printer one day uses 4 bundles 1 ream 15 quires 20 sheets of paper, the next day 3 bundles 1 ream 10 quires 10 sheets, and the next, 2 bundles 13 sheets, how much does he use in the three days?

Ans. 10 bundles 1 ream 6 quires 19 sheets.

7. A tailor used, in one year, 2 gross 5 doz. 10 buttons, another year 3 gross 7 doz. 9, and another year 4 gross 6 doz. 11. How many did he use in the three years?

Ans. 10 gross 8 doz. 6.

8. A ship, leaving New York, sailed east the first day 3° 45′ 50″; the second day, 4° 50′ 10″; the third, 2° 10′ 55″; the fourth, 2° 39″. How far was she then east from the place of starting? *Ans.* 12° 47′ 34″.

9. A man, in digging a cellar, removed 127 cu. yd. 20 cu. ft. of earth; in digging a drain, 6 cu. yd. 25 cu. ft.; and in digging a cistern, 17 cu. yd. 18 cu. ft. What was the amount of earth removed, and what the cost at 16 cents a cu. yd.? *Ans.* 152⅛ cu. yd.; $24.37½.

10. A farmer received 80 cents a bushel for 4 loads of corn, weighing as follows: 2564 lb., 2713 lb., 3000 lb., and 3109 lb. How much did he receive for the whole?

218. To add denominate fractions.

1. Add $\frac{5}{6}$ of a mile to $13\frac{1}{3}$ rods.

OPERATION.

$\frac{5}{6}$ mi. = 266 rd. 11 ft.
$13\frac{1}{3}$ rd. = 13 rd. $5\frac{1}{2}$ ft.
Ans. 280 rd. 0

Or, $13\frac{1}{3}$ rd. $\div$ 320 = $\frac{1}{24}$ mi.
$\frac{1}{24}$ mi. + $\frac{5}{6}$ mi. = $\frac{21}{24}$ mi. = 280 rd.

SOLUTION. — We find the value of each fraction in integers of lower denominations (**213**), and then add their values as in compound numbers (**217**).

Or, we reduce the given fractions to fractions of the same denomination, then add them, and find the value of their sum in lower denominations.

2. Add $\frac{3}{4}$ of a rod to $\frac{3}{4}$ of a foot. *Ans.* 13 ft. $1\frac{1}{2}$ in.

3. What is the sum of $\frac{7}{8}$ of a mile, $28\frac{1}{2}$ rods?

4. What is the sum of $\frac{2}{3}$ of a pound and $\frac{5}{9}$ of a shilling? *Ans.* 13*s.* 10*d.* $2\frac{2}{3}$ far.

5. What is the sum of $\frac{3}{5}$ of a ton and $\frac{3}{7}$ of 1 cwt.?
Ans. 12 cwt. 42 lb. $13\frac{5}{7}$ oz.

6. What is the sum of $\frac{3}{8}$ of a day added to $\frac{1}{2}$ an hour?
Ans. 9 h. 30 min.

7. What is the sum of $\frac{1}{6}$ of a week, $\frac{3}{4}$ of a day, and $\frac{1}{4}$ of an hour? *Ans.* 1 da. 22 h. 15 min.

8. Add $\frac{6}{7}$ of a hhd. to $\frac{3}{4}$ of a gal.

9. What is the sum of $\frac{4}{7}$ of a cwt., $8\frac{5}{8}$ lb., and $3\frac{9}{10}$ oz. by long ton table? *Ans.* 73 lb. $1\frac{7}{80}$ oz.

10. What is the sum of $\frac{3}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot?

11. I sold 4 village lots; the first contained $\frac{1}{2}$ of $\frac{1}{3}$ of an acre; the second, $60\frac{3}{4}$ rods; the third, $\frac{2}{7}$ of an acre; and the fourth, $\frac{3}{8}$ of $\frac{2}{9}$ of an acre. How much land was there in the four lots? *Ans.* 146 sq. rd. $126\frac{45}{112}$ sq. ft.

12. A farmer sold three loads of hay; the first weighed $1\frac{1}{6}$ T., the second, $1\frac{3}{16}$ T., and the third, $18\frac{5}{6}$ cwt. What was the aggregate weight of the three loads?
Ans. 3 T. 5 cwt. 91 lb. $10\frac{2}{3}$ oz.

SUBTRACTION.

EXAMPLES.

219. 1. If a grocer buys 25 gal. 2 qt. 1 pt. of vinegar and sells 18 gal. 3 qt., how much has he left?

OPERATION.

	gal.	qt.	pt.
	25	2	1
	18	3	0
Ans.	6	3	1

SOLUTION. — Writing the subtrahend under the minuend, placing units of the same denomination under one another, we begin at the right hand, or lowest denomination. The remainder is 1 pt. As we cannot take 3 qt. from 2 qt., we add 1 gal. or 4 qt. to 2 qt., making 6 qt., and taking 3 qt. from 6 qt., we write the remainder, 3 qt., under the denomination of quarts. Adding 1 gal. to 18 gal., we subtract 19 gal. from 25 gal., as in simple numbers, and write the remainder, 6 gal., under the column of gallons.

The example may be performed by reducing the lower denominations to fractions of a gallon, according to the usual custom in business, as illustrated below:

$25\frac{5}{8}$ gal.	25.625 gal.
$18\frac{3}{4}$	18.75
$6\frac{7}{8}$	6.875

RULE. — I. *Write the subtrahend under the minuend, so that units of the same denomination stand under each other.*

II. *Beginning at the right hand, subtract each denomination separately, as in simple numbers.*

III. *If the number of any denomination in the subtrahend exceeds that of the same denomination in the minuend, add to the number in the minuend as many units as make one of the next higher denomination, and then subtract; in this case add* 1 *to the next higher denomination of the subtrahend before subtracting. Proceed in the same manner with each denomination.*

2.

	lb.	oz.	pwt.	gr.
From	18	6	10	14
Take	10	5	4	6
Rem.	8	1	6	8

3.

	A.	P.
From	25	96.9
Take	19	145.14
Rem.	5	111.76

4.

T.	cwt.	lb.
14	11	$69\frac{3}{4}$
10	12	$98\frac{7}{8}$

5.

yr.	da.	h.	min.	sec.
38	187	16	45	50
17	190	20	50	40
20	361	19	55	10

6. A Boston merchant bought English goods to the amount of £4327 13*s.* $7\frac{1}{2}$*d.*, and he paid £1374 10*s.* $11\frac{3}{4}$*d.* How much did he then owe?

7. From 300 miles take 198 mi. 305 rd. 2 yd. 1 ft. 10 in. *Ans.* 101 mi. 14 rd. 2 yd. 2 ft. 8 in.

8. What is the difference in the longitude of two places, one 75° 20′ 30″ west, and the other 71° 19′ 35″ west? *Ans.* 4° 55″.

9. From 1892 yr. 5 mo. 19 da. take 1846 yr. 6 mo. 13 da.

10. The apparent periodic revolution of the sun is made in 365 da. 6 h. 9 min. 9 sec., and that of the moon in 29 da. 12 h. 44 min. 3 sec. What is the difference? *Ans.* 335 da. 17 h. 25 min. 6 sec.

11. From 160 acres of land, a farmer sells 16 acres 120 rods. How much does he retain?

12. A section of land containing 640 acres is owned by four men; the first owns 196 A. $96\frac{1}{4}$ P.; the second, 200 A. 60 P.; the third, 177 A. 36 P. How much does the fourth own? *Ans.* 65 A. 127.75 P.

13. From a pile of wood containing $75\frac{3}{4}$ Cd. was sold at one time 16 Cd. 5 cd. ft.; at another, 24 Cd. 6 cd. ft.

12 cu. ft.; at another, 27 Cd. 112 cu. ft. How much remained in the pile? *Ans.* 6 Cd. 3 cd. ft. 4 cu. ft.

14. If from a hogshead of molasses 10 gal. 1 qt. 1 pt. are drawn at one time, 15 gal. 1 pt. at another, and 14 gal. 3 qt. at another, how much will remain?

15. An excavation 58 ft. long 37 ft. wide and 6 ft. deep is to be made for a cellar. After 471 cu. yd. 16 cu. ft. 972 cu. in. of earth have been removed, how much more remains to be taken out?

In subtracting time when hours are to be obtained, we reckon from 12 at night, and if minutes and seconds, we write them at the right of hours.

In finding the time between two dates, or in computing interest, 12 months are considered a year, and 30 days a month.

When the *exact number of days* is required for any period not exceeding one ordinary year, it may be readily found by the following table:

TABLE.

Showing the number of days from any day of one month to the same day of any other month within one year.

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT.											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

If the days of the different months are not the same, the number of days of difference should be *added* when the earlier day belongs to the month *from* which we reckon, and *subtracted* when it belongs to the month *to* which we find the time. If the 29th of February is to be included in the time computed, one day must be added to the result.

220. To find the difference in dates.

1. What length of time elapsed from the discovery of America by Columbus, Oct. 21, 1492, to the Declaration of Independence, July 4, 1776?

OPERATION.

yr.	mo.	da.
1776	7	4
1492	10	21
283	8	13

SOLUTION. — We place the earlier date under the later, writing first on the left the number of the year from the Christian era, next the number of the month, counting January as the first month, and next the number of the day from the first day of the month, then subtract.

2. George Washington was born Feb. 22, 1732, and died Dec. 14, 1799. What was his age?

Ans. 67 yr. 9 mo. 22 da.

3. How much time has elapsed since the Declaration of Independence of the United States?

4. How many years, months, and days is it from your birthday to this date; or what is your age?

5. How long was it from the battle of Bunker Hill, June 17, 1775, to the battle of Waterloo, June 18, 1815?

Ans. 40 yr. 1 da.

6. What length of time will elapse from 20 minutes past 2 o'clock, P.M., June 24, 1892, to 10 minutes before 9 o'clock, A.M., January 3, 1897?

Ans. 4 yr. 6 mo. 8 da. 18 h. 30 min.

7. How many days is it from any day of April to the same day of August? Of December? Of February?

8. How many days is it from the 6th of November to the 15th of April? *Ans.* 160 days.

9. How many days is it from the 20th of August to the 15th of the following June? *Ans.* 299 days.

10. What length of time elapsed from the death of Henry Wadsworth Longfellow, March 24, 1882, to that of James Russell Lowell, Aug. 12, 1891?

221. To subtract denominate fractions.

1. From $\frac{3}{8}$ of an oz. take $\frac{7}{8}$ of a pwt.

OPERATION.

$\frac{3}{8}$ oz. $= 7$ pwt. 12 gr.
$\frac{7}{8}$ pwt. $=$ 21 gr.
6 pwt. 15 gr., *Ans.*

Or, $\frac{3}{8}$ oz. $\times 20 = \frac{60}{8}$ pwt.
$\frac{60}{8} - \frac{7}{8} = \frac{53}{8}$ pwt. $= 6$ pwt. 15 gr.

SOLUTION.—We perform the same reductions as in addition of denominate fractions (**218**), and then subtract the less value from the greater.

2. What is the difference between $\frac{1}{2}$ rod and $\frac{3}{4}$ of a foot? *Ans.* 7 ft. 6 in.

3. From £$\frac{5}{9}$ take $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling.

4. From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile.
Ans. 1 mi. 193 rd. 11. ft. 7.9 in. +

5. From $8\frac{9}{10}$ cwt. take $27\frac{3}{7}$ lb.
Ans. 8 cwt. 62 lb. $9\frac{1}{7}$ oz.

6. From $\frac{1}{5}$ of a week take $\frac{1}{5}$ of a day.
Ans. 1 da. 4 h. 48 min.

7. Two persons, A and B, start from two places 120 miles apart, and travel towards each other; after A travels $\frac{2}{3}$, and B $\frac{3}{7}$, of the distance, how far are they apart? *Ans.* 41 mi. 289 rd. 8 ft. $7\frac{5}{7}$ in.

8. From a cask containing 96 gallons, $\frac{1}{5}$ leaked out, and $\frac{2}{3}$ of the remainder was sold. How much still remained in the cask? *Ans.* 25 gal. 2 qt. $3\frac{1}{5}$ gi.

9. From $\frac{5}{8}$ of a gross subtract $\frac{2}{3}$ of a dozen.

10. From $\frac{7}{9}$ of a mile take $\frac{21}{32}$ of a rod.

11. From the sum of $\frac{2}{7}$ of $3\frac{3}{4}$ mi. and $17\frac{1}{7}$ rd. take $5\frac{1}{8}$ fur.

12. From the sum of $\frac{4}{5}$ lb. $4\frac{5}{6}$ oz. and $13\frac{1}{3}$ pwt. take the difference between $\frac{3}{5}$ oz. and 7 pwt.

13. From the sum of $\frac{5}{9}$ of $365\frac{1}{4}$ da. and $\frac{3}{4}$ of $5\frac{5}{9}$ wk. take $49\frac{1}{7}$ min.

MULTIPLICATION.

EXAMPLES.

222. 1. A farmer has 8 fields, each containing 4 A. 107 P. How much land has he in all?

OPERATION.	
A.	P.
4	107
	8
37	56

SOLUTION. — Eight times 107 P. are 856 P., equal to 5 A. 56 P. We write the 56 P. under the number multiplied. Eight times 4 A. are 32 A., and 5 A. added make 37 A., which we write under the same denomination in the multiplicand, and the answer is 37 A. 56 P.

RULE. — I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply as in simple numbers, and carry as in addition of compound numbers.*

2. How many acres of land are there in 4 fields, each containing 2 acres 160 rods?

It is generally better to reduce the lower denominations to the form of a fraction of the higher.

3. How many square yards of cloth are there in a piece 40 yards long and 1 yard 18 inches wide?

4. In 6 barrels of grain, each containing 2 bu. 3 pk. 5 qt., how many bushels are there? *Ans.* 17 bu. 1 pk. 6 qt.

5. If a man travels 29 mi. 150 rd. 15 ft. in 1 day, how far will he travel in 8 days?

6. If a woodchopper can cut 3 Cd. 48 cu. ft. of wood in one day, how many cords can he cut in 12 days?

Ans. 40½ Cd.

7. What is the weight of 48 loads of hay, each weighing 1 T. 3 cwt. 50 lb.?

8. If 1 acre of land produces 45 bu. 3 pk. 6 qt. 1 pt. of corn, how much will 64 acres produce? *Ans.* 2941 bu.

9. How much will 120 yards of cloth cost, at £1 9*s.* per yard?

DIVISION.

EXAMPLES.

223. 1. If 4 acres of land produce 102 bu. 2 pk. of wheat, how much will 1 acre produce?

OPERATION.

	bu.	pk.	qt.
4)	102	2	0
	25	2	4

SOLUTION. — We divide 102 bu. by 4, and obtain a quotient of 25 bu., and a remainder of 2 bu. We write the 25 bu. under the denomination of bushels, and reduce the 2 bu. to pecks, making 8 pk., and the 2 pk. of the dividend added make 10 pk. Dividing 10 pk. by 4, we obtain a quotient of 2 pk. and a remainder of 2 pk.; writing the 2 pk. under the order of pecks, we next reduce 2 pk. to quarts, making 16 qt., which divided by 4 gives a quotient of 4 qt.

By using the fractional form, the following more simple operation may be performed.

$$4)\overline{102\tfrac{1}{2}}\text{ bu.} \qquad 25\tfrac{5}{8}$$

$$4)\overline{102.50}\text{ bu.} \qquad 25.625$$

RULE. — I. *Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner if there is no remainder.*

II. *If there is a remainder after dividing any denomination, reduce it to the next lower denomination, adding in the given number of that denomination, if any, and divide as before.*

III. *The several partial quotients will be the quotient required.*

1. When the divisor is large and is a *composite* number, we may shorten the work by dividing by the factors.

2. When the divisor and dividend are both compound numbers, they must both be reduced to the same denomination before dividing, and then the process is the same as in simple numbers.

2.

	£	s.	d.
5)	25	8	4
	5	1	8

3.

	T.	cwt.	lb.
7)	45	15	25
	6	10	75

4. Divide 358 A. 57 P. 6 sq. yd. 2 sq. ft. by 7.

Ans. 51 A. 31 P. 8 sq. ft.

5. Divide 336 yd. 4 ft. $3\frac{1}{3}$ in. by 21. *Ans.* 16 yd. $2\frac{4}{9}$ in.

6. Divide 77 sq. yd. 5 sq. ft. 82 sq. in. by 13.

Ans. 5 sq. yd. 8 sq. ft. 106 sq. in.

7. Divide 1986 mi. 3 fur. 20 rd. 1 yd. by 108.

8. Divide 365 da. 6 h. by 240.

9. Divide 3794 cu. yd. 20 cu. ft. $709\frac{1}{3}$ cu. in. by $33\frac{1}{3}$.

10. Divide 121 lb. 3 ℥ 2 ʒ 1 ℈ 4 gr. by $13\frac{3}{4}$.

11. Divide 28° 51′ 27.765″ by 2.754.

12. Divide 1950 da. 15 h. $15\frac{5}{9}$ min. by 100.

13. How many casks, each containing 2 bu. 3 pk. 6 qt., can be filled from 356 bu. 3 pk. 5 qt. of cherries?

14. I bought 6 large silver spoons, which weighed 11 oz. 3 pwt. What was the weight of each spoon?

15. A man traveled by railroad 1000 miles in one day. What was the average rate per hour?

Ans. 41 mi. 213 rd. 5 ft. 6 in.

16. If a family uses 10 bbl. of flour in a year, what is the average amount used each day? *Ans.* 5 lb. $5\frac{87}{146}$ oz.

17. The aggregate weight of 123 hogsheads of sugar is 57 T. 19 cwt. 42 lb. 14 oz. What is the average weight per hogshead? *Ans.* 9 cwt. 42 lb. 10 oz.

18. How many times are £5 10*s.* 10*d.* contained in £537 10*s.* 10*d.*? *Ans.* 97 times.

19. A cellar 50 feet long, 30 ft. wide, and 6 ft. deep was excavated by 5 men in 6 days. How many cubic yards did each man excavate daily?

Ans. 11 cu. yd. 3 cu. ft.

20. If a town 5 miles square is divided equally into 150 farms, what will be the size of each farm?

Ans. 106 A. 106 P. 20 sq. yd. 1 sq. ft. 72 sq. in.

21. How many times are 4 bu. 3 pk. 2 qt. contained in 336 bu. 3 pk. 4 qt.? *Ans.* 70 times.

LONGITUDE AND TIME.

224. Every circle is supposed to be divided into 360 equal parts, called *degrees*.

Since the sun appears to pass from east to west round the earth, or through 360°, once in every 24 hours, it will pass through $\frac{1}{24}$ of 360°, or 15° of the distance, in 1 hour; and 1° of distance in $\frac{1}{15}$ of 1 hour, or 4 minutes; and 1′ of distance in $\frac{1}{60}$ of 4 minutes, or 4 seconds, etc.

TABLE OF LONGITUDE AND TIME.

360° of longitude	=	24 hours, or 1 day	of time.
15° " "	=	1 hour	" "
15′ " "	=	1 minute	" "
15″ " "	=	1 second	" "
1° " "	=	4 minutes	" "
1′ " "	=	4 seconds	" "

EXAMPLES.

225. To find the difference of time between two places, when their longitudes are given.

1. The longitude of Boston is 71° 3′, and of Chicago 87° 30′. What is the difference of time between these two places?

OPERATION.

$$\begin{array}{rr} 87° & 30' \\ 71° & 3' \\ \hline 16° & 27' \\ & 4 \\ \hline & 65' \quad 48'' = \end{array}$$

1 h. 5 min. 48 sec.

Or,

$$15 \overline{)\, 16° \quad 27'}$$

1	5	48
h.	min.	sec.

SOLUTION. — By subtraction of compound numbers we first find the difference of longitude between the two places, which is 16° 27′. Since 1° of longitude makes a difference of 4 minutes of time, and 1′ of longitude a difference of 4 seconds of time, we multiply 16° 27′, the difference in longitude, by 4, and we obtain the difference of time in minutes and seconds, which, reduced to higher denominations, gives 1 h. 5 min. 48 sec., the difference in time. Or, since places distant from each other 15° of longitude differ 1 h. in time; 15′ 1 minute in time, etc., $\frac{1}{15}$ of 16° 27′ is the difference in hours, minutes, and seconds of time.

RULE. — *Multiply the difference of longitude in degrees and minutes by* 4, *and the product will be the difference of time in minutes and seconds, which may be reduced to hours.* Or,

Divide the difference of longitude in degrees, minutes, and seconds by 15, *and the quotient will be the difference of time in hours, minutes, and seconds.*

1. Multiplying by 4 gives an answer 60 times as great as dividing by 15, but since by the first operation the answer is in minutes (which we then reduce to hours), and by the second in hours, the first number should be 60 times as great as the second, hence we see that both the above methods give the same result.

2. If one place is in east, and the other in west longitude, the difference of longitude is found by *adding* them, and if the sum is greater than 180°, it must be subtracted from 360°.

2. New York is 74° 1′, and Cincinnati 84° 24′ west longitude. What is the difference of time?

Ans. 41 min. 32 sec.

3. The Cape of Good Hope is 18° 28′ east, and the Sandwich Islands 155° west longitude. What is the difference of time? *Ans.* 11 h. 33 min. 52 sec.

4. If a message is sent by telegraph without any loss of time, at 12 M. from London, 0° 0′ longitude, to Washington, 77° 1′ west, what is the time of its receipt at Washington?

Since the sun appears to move from east to west, when it is exactly 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, knowing the difference of time between two places and the exact time at one of them, the exact time at the other will be found by *adding* their difference to the given time if it is *east*, and by *subtracting* if it is *west*.

Ans. 6 h. 51 min. 56 sec. A.M.

5. Washington is 77° 1′ west, and St. Petersburg 30° 19′ east longitude. What is their difference of time?

Ans. 7 h. 9 min. 20 sec.

6. A steamer arrives at Halifax, 63° 36′ west, at 4 o'clock, P.M.; the fact is telegraphed to St. Louis, 90° 15′ west, without loss of time. What is the time of its receipt at St. Louis? *Ans.* 2 h. 13 min. 24 sec. P.M.

7. If Pekin is 118° east, and San Francisco 122° west longitude, what is their difference of time?

8. If, at a presidential election, the voting begins at sunrise and ends at sunset, how much sooner will the polls open and close at Eastport, Me., 67° west, than at Portland, Oregon, 124° west? *Ans.* 3 h. 48 min.

9. When it was 1 o'clock, A.M., on the first day of January, 1891, at Bangor, Me., 68° 47′ west, what was the time at the city of Mexico, 99° 5′ west?

Ans. Dec. 31, 1890, 58 min. 48 sec. past 10, P.M.

226. To find the difference of longitude between two places, when the difference of time is known.

1. If the difference of time between New York and Cincinnati is 41 min. 32 sec., what is the difference of longitude?

OPERATION.

min.	sec.
4) 41	32
10°	23′, *Ans.*

Or,

min.	sec.
41	32
	15
623′	0″ =
10°	23′, *Ans.*

SOLUTION. — Since 4 minutes of time make a difference of 1° of longitude, and 4 seconds of time, a difference of 1′ of longitude, there will be $\frac{1}{4}$ as many degrees of longitude as there are minutes of time, and $\frac{1}{4}$ as many minutes of longitude as there are seconds of time. Or, since there are 15 times as many degrees, minutes, and seconds of longitude as there are hours, minutes, and seconds of time, we multiply the time by 15.

RULE. — *Reduce the difference of time to minutes and seconds, and then divide by 4; the quotient will be the difference in longitude, in degrees and minutes.* Or,

Multiply the difference in minutes and seconds of time by 15, *the product will be the difference in longitude in minutes and seconds.*

Dividing by 4 gives an answer $\frac{1}{60}$ as great as multiplying by 15, but since by the first operation the answer is in degrees, and by the second in minutes (which we then reduce to degrees), the first number should be $\frac{1}{60}$ as great as the second; hence we see that both methods are correct and give the same result.

2. What is the difference of longitude between the Cape of Good Hope and the Sandwich Islands, if the difference of time is 11 h. 33 min. 52 sec.?

Ans. 173° 28′.

3. What is the difference of longitude between Washington and St. Petersburg, if their difference of time is 7 h. 9 min. 20 sec.? *Ans.* 107° 20′.

STANDARD TIME.

227. To avoid the inconvenience arising from the difference in time between places comparatively near (as the difference of 12 minutes between New York and Boston), the following standard time divisions have been adopted by the railroads, and local time is now generally superseded by standard time.

228. Standard Time Divisions as adopted by the Railroads. — EASTERN STANDARD. — 75th meridian. Canada, between Quebec and Detroit; U.S. east of Buffalo, N.Y.; Pittsburgh, Pa.; Wheeling and Huntington, W.Va.; Bristol, Tenn.; Charlotte, N.C., and Augusta, Ga.

CENTRAL STANDARD. — 90th meridian. West from "Eastern" limits, as above, to Broadview, Canada; to the Missouri River in Dakota; North Platte and McCook, Neb.; Wallace and Dodge City, Kan.; Toyah and Sanderson, Tex.

MOUNTAIN STANDARD. — 105th meridian. West from "Central" limits to Heron, Mont.; Ogden, Utah; Needles and Yuma, Ariz.

PACIFIC STANDARD. — 120th meridian. West from "Mountain" limits to coast.

229. As a matter of fact, standard time coincides with local time in very few places in the United States, and therefore for accurate calculations the correction is constantly needed.

Eastern standard time — that of the 75th meridian — coincides very nearly with local time at Potsdam and Herkimer, N.Y.; Camden and Cape May, N.J., and Philadelphia, Pa.

Central standard time — that of the 90th meridian — is practically local time for Bessemer, Mich.; Mt. Carrol, Ill.; St. Louis, Mo.; Memphis, Tenn.; Jackson, Miss.; and New Orleans, La.

Mountain time is very nearly local time at Cheyenne, Wy.; Denver, Col.; and Las Vegas, N.M.

Pacific time is very close to local time at Reno and Carson City, Nev., and Santa Barbara, Cal.

While the standard meridians are exactly 15° apart, the belts using the several meridian times vary, according to convenience, from 10° to 25° in width. This results in some singular variations. For instance, El Paso, Tex., has the same standard time as Savannah, Ga., though there is a difference of 1 h. 45 min. in local time. On the other hand, San Diego, Cal., has two hours later standard time than El Paso, although there is a difference of but 42 min. in local time.

By knowing the meridian, which is the standard for any place, and the distance of the meridian from the place in degrees east or west, local time can be converted into standard time.

For a place *east* of its standard meridian, the time of the rising or setting of any heavenly body may be expressed in standard time by subtracting from the almanac calendar time one minute of time for every 15′ of longitude, or 4 minutes of time for every degree of longitude that the place is east from the standard meridian.

For a place *west* of its standard meridian, the time of the rising or setting of the sun or moon may be found by adding to the almanac calendar time one minute of time for every 15′ of longitude, or 4 minutes of time for every degree of longitude that the place is west of the standard meridian.

230. Corrections for the following Cities.

Eastern Standard. 75° Longitude.		Central Standard. 90° Longitude.		Mountain Standard. 105° Longitude.	
	Min.		Min.		Min.
Bangor, Me.	− 25	Cleveland, Ohio	− 33	Denver, Col.	
Augusta, Me.	− 21	Columbus, Ohio	− 28	Salt Lake City, Utah.	+ 28
Portland, Me.	− 19	Detroit, Mich.	− 28		
Boston, Mass.	− 16	Toledo, Ohio	− 26	Pacific Standard. 120° Longitude.	
Newport, R.I.	− 15	Dayton, Ohio	− 23		
Providence, R.I.	− 14	Cincinnati, Ohio	− 22	Sacramento, Cal.	+ 6
Concord, N.H.	− 14	Louisville, Ky.	− 18	San Francisco, Cal.	+ 10

EXAMPLES.

1. Buffalo is about 79° west longitude. What is the difference between standard time and actual time in that city?

2. Cincinnati is 84° 24′ west longitude. When it is noon by actual time, what is the hour by standard time?

3. Washington is about 77° west longitude. When it is noon by standard time, what is the actual time?

4. What is the longitude of Bangor? Of Newport? Of Louisville? Of Detroit? Of San Francisco?

NOTE. — Refer to table of corrections.

5. When the sun rises in Boston at 6 o'clock in the morning, as indicated by the almanac, at what hour does it rise by standard time?

6. The longitude of Springfield, Mass., is 72° 35′ 45″ W., and of Galveston, Tex., 94° 46′ 34″ W.; when it is 20 min. past 6 o'clock, A.M., at Springfield, what time is it at Galveston? What is the difference between the actual time and standard time?

PROMISCUOUS EXAMPLES.

1. In 115200 grains troy, how many pounds are there?

2. In 365 da. 5 h. 48 min. 46 sec., how many seconds are there? *Ans.* 31556926.

3. A man wishes to ship 1560 bushels of potatoes in barrels containing 3 bu. 1 pk. each. How many barrels will be required? *Ans.* 480 bbl.

4. Reduce 295218 inches to miles.

5. Reduce 456575 grains to pounds, troy weight.

6. How many sheets are there in 3 reams of paper?

7. What is the value of 4 piles of wood, each 20 ft. long, 6 ft. wide, and 10 ft. high, at $3.25 per cord?

Ans. $121.87½.

8. How many bottles, each holding 1 qt. 1 gi., can be filled from a barrel of cider? *Ans.* 112 bottles.

9. At $26.40 per sq. rd. for land, what will be the cost of a village lot 8¼ rd. long and 4½ rd. wide?

Ans. $980.10.

10. Divide 259 A. 50 P. of land into 36 equal lots.

Ans. 7 A. 32½ P.

11. How many times can a box holding 4 bu. 3 pk. 2 qt. be filled from 336 bu. 3 pk. 4 qt.? *Ans.* 70 times.

12. What is the value of .875 of a gallon?

13. What part of a mile is 116 rd. 2 yd.? *Ans.* $\frac{4}{11}$ mi.

14. What part of 2 days is 13 h. 26 min. 24 sec.?

15. From 26 A. 80 P. of land, 5 A. 120 P. were sold. What part of the whole piece remained unsold?

Ans. $\frac{83}{106}$.

16. What is the difference between $\frac{3}{5}$ of a pound sterling and 5¼ pence? *Ans.* 11$\frac{1}{2}$, 6¾*d.*

17. What is the sum of $\frac{1}{7}$ of a yard, $\frac{1}{7}$ of a foot, and $\frac{1}{7}$ of an inch? *Ans.* 7 inches.

18. Reduce 3 cwt. 1 qr. of coal to the decimal of a ton.

19. Benjamin Franklin was born Jan. 18, 1706, and George Washington Feb. 22, 1732. How much older was Franklin than Washington? *Ans.* 26 yr. 1 mo. 4 da.

20. The longitude of Boston is 71° 4′ west, and that of Chicago 87° 30′ west. When it is 12 M. at Boston, what is the time in Chicago? *Ans.* 10 h. 54 min. 16 sec. A.M.

21. If the difference of time between New York and New Orleans is 1 h. 4 sec., what is the difference in longitude? *Ans.* 15° 1′.

22. Add $\frac{2}{3}$ of a mile, $\frac{1}{16}$ of a mile, and $\frac{3}{16}$ of a rod together. *Ans.* 233 rd. 8 ft. 3 in.

23. If a bushel of barley costs \$.80, what will 20 bu. 3 pk. 6 qt. cost? *Ans.* \$16.75.

24. Change .875 of a gross to dozens. *Ans.* $10\frac{1}{2}$ doz.

25. How many acres are there in a field $56\frac{1}{2}$ rods long, and 24.6 rods wide? *Ans.* 8 A. 109.9 P.

26. How many perches of masonry are there in the wall of a cellar which is 20 ft. square on the inside, 8 ft. high, and $1\frac{1}{2}$ ft. in thickness? *Ans.* 44.6+ P.

27. A, B, and C rent a farm, and agree to work it upon shares; they raise 640 bu. 3 pk. of grain, which they divide as follows: one fourth is given for the rent; of the remainder A takes $10\frac{1}{2}$ bu. more than one third, after which B takes one half of the remainder less 7 bushels, and C has what is left. How much is C's share?

Ans. 161 bu. 3 pk. 6 qt.

28. What is the value in troy weight of 13 lb. 8 oz. avoirdupois weight? *Ans.* 16 lb. 4 oz. 17 pwt. 12 gr.

29. If 154 bu. 1 pk. 6 qt. cost \$173.74, how much will 1.5 bushels cost? *Ans.* \$1.687.

30. Reduce .0125 of a ton to pounds. *Ans.* 25 lb.

31. What fraction of 3 bushels is $\frac{7}{12}$ of 2 bu. 3 pk.?

Ans. $\frac{77}{144}$.

32. How many wine gallons are there in a water tank 4 ft. long, $3\frac{1}{2}$ ft. wide, and 1 ft. 8 in. deep? *Ans.* $174\frac{6}{11}$.

33. How many bushels will a bin contain that is $7\frac{1}{2}$ ft. square, and 6 ft. 8 in. deep? *Ans.* 301.339+ bu.

34. How much must be paid for lathing and plastering overhead a room 36 ft. long and 20 ft. wide, at 26 cents a square yard?

35. How many shingles will it take to cover the roof of a building 46 ft. long, each of the two sides of the roof being 20 ft. wide, allowing each shingle to be 4 in. wide, and to lie 5 in. to the weather? *Ans.* 13248.

36. John Young was born at a quarter before 4 o'clock, A.M., Sept. 4, 1888. What will be his age at half-past 6 o'clock, P.M., April 20, 1945?

Ans. 57 yr. 7 mo. 16 da. 14 h. 45 min.

37. How many cubic yards of earth were removed in digging a cellar 28 ft. 9 in. long, 22 ft. 8 in. wide, and 7 ft. 6 in. deep? *Ans.* $181\frac{1}{54}$ cu. yd.

38. What will 30 bu. 54 lb. of wheat cost, at $\$1.37\frac{1}{2}$ per bushel? *Ans.* \$ 42.4875.

39. What is the cost of 54 bu. 8 lb. of barley, at 84 cents per bushel? *Ans.* \$ 45.50.

40. How many square yards of carpeting will it take cover a floor 24 ft. 8 in. long and 18 ft. 6 in. wide?

Ans. $50\frac{19}{27}$ sq. yd.

41. What is the area of a rectangle 17 cm. by 19 cm.?

42. What is the depth of a lot that has 120 ft. front, and contains 18720 square feet?

43. If I travel 789.7 km. a day, how many miles will I travel in 10 days?

44. How many steps of 30 in. each must a person take in walking 21 miles?

45. How many steres of wood are there in a pile 4 m. long, 3.5 m. thick, and 32.25 m. high?

MEASUREMENTS.

LAND.

230a. The **Unit** of land measure is the *acre.*

Measurements of land are commonly recorded in *square miles*, *acres*, and *hundredths* of an acre. The ordinary table of land measure with examples will be found on pp. 173 to 177.

230b. Government Lands are usually surveyed into rectangular tracts, bounded by lines conforming to the cardinal points of the compass.

A *Base Line* on a parallel of latitude, and a *Principal Meridian* intersecting it, are first established. Other lines are then run *six miles* apart, each way, as nearly as possible.

The tracts thus formed are called *Townships*, and contain nearly 23040 acres. A line of townships extending north and south is called a *Range.*

The *ranges* are designated by their number east or west of the *principal meridian.* The *townships* in each range are designated by their number north or south of the *base line.*

Since the earth's surface is convex, the principal meridians converge as they proceed northward. This tends to throw the townships and sections out of square, and necessitates occasional lines of offset, called "*correction lines:*"

Townships are subdivided into *Sections*, and sections into *Half-Sections*, *Quarter-Sections*, *Half-Quarter-Sections*, *Quarter-Quarter-Sections*, and *Lots.*

DIAGRAM No. 1.

A TOWNSHIP.

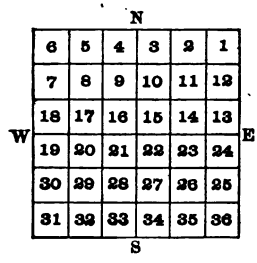


DIAGRAM No. 2.

A SECTION.

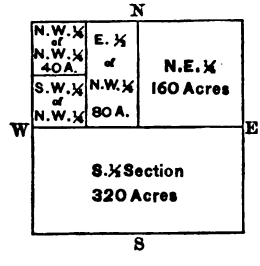


Diagram No. 1 shows the subdivisions of a *Township* into Sections, and how they are *numbered*, commencing at the N.E. corner.

Diagram No. 2 shows the subdivisions of a *Section*, on an enlarged scale, and how they are *named*.

TABLE.

6 mi.	× 6 mi.	= 36 sq. mi.	=	23040	Acres	= 1 Township.
1 "	× 1 "	= 1 "	=	640	"	= 1 Section.
1 "	× ½ "	= ½ "	=	320	"	= 1 Half-Section.
½ "	× ½ "	= ¼ "	=	160	"	= 1 Quarter-Section.
½ "	× ¼ "	= ⅛ "	=	80	"	= 1 Half-Quarter-Section.
¼ "	× ¼ "	= 1/16 "	=	40	"	= 1 Quarter-Quarter-Section.

A *Lot* is a subdivision of a section, usually of irregular form, on account of bordering upon a navigable river or lake — containing as nearly as possible the area of a Quarter-Quarter-Section, and described as lot No. 1, 2, 3, etc., of a particular section.

City and village plots are usually subdivided into *Blocks*, and these into *Lots*.

EXAMPLES.

1. If a township of land is equally divided among 288 families, how many acres does each receive? What part of a section?

2. What number of rails will inclose a quarter-section of land with a fence 6 rails high, and 3 lengths for every 2 rods; and what will be the cost of the rails, at $40 per thousand?

3. A man bought the S. $\frac{1}{2}$ of a section of land at $\$2\frac{1}{4}$ an acre, and afterward sold the E. $\frac{1}{2}$ of what he bought at $\$4.37\frac{1}{2}$ an acre. What did he gain on what he sold?

Ans. $\$340.$

4. If I buy the N. E. $\frac{1}{4}$ and the E. $\frac{1}{2}$ of N. W. $\frac{1}{4}$ of a section of land, how many acres do I purchase? What part of a whole section? How are the parts located in respect to each other?

5. A speculator bought of the Ill. Central R.R. Co., the S. $\frac{1}{2}$ of Section 4, township 10 north, range 6 east, at $\$2$ an acre. He afterward sold the E. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ at $\$2.75$ an acre; the N. W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$ at $\$3\frac{1}{2}$ an acre; and the N. $\frac{1}{2}$ of S. W. $\frac{1}{4}$ at $\$3.84$ an acre. How many acres has he left? What was his gain on the purchase price of the whole? Draw diagram. *Ans.* $\$27.20.$

230c. The old **French Linear,** and **Land Measure,** is still used to some extent in Louisiana, and in other French settlements in the United States.

TABLE.

12 Lines = 1 Inch.	6 Feet = 1 Toise.
12 Inches = 1 Foot.	32 Toises = 1 Arpent.
900 Square Toises = 1 Square Arpent.	

The *French Foot* equals 12.8 inches, American, nearly.

The *Arpent* is the old French name for *Acre*, and contains nearly $\frac{5}{6}$ of an English Acre.

In Texas, New Mexico, and in other Spanish settlements of the United States, the following denominations are still used:

TABLE.

1000000 Square Varas	= 1 Labor	= 177.136	Acres (American).
25 Labors	= 1 League	= 4428.4	Acres "

The *Spanish Foot* = 11.11 + in. (Am.); 1 Vara = $33\frac{1}{3}$ in. (Am.); 108 Varas = 100 Yards, and 1900.8 Varas = 1 Mile.

MASONRY AND PAVING.

230d. Masonry is estimated by the *cubic foot*, and by the *perch;* also by the *square foot* and the *square yard.*

1. *Materials* are usually estimated by cubic measure; the *work* by cubic or square measure.

2. A *Perch* of stone, or of masonry, is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, and is equal to 24.75 cu. ft.

3. When stone is built into a wall, 22 cu. ft. make a perch, $2\frac{3}{4}$ cu. ft. being allowed for mortar and filling.

4. *Embankments* and *Excavations* are estimated by the *cubic yard.*

5. A cubic yard of common earth is called a *load.*

6. *Brickwork* is generally estimated by the *thousand bricks;* sometimes in *cubic feet.* In *walls*, brickwork is estimated at the rate of a brick and a half thick.

7. North River bricks are 8 in. $\times$ $3\frac{1}{2}$ $\times$ $2\frac{1}{4}$; Maine bricks are $7\frac{1}{2}$ in. $\times$ $3\frac{3}{8}$ $\times$ $2\frac{3}{8}$; Philadelphia and Baltimore bricks are $8\frac{1}{4}$ in. $\times$ $4\frac{1}{8}$ $\times$ $2\frac{3}{8}$; and Milwaukee bricks, $8\frac{1}{2}$ in. $\times$ $4\frac{1}{8}$ $\times$ $2\frac{3}{8}$.

8. In estimating *material*, allowance is made for doors, windows, and cornices.

9. In estimating the *work*, masons measure each wall on the *outside*, and ordinarily no allowance is made for doors, windows, and corners; but sometimes an allowance of *one half* is made, this being, however, a matter of *contract.*

EXAMPLES.

230e. To find the number of bricks in a cu. ft. of masonry.

RULE. — I. *Add to the face dimensions of the kind of bricks used, one half the thickness of the mortar or cement in which they are laid, and compute the area.*

II. *Multiply this area by the quotient of the thickness of the wall divided by the number of bricks of which it is composed; the product will be the volume of a brick and its mortar in cubic inches.*

III. *Divide* 1728 *by this volume, and the quotient will be the number of bricks in a cubic foot.*

1. How many Milwaukee bricks are there in a cubic foot of wall $12\frac{3}{4}$ in. wide, laid in courses of mortar $\frac{1}{4}$ of an inch thick?

OPERATION.

8.5 + .25 = 8.75 in. = *length* of brick and joint.
2.375 + .25 = 2.625 in. = *thickness* of brick and joint.
8.75 × 2.625 = 22.96875 sq. in. = *area* of its face.
12.75 ÷ 3 (number of bricks in width of wall) = 4.25 in. = *width* of brick and mortar.
22.96875 × 4.25 = 97.617+ = *cubic inches* in a brick.
1728 ÷ 97.617+ = 17.7+ = *number of bricks* in a cubic foot.

2. How many bricks, 8 in. × 4 × 2, will be required to build a wall 42 ft. long, 24 ft. high, and $16\frac{1}{2}$ in. thick, laid in courses of mortar $\frac{1}{4}$ of an inch thick?

3. How many perches of stone, laid dry, will build a wall 60 ft. long, $16\frac{1}{2}$ ft. high, and 18 in. thick?

RULE. — I. *Multiply the number of cubic feet in the wall, or work to be done, by the number of bricks in a cu. ft.; the product will be the number of bricks required.*

II. *Divide the number of cubic feet in the work to be done by* 24.75 ; *the quotient will be the number of perches.*

4. How many perches of masonry are there in a wall 120 ft. long, 6 ft. 9 in. high, and 18 in. thick?

5. What will be the cost of building a wall 60 ft. long, $21\frac{7}{8}$ ft. high, and 17 in. thick, of Philadelphia bricks, laid in courses of mortar $\frac{1}{4}$ of an inch thick, at $\$12\frac{1}{2}$ per M.? *Ans.* $\$423.52$.

6. A street 650 ft. long and 72 ft. wide, averages 4.5 ft. below grade. Find the cost of filling it in, at $\$.42$ a cubic yard.

7. Find the cost of paving a sidewalk 4 ft. wide, and 63 ft. long, at $\$.21$ per sq. ft. *Ans.* $\$52.92$.

8. What will it cost to pave a sidewalk 250 ft. × 12 ft. with North River bricks, 42 to a square yard, laid flat, at $\$9$ per M. *Ans.* $\$126$.

BOARDS AND TIMBER.

230f. A Board Foot is 1 ft. long, 1 ft. wide, and 1 in. thick. Hence 12 board ft. make 1 cu. ft.

Board feet are changed to cubic feet by dividing by 12; and cubic feet are changed to board feet by multiplying by 12.

1. In *Board Measure* all boards are assumed to be 1 inch thick.

2. *Lumber* and *sawed timber*, as plank, scantling, etc., are usually estimated in *board* measure, *hewn* and *round timber* in *cubic* measure.

RULE. — I. *When lumber is not more than* 1 *inch thick, multiply the length in feet by the width in inches, and divide the product by* 12.

II. *When it is more than* 1 *inch thick, multiply the length in feet by the width and thickness in inches, and divide the product by* 12.

EXAMPLES.

1. Find the contents of a board 15 ft. long, 8 in. wide.

OPERATION.

$\overline{15 \times 8} \div 12 = 10$ board feet.

2. What are the contents, in board measure, of a joist 16 ft. long, 10 in. wide, and 3 in. thick?

OPERATION.

$\overline{3 \times 10 \times 16} \div 12 = 40$ board feet.

3. How many board feet are there in 4 boards 16 ft. long, 10 in. wide?

4. How many board feet are there in 2 joists 17 ft. long, 11 in. wide, and 3 in. thick?

5. Find the contents of a board 18 ft. long, 1 ft. 8 in. wide at one end, and 14 in. at the other.

OPERATION.

$\overline{20 \text{ in.} + 14 \text{ in.}} \div 2 = 17$ in.; $\overline{18 \times 17} \div 12 = 25\frac{1}{2}$ board ft.

6. Find the cost of 5 boards 12 ft. long, 17 in. wide at one end and 11 in. at the other, at 6 cents a square foot.

7. Find the cost of 10 planks, each 15 ft. long, 16 in. wide, and $3\frac{1}{2}$ in. thick, at \$2.25 per hundred feet.

8. What length of board 9 in. wide contains 8 board ft.?

OPERATION.

$\overline{144 \times 8} \div 9 = 128$; $128 \div 12 = 10\frac{2}{3}$ ft., the length.

9. What length may be cut from a board 15 ft. long and 20 in. wide, so as to leave 15 board feet?

10. What must be the width of a board 16 ft. long, to contain 12 board feet?

OPERATION.

16 ft. = 192 in.; $\overline{144 \times 12} \div 192 = 9$ in., the width.

11. What must be the width of a piece of board 5 ft. 3 in. long, to contain 7 square feet?

12. Find the cost of 3 pieces of timber, each 26 ft. long and 6 in. by 9 in., at \$1.75 per hundred board feet.

13. Find the cost of 8 pieces of scantling, 3 in. by 4 in. and 14 ft. long, at \$9.50 per thousand board feet.

14. What length of a piece of timber 6 in. by 9 in., will contain 3 cubic feet?

OPERATION.

$\overline{1728 \times 3} \div \overline{9 \times 6} = 96$; $96 \div 12 = 8$ ft., the length.

15. A piece of timber is 10 in. by 16 in. What length of it will contain 15 cubic feet?

16. What amount of inch lumber will make a box 4 ft. by 3 ft. 6 in. by 2 ft. 6 in. on the outside?

17. Find the cost of 36 boards, 12 ft. long, 11 in. wide, @ \$$2\frac{1}{2}$ per C.

18. Find the cost of 16 planks, $14\frac{1}{2}$ ft. long, 10 in. wide, and 3 in. thick, @ \$$16\frac{1}{4}$ per M.

CAPACITY OF BINS, CISTERNS, ETC.

230g. The **Standard Bushel** of the United States contains 2150.42 cubic inches, and is a cylindrical measure $18\frac{1}{2}$ inches in diameter and 8 inches deep **(203)**.

1. Measures of capacity are all *cubic* measures, solidity and capacity being measured by different units, as seen in the tables.

2. Grain is shipped from New York by the *quarter* of 480 lb. (8 U. S. bu.), or by the *ton* of $33\frac{1}{3}$ U. S. bushels.

3. It is sufficiently accurate in practice to call 5 *stricken* measures equal to 4 *heaped* measures.

4. Ordinary anthracite coal measures from 36 to 40 cu. ft. to the ton; bituminous coal, from 36 to 45 cu. ft. to the ton.

5. Lehigh, white ash, egg size, measures about $34\frac{1}{2}$ cu. ft. to the ton (2000 lb.); Schuylkill, white ash, 35 cu. ft.; and of gray or red ash, 36 cu. ft. to the ton.

6. Coal is bought and sold in large quantities by the ton; in small quantities by the bushel, the conventional rate being 28 bu. (5 pecks) to a ton, or about 43.5 cu. ft.

EXAMPLES.

230h. To find the exact capacity of a bin in bushels.

RULE.—I. *Divide the contents in cubic inches by* 2150.42; *the quotient will represent the number of bushels.*

Since a standard bushel contains 2150.42 cu. in., and a cubic foot contains 1728 cu. in., a bushel is to a cubic foot nearly as 4 to 5; or a bushel is equal to $1\frac{1}{4}$ cu. ft., nearly. Hence for all practical purposes,

II. *Any number of cubic feet diminished by* $\frac{1}{5}$ *will represent an equivalent number of bushels.*

Thus, 250 cu. ft. $-\frac{1}{5}$ of 250 cu. ft., or 50 cu. ft. = 200, the number of bushels in 250 cu. ft.

III. *Any number of bushels increased by* $\frac{1}{4}$ *will represent an equivalent number of cubic feet.*

Thus, 200 bu. $+\frac{1}{4}$ of 200 bu. or 50 bu. = 250, the number of cubic feet in 200 bushels.

1. How many bushels of wheat can be put in a bin 8 ft. long, 6 ft. 6 in. wide, and 3 ft. 4 in. deep?

2. What must be the depth of a bin to contain 240 bu., its length being 10 feet and its width 5 feet?

OPERATION.

240 bu. + 60 bu. = 300. $300 \div \overline{10 \times 5} = 6$ ft., the *depth*.

RULE. — *Divide the contents in cubic feet or inches by the product of the two dimensions, in the same denomination.*

3. What must be the length of a bin that is 6 feet wide and $4\frac{1}{2}$ feet deep, to contain 324 bushels? *Ans.* 15 ft.

4. How many bushels of apples will a bin hold, that is 12 ft. long, 3 ft. wide, and 2 ft. 6 in. deep? How many of barley?

5. A bin 20 ft. long, 12 ft. wide, and 5 ft. deep, is full of wheat. What is its value at $2 a bushel?

6. A bin 7 ft. long, 6 ft. wide, and 5 ft. deep, is $\frac{3}{4}$ full of rye. What is its value at $1.37\frac{1}{2}$ a bushel?

7. A crib, the inside dimensions of which are 15 ft. long, 7 ft. 4 in. wide, and 8 ft. high, is full of corn in the ear. If 2 bushels of ears make 1 bushel of shelled corn, what is the value of the whole, when shelled, at $.92 a bushel? *Ans.* $323.84.

8. If 1 bu. or 60 lb. of wheat make 48 lb. of flour, how many barrels of flour can be made from the contents of a bin 10 ft. long, 5 ft. wide, and 4 ft. deep, filled with wheat? *Ans.* $39\frac{9}{49}$ bbl.

9. How many tons of red ash coal, egg size, will a bin 17 ft. long, 6 ft. wide, and 3 ft. deep contain?

10. A bin 6 ft. long, 4 ft. deep, and 5 ft. 9 in. wide is full of Lehigh white ash coal. Find its value at $6.75 a ton.

11. Find its value if full of Schuylkill white ash, at $5.90 a ton. If full of Schuylkill red ash, at $5.50 a ton.

230i. To find the exact capacity of a vessel or space in gallons.

1. How many gallons of water will a cistern hold, that is 4 feet square and 6 feet deep?

OPERATION.

$(4 \times 4 \times 6 \times 1728) \div 231 = 718\frac{10}{11}$ gal., *capacity.*

RULE. — *Divide the contents in cubic inches by* 231 *for liquid gallons, or by* 268.8 *for dry gallons.*

2. How many cubic feet are there in a space that holds 48 hhd.?

3. How many hogsheads will a cistern 11 ft. long, 6 ft. wide, and 7 ft. deep contain? *Ans.* $54\frac{6}{7}$ hhd.

4. How many gallons will a space contain that is 22.5 ft. long, 3.25 ft. wide, and 6.4 ft. deep?

5. A man constructed a cistern to hold 32 hogsheads, the bottom being 6 ft. by 8 ft. What was its depth?

6. A tank in the attic of a house is 6 ft. 6 in. long, 4 ft. wide, and 3 ft. 6 in. deep. How many gallons of water will it hold?

7. If 64 quarts of water are put into a vessel that will exactly hold 64 quarts of wheat, how much will the vessel lack of being full? *Ans.* 604.8 cu. in.

8. If a man buys 10 bu. of chestnuts at $5 a bushel, dry measure, and sells the same at 25 cents a quart, liquid measure, how much does he gain? *Ans.* $43.09.

9. A cistern 5 ft. by 4 ft. by 3 ft. is full of water. If it is emptied by a pipe in 1 h. 30 min., how many gallons are discharged through the pipe in a minute?

10. A vat that will hold 5000 gallons of water, will hold how many bushels of corn? *Ans.* $537\frac{7}{64}$ bu.

11. A cellar 40 ft. long, 20 ft. wide, and 8 ft. deep is half full of water. What will be the cost of pumping it out, at 6 cents a hogshead? *Ans.* $22.80.

PLASTERING, PAINTING, KALSOMINING.

230j. Plastering, painting, and kalsomining are generally computed by the square yard.

The processes of calculating the cost of plastering, painting, and kalsomining vary so much in different localities that it is impossible to lay down any rule. Usually some allowance is made for doors, windows, etc., on which no work is done; but sometimes the measurements of walls are taken regardless of such openings. At other times, one half the area of the openings is deducted.

EXAMPLES.

1. What would it cost to plaster the walls and ceilings of a room 16 ft. long, 15 ft. wide, and 11 ft. high, in which there are 2 doors, 2 ft. 8 in. by 7 ft., and 2 windows, 2 ft. 10 in. by 6 ft. 2 in., at $.37 a sq. yd., deducting half the area for doors and windows?

OPERATION.

$2 \times 16 \times 11 = 352$
$2 \times 15 \times 11 = 330$
$16 \times 15 = 240$
Total area, 922 sq. ft.
$2 \times 2\frac{2}{3} \times 7 = 37.3$
$2 \times 2\frac{5}{6} \times 6\frac{1}{6} = 34.9$
$72.2 \div 2 = 36.1$ sq. ft. area to be deducted.
$922 - 36.1 = 885.9$ sq. ft.
$885.9 \div 9 = 98.4$ sq. yd.
$98.4 \times \$.37 = \$36.41$, *Ans.*

SOLUTION. — Two of the walls will be 16 × 11 ft., two, 15 × 11 ft., and the ceiling, 16 × 15 ft. Adding these, we find the total area to be 922 sq. ft. The area of the two doors will be twice $7 \times 2\frac{2}{3}$ ft., and of the two windows twice $2\frac{5}{6} \times 6\frac{1}{6}$ ft. Adding these, the area of doors and windows is 72.2. We deduct half this area, 36.1, from 922, divide the remainder, 885.9 sq. ft., by 9, to reduce it to square yards, multiply $.37 by the result, 98.4 sq. yd., and find the answer $36.41.

2. What would it cost to plaster the walls and ceiling of a room 12 ft. long, 17 ft. wide, and 10 ft. high, at $.35 a square yard, making no allowance for openings?

3. What will it cost to kalsomine a hall 6 ft. 6 in. × 27 ft. × 11 ft., with 2 coats, at 3¢ a sq. yd. each?

4. What will it cost to paint the walls and ceiling of a room 18 ft. long, 14 ft. wide, 9 ft. high, with 3 coats, at $.07 a sq. yd. each, making no allowance for openings?

PAPERING AND CARPETING.

WALL PAPER.

230k. Wall paper is sold only by the roll, and any part of a roll is considered a whole roll.

American paper is commonly $\frac{1}{2}$ yd. wide, and has 8 yd. in a roll. Foreign papers vary in width and length to the roll. Borders and friezes are sold by the yard, and vary in width from 3 in. to 18 in.

Paper is also often put up in double rolls 16 yards long.

It is not possible to find in advance the exact cost of papering a room, since there is frequently much waste, and a paper-hanger will charge for the number of rolls actually used in doing the work; but it is well to make an approximate estimate.

EXAMPLES.

1. What would it cost to paper the walls of a room 14 ft. long, 12 ft. wide, and 8 ft. high from baseboard to ceiling, with paper 8 yd. long, and $\frac{1}{2}$ yd. wide, at \$.30 a roll?

OPERATION.

$2 \times 14 = 28$
$2 \times 12 = 24$
52 ft.
$52 \div 3 = 14$ yd. =
28 half-yards, or strips.
$24 \div 8 = 3$ strips to a roll.
$28 \div 3 = 9+$, or 10 rolls.
\$.30 $\times$ 10 = \$ 3.00, *Ans.*

SOLUTION. — The distance around the room is 2 × 14 ft. + 2 × 12 ft. = 52 ft. = 14 yd. = 28 half yards, the number of strips required. Since the height is 8 ft., and there are 24 ft. on a roll, 1 roll will make 3 strips; and it will take as many rolls to make 28 strips as 3 is contained times in 28, which is 9+. Therefore 10 rolls will be required. At \$.30 a roll, they will cost \$ 3.00.

RULE. — I. *Find the entire distance around the room in yards. Multiply this by 2 to find the number of half-yards, or strips, since the paper is only half a yard wide.*

II. *Divide the number of half-yards by the number of strips that can be cut from a roll, and the result will be the number of rolls required.*

Since there are 24 ft. in a roll 8 yards long, if the distance from baseboard to ceiling is 8 ft. or less, 3 strips can be cut from a roll; if more than 8 ft. and not more than 12 ft., 2; etc. In the former case, the divisor would be 3; in the latter, 2.

2. Find the cost of papering a room 16 ft. long, 15 ft. wide, and 11 ft. high, with American paper 8 yd. long, and $\frac{1}{2}$ yd. wide, at \$.50 a roll, allowing 1 ft. for baseboard, and making no deductions for openings.

Ans. \$10.50.

3. How many double rolls of paper 16 yd. long, $\frac{1}{2}$ yd. wide, will be required to paper the walls of a room 30 ft. long, 30 ft. wide, and 12 ft. from baseboard to ceiling?

Ans. 20 rolls.

CARPETS.

2301. Carpets are usually 1 yd. wide or $\frac{3}{4}$ yd. wide, and are sold by the yard.

It is not often possible to find the number of yards of carpet needed by calculating the number of square yards in a floor. It is cut up into breadths which must be matched and seamed together, and more or less of the carpet is wasted in matching. A carpet with small figures usually loses less in matching than one with large figures.

If neither the length nor the width of the room is a multiple of the width of the carpet, a certain amount must be turned under at the end or side, which causes another loss.

EXAMPLES.

1. How many yards of carpet will be required to carpet a room 20 feet long by $13\frac{1}{2}$ ft. wide, with carpet $\frac{3}{4}$ yd. wide, if the strips run lengthwise? How much if the strips run across the room?

OPERATION.

$13\frac{1}{2}$ ft. $= 162$ in. 20 ft. $= 240$ in.
$162 \div 27 = 6$ strips.
$240 \div 36 = 6\frac{2}{3}$, length of strips.
$6 \times 6\frac{2}{3} = 40$ yd., lengthwise, *Ans.*
$240 \div 27 = 8 + = 9$ strips.
$162 \div 36 = 4\frac{1}{2}$, length of strips.
$9 \times 4\frac{1}{2} = 40\frac{1}{2}$ yd., crosswise, *Ans.*

SOLUTION. — Since the carpet is 27 in. wide and the room $13\frac{1}{2}$ ft. or 162 in. wide, when laid lengthwise we will require 6 breadths of carpet of the required length. And since the room is 20 ft. or 240 in. long, it will require as many yards for each breadth as 36 in., or 1 yd., is contained times in 240 in., which is $6\frac{2}{3}$ yd. Hence for 6 breadths, or strips, it will require $6 \times 6\frac{2}{3}$ yd., or 40 yd.

If laid crosswise, we will need as many strips as 27 in., the width of the carpet, is contained times in 240 in., the length of the room, which is $8\frac{8}{9}$; therefore we will need 9 strips, and $\frac{1}{9}$ will be turned under. We will need as many yards for each strip as 36 in., or 1 yd., is contained times in 162 in., which is $4\frac{1}{2}$ yd. Hence for 9 strips it will require 9 times $4\frac{1}{2}$ yd., or $40\frac{1}{2}$ yd.

RULE. — *Find the number of breadths or strips required, and the length of each strip.*

2. How many yards of carpet, 1 yd. wide, will be required for a room 18 ft. long and 18 ft. wide, if the strips run lengthwise? If they run crosswise? *Ans.* 36 yd.

3. Compute the cost of carpeting a room $16\frac{1}{2}$ ft. long and 15 ft. wide with carpet $\frac{3}{4}$ of a yard wide, running lengthwise of the room, at $1.30 per yard.

4. Find the cost of a carpet $\frac{3}{4}$ yd. wide, at $1.50 per yard, for a room 17 ft. long and $14\frac{1}{2}$ ft. wide, there being a waste of 1 yd. in matching the pattern, and the carpet-layer having been instructed to lay the carpet in the most economical way. *Ans.* $59.50.

PERCENTAGE.

231. The term per cent is from the Latin *per centum*, and means by the hundred. Any per cent of a number is a certain number of each *one hundred* parts. By 5 per cent is meant 5 cents of every 100 cents, $5 of every $100, 5 bushels of every 100 bushels, etc. Therefore, 5 per cent equals 5 hundredths $= .05 = \frac{5}{100} = \frac{1}{20}$; 8 per cent = 8 hundredths $= .08 = \frac{8}{100} = \frac{2}{25}$.

232. Rate per cent, or rate, is the decimal which denotes how many hundredths of a number are to be taken. The sign of per cent is %.

Since per cent is any number of hundredths, it is usually expressed in the form of a *decimal;* but it may be expressed either as a *decimal* or as a *common fraction.*

233. Percentage is such a part of a number as is indicated by the rate per cent.

Thus, 5 % of 100 = 5. 5 % is the rate per cent, and 5 is the percentage.

234. The **Base** of percentage is the number on which the percentage is computed.

1. The **amount** is the *sum* obtained by *adding* the base and percentage. It is equal to 100 % + the rate per cent.
2. The **difference** is the *remainder* obtained by *subtracting* the percentage from the base. It is equal to 100 % − the rate per cent.

TABLE.

		Decimals.		Common Fractions.		Lowest Terms.
1%	=	.01	=	$\frac{1}{100}$	=	$\frac{1}{100}$
2%	=	.02	=	$\frac{2}{100}$	=	$\frac{1}{50}$
4%	=	.04	=	$\frac{4}{100}$	=	$\frac{1}{25}$
5%	=	.05	=	$\frac{5}{100}$	=	$\frac{1}{20}$
6%	=	.06	=	$\frac{6}{100}$	=	$\frac{3}{50}$
7%	=	.07	=	$\frac{7}{100}$	=	$\frac{7}{100}$
8%	=	.08	=	$\frac{8}{100}$	=	$\frac{2}{25}$
10%	=	.10	=	$\frac{10}{100}$	=	$\frac{1}{10}$
16%	=	.16	=	$\frac{16}{100}$	=	$\frac{4}{25}$
20%	=	.20	=	$\frac{20}{100}$	=	$\frac{1}{5}$
25%	=	.25	=	$\frac{25}{100}$	=	$\frac{1}{4}$
50%	=	.50	=	$\frac{50}{100}$	=	$\frac{1}{2}$
100%	=	1.00	=	$\frac{100}{100}$	=	1
125%	=	1.25	=	$\frac{125}{100}$	=	$\frac{5}{4}$
$\frac{1}{2}$%	=	.005	=	$\frac{5}{1000}$	=	$\frac{1}{200}$
$\frac{3}{4}$%	=	.0075	=	$\frac{75}{10000}$	=	$\frac{3}{400}$
$12\frac{1}{2}$%	=	.125	=	$\frac{125}{1000}$	=	$\frac{1}{8}$
$16\frac{1}{4}$%	=	.1625	=	$\frac{1625}{10000}$	=	$\frac{13}{80}$

EXAMPLES.

1. Express, using the sign: 3 per cent; 6 per cent; 9 per cent; 14 per cent; 24 per cent; 40 per cent; $122\frac{1}{2}$ per cent; 150 per cent.

2. Express, using the sign: $6\frac{1}{4}$ per cent; $8\frac{3}{4}$ per cent; $33\frac{1}{3}$ per cent; $7\frac{1}{2}$ per cent; $10\frac{2}{5}$ per cent; $9\frac{5}{8}$ per cent; $103\frac{1}{2}$ per cent; 225 per cent.

3. Express decimally $\frac{1}{4}$ per cent; $\frac{3}{4}$ per cent; $\frac{2}{5}$ per cent; $\frac{4}{5}$ per cent; $\frac{5}{8}$ per cent; $1\frac{1}{4}$ per cent; $2\frac{4}{5}$ per cent; $4\frac{1}{3}$ per cent; $5\frac{3}{4}$ per cent; $7\frac{1}{8}$ per cent; $12\frac{1}{5}$ per cent; $25\frac{3}{8}$ per cent.

4. Express in the form of common fractions, in their lowest terms: 6 per cent; 8 per cent; 12 per cent; $14\frac{1}{2}$ per cent; $18\frac{3}{8}$ per cent; $21\frac{4}{5}$ per cent; $31\frac{1}{4}$ per cent; $37\frac{1}{2}$ per cent; $40\frac{3}{4}$ per cent; 112 per cent; 225 per cent.

235. To find the percentage, amount, and difference of any number.

1. What is 4% of $125, and what are the amount and difference?

OPERATION.

$125
.04
$5.00, Percentage. Or, $\frac{1}{25}$ of $125 = $5, Percentage.

$125 + $5 = $130, Amt. $125 − $5 = $120, Diff.

Or,

100% + 4% = 104% = 1.04 of $125 = $130, Amt.
100% − 4% = 96% = .96 of $125 = $120, Diff.

SOLUTION. — 4% of $125 = .04 of $125, or $5; or 4% = $\frac{1}{25}$ of $125 = $5, percentage.

Since the amount is the base plus the percentage, it is $125 + $5 = $130; and since the difference is the rate minus the percentage, it is $125 − $5 = $120. Or, the base 100% + the rate 4% = 104%, the amount expressed in per cent. 104% of $125 = 1.04 of $125 = $130, Amt.; and the base 100% − the rate 4% = 96%, the difference expressed in per cent. 96% of $125 = .96 of $125 = $120, Diff.

RULE. — I. *Multiply the given number or quantity by the rate per cent, expressed decimally, and point off as in decimals; or, reduce the decimal to a common fraction, and take that fractional part of the given number, and the result will be the percentage.*

II. *Add the percentage and base to find the amount, and subtract the percentage from the base to find the difference.* Or,

To find the amount or difference directly from the base; add the rate per cent to 100% *and multiply the base by this per cent, expressed decimally, to find the amount, and subtract the rate from* 100% *and multiply the base by the result to find the difference.*

2. What is 6% of $320? *Ans.* $19.20.
3. What is 7¼% of $56.75? *Ans.* 4.11\frac{7}{16}$.

4. What are the amount and difference of $320 at 6%? *Ans.* Amt. $339.20; Diff. $300.80.

5. What is 2450 pounds + 12½%?

6. What is 846 gallons less 33⅓%? *Ans.* 564 gallons.

7. What is 105% of $5760? *Ans.* $6048.

8. What is 25% of ⅞?

Ans. 25% equals $\frac{25}{100} = \frac{1}{4}$, and $\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$.

9. What is 15% of ⅝? *Ans.* $\frac{1}{12}$.

10. What is 2½% of 6⅔? *Ans.* $\frac{1}{6}$.

11. A man has a capital of $24500; he invests 18% of it in bank stock, 30% of it in railroad stocks, and the remainder in bonds and mortgages. How much does he invest in bonds and mortgages? *Ans.* $12740.

12. A speculator bought 1576 barrels of apples, and upon opening them he found 12½% of them spoiled. How many barrels did he lose?

13. Two men engaged in trade, each with $2760. One of them gained 33⅓% of his capital, and the other gained 75%. How much more did the one gain than the other?

236. To find what per cent one number is of another.

1. A man, having $125, lost $5. What per cent of his money did he lose?

OPERATION.

$5 \div 125 = .04 = 4\%$.

Or,

$\frac{5}{125} = \frac{1}{25}$ of $100\% = 4\%$.

SOLUTION. — He lost $\frac{5}{125} = \frac{1}{25}$ of his money. Reducing this to a decimal, we have $.04 = 4\%$ loss. We divide the percentage by the base to obtain the rate per cent. Or, since $125 is 100% of his money, $5 is $\frac{5}{125}$, or $\frac{1}{25}$ of 100%, which is 4%.

RULE. — *Divide the percentage by the base, and the quotient will be the rate per cent expressed decimally.* Or,

Take such a part of 100% *as the percentage is part of the base.*

2. What per cent of $450 is $90? *Ans.* 20%.

3. What per cent of $1400 is $175? *Ans.* 12½%.

4. What per cent of $750 is $165?

5. What per cent of $240 is $13.20? *Ans.* 5½%.

6. What per cent of $2 is 15 cents?

7. What per cent of 6 bushels 1 peck is 4 bushels 2 pecks 6 quarts? *Ans.* 75%.

8. What per cent of 15 pounds is 5 pounds 10 ounces, avoirdupois weight? *Ans.* 37½%.

9. What per cent of 250 head of cattle is 40 head?

10. From a hogshead of sugar containing 760 pounds, 100 pounds were sold at one time, and 90 pounds at another. What per cent of the whole was sold?

11. A man, having 600 acres of land, sold ¼ of it at one time, and ⅓ of the remainder at another time. What per cent remained unsold? *Ans.* 50%.

237. To find a number when a certain per cent of it is given.

1. A man lost $5, which was 4% of all the money he had. How much had he at first?

OPERATION.

$5 ÷ .04 = $125.

Or,

$\$\frac{5}{4} \times 100 = \$125$.

SOLUTION. — We are here required to find the base of which $5 is the percentage. Since the percentage equals the base multiplied by the rate per cent (**235**), the base must equal the percentage divided by the rate per cent. Dividing $5 by .04, we have $125, the base. Or, since $5 is 4% of all he had, 1% of all he had will be ¼ of $5, or $\$\frac{5}{4}$, and 100% (or all he had) will be 100 times $\$\frac{5}{4} = \$125$.

RULE. — *Divide the percentage by the rate per cent, expressed decimally, and the quotient will be the base, or number required.* Or,

Take as many times 100 *as the number expressing the percentage is times that expressing the rate per cent.*

2. 16 is 8% of what number? *Ans.* 200.
3. 42 is 7% of what number?
4. 75 is 12½% of what number? *Ans.* 600.
5. 33 is 2¾% of what number? *Ans.* 1200.
6. $281.25 is 37½% of what sum of money? *Ans.* $750.
7. A farmer sold 50 sheep, which was 20% of his whole flock. How many sheep had he at first?
8. I loaned a man a certain sum of money; at one time he paid me $59.75, which was 12½% of the whole sum loaned to him. How much did I loan him?
9. A merchant invested $975 in dry goods, which was 15% of his entire capital. What was the amount of his capital? *Ans.* $6500.

238. To find a number when the number, increased by a certain per cent of itself, is given.

1. A man's income this year is $525, which is 5% more than it was last year. What was it last year?

OPERATION.

$525 ÷ 105 = $5 × 100 = $500.

Or,

$525 ÷ 1.05 = $500.

SOLUTION. — Since his income this year is .05 more than it was last year, this year's income must be 1.05 times the income of last year. Since $525 is $\frac{105}{100}$ of last year's income, $\frac{1}{100}$ will be $\frac{1}{105}$ of $525 = $5, and $\frac{100}{100}$, or his last year's income, will be 100 times $5, or $500. Or, we divide this year's income by 1.05 and it gives the income of last year.

RULE. — *Divide the amount by* 1 *plus the rate expressed decimally, and the quotient will be the base or number required.*

2. What number increased by 18% of itself is equal to 1475? *Ans.* 1250.
3. A merchant sells broadcloth for $4 per yard, and thereby gains 25%. What did the cloth cost him?

4. A expended a certain sum for a house, and 15% of the purchase price on repairs, and then found that the whole cost was $6900. What was the purchase price?

5. A certain manufacturing company have sold this year $432250 worth of goods, which is $8\frac{1}{16}\%$ more than they sold last year. How much did they sell last year?

Ans. $400000.

239. To find a number when the number, diminished by a certain per cent of itself, is given.

1. A man lost 8% of his sheep and had 368 left. How many had he at first?

OPERATION.

$$368 \div 92 = 4 \times 100 = 400.$$

Or,

$$368 \div .92 = 400.$$

SOLUTION. — Since the man lost 8% of his sheep, he has 92% left; hence 368 is $\frac{92}{100}$ of his original flock. $\frac{1}{100}$ will be $\frac{1}{92}$ of 368 $= 4$ and $\frac{100}{100}$, or his original flock, will be 100 times $4 = 400$. Or, we divide the remainder, 368, by .92 and obtain the required number of sheep.

RULE. — *Divide the given number by* 1 *minus the rate expressed decimally, and the quotient will be the base or number required.*

2. What number diminished by 15% of itself is equal to 340? *Ans.* 400.

3. A having a certain sum on deposit drew out 20%, and then found he had $1000 left. How much had he on deposit at first? *Ans.* $1250.

4. My income this year is $4028, which is 24% less than it was last year. How much was it last year?

5. What number diminished by $\frac{1}{2}\%$ of itself is equal to $298\frac{1}{2}$? *Ans.* 300.

COMMISSION AND BROKERAGE.

240. An **Agent, Commission Merchant,** or **Broker** is a person who transacts business for another, or buys and sells money, stocks, notes, etc.

241. Commission is the percentage, or compensation allowed an agent, or commission merchant, for buying and selling goods or produce, collecting money, and transacting other business.

Brokerage is the fee, or allowance paid to a broker or dealer in money, stocks, or bills of exchange, for making exchanges of money, buying and selling stocks, negotiating bills of exchange, or transacting other like business.

The rates of commission and brokerage are not regulated by law, but are usually reckoned at a certain per cent upon the money employed in the transaction.

When an agent sells, his commission is some per cent of the sales; and when he buys, it is some per cent of the purchase price.

1. The merchandise sent to a commission merchant to be sold is called a *consignment*. The person who sends it is called the *consignor*, and the person to whom it is sent the *consignee*.

2. Commission corresponds to *percentage*.

3. The sum received by the agent as the price of the property sold, or the sum invested in the purchase or exchange of property, corresponds to the *base*.

4. The sum remitted to an agent, and including both the purchase money and the agent's commission, corresponds to the *amount*.

5. The sum due the employer or consignor, as the *net proceeds* of a sale or collection, corresponds to the difference.

242. Net proceeds is the sum left after commission and other expenses have been paid.

Thus, if a man receives $5000 to invest, and his commission is $200 and other expenses $100, the net proceeds will be $5000 − $300.

EXAMPLES.

243. To find the commission or brokerage on any sum of money.

1. A commission merchant sells butter and cheese to the amount of $1540. What is his commission at 5%?

OPERATION.

$1540 × .05 = $77, *Ans.*

Or, $\frac{5}{100} = \frac{1}{20}$, and $\frac{1}{20}$ of $1540 = $77.

SOLUTION.— Since the commission on $1 is 5 cents or .05 of a dollar, on $1540 it is $1540 × .05 = $77.

RULE. — *Multiply the given sum by the rate per cent expressed decimally, and the result will be the commission or brokerage.* Or,

Reduce the per cent to a common fraction, and take that fractional part of the given sum.

2. A commission merchant sells goods to the amount of $6756. What is his commission at 2%?

Ans. $135.12.

3. What commission must be paid for collecting $17380, at $3\frac{1}{2}$%? *Ans.* $608.30.

4. An agent in Chicago purchased 4700 bushels of wheat at 75 cents a bushel. What was his commission at $1\frac{1}{2}$% on the purchase money?

5. A broker in New York exchanged $25875 on the Suffolk Bank, Boston, at $\frac{1}{8}$%. How much brokerage did he receive? *Ans.* $32.34375.

6. An auctioneer sold at auction a house for $3284, and the furniture for $2176.50. What did his fees amount to at $2\frac{1}{4}$%?

7. An agent buys for a manufacturing company 26750 pounds of wool, at 32 cents a pound, and receives a commission of $2\frac{3}{4}$%. What amount does he receive?

Ans. $235.40.

244. To find the commission or brokerage, when it is to be deducted from the given sum, and the balance is to be invested.

1. A merchant sends his agent $1260 with which to buy merchandise, after deducting his commission of 5%. What is the sum invested, and how much is the commission?

OPERATION.

$1260 ÷ 1.05 = $1200, invested.

$1260 − $1200 = $60, commission.

SOLUTION. — Since the agent receives a commission of 5 per cent for his services, it requires $1.05 to purchase every dollar's worth of goods; therefore, he can invest as many dollars as $1.05 is contained times in $1260, or $1200; and the difference between the given sum and the sum invested is his commission.

RULE. — I. *Divide the given amount by* 1 *increased by the rate per cent of commission, and the quotient is the sum invested.*

II. *Subtract the investment from the given amount, and the remainder is the commission.*

2. A man sends $3246.20 to his agent in Boston, requesting him to lay it out in shoes, after deducting his commission of 2%. What is his commission?

3. What amount of stock can be bought for $9682, and allow 3% brokerage? *Ans.* $9400 worth.

4. A flour merchant sent $10246.50 to his agent in Chicago, to invest in flour, after deducting his commission of 3½%. How many barrels of flour could he buy at $5.50 per barrel? *Ans.* 1800 barrels.

5. An agent receives a remittance of $4908, with which to purchase grain, at a commission of 4½%. What will be the amount of the purchase?

6. A commission merchant receives $9376.158 with orders to purchase grain; his commission is 3%, and he charges 1½% additional for guaranteeing its delivery at a specified time. How much will he pay out, and what are his fees? *Ans.* Fees, $403.758.

7. A broker received $10650, to be invested in stocks after deducting ⅛% for brokerage. What amount of stock did he purchase?

245. To find the amount of sale when the net proceeds and the rate are given.

1. Find the amount of sales when the agent remits to the owner $3800 after deducting his commission of 5%.

OPERATION.

Sale.	Com.	Net Proceeds.
100%	− 5%	= 95%.
$3800		= 95% of sale.
$3800 ÷ .95 = $4000, sale.		

SOLUTION. — The agent's commission is some per cent of the sale, and $3800 is the net proceeds after the 5% commission has been deducted. It is, therefore, 100% − 5% = 95% of the sale. If $3800 is 95% of the sale, the sale must be $3800 ÷ .95 = $4000, *Ans.*

RULE. — *Divide the net proceeds by* 1 *diminished by the rate per cent of commission, and the quotient will be the amount of sale.*

2. A real estate agent sold a house for a certain sum of money and remitted $19600 to the owner, after deducting his commission of 2%. For how much did the agent sell the house? *Ans.* $20000.

3. An agent sold a piano. After deducting his commission of 10% he sent the owner $450. How much did the purchaser pay for it? *Ans.* $500.

4. An agent sold a grocer 40 bbl. flour and sent the miller, as the proceeds of the sale, $82.32, after deducting his commission of 2%. What price per bbl. did the grocer pay for the flour? *Ans.* $2.10.

STOCKS AND BONDS.

246. A **Corporation** is a body authorized by a general law, or by a special charter, to transact business as a single individual.

A charter is the legal act of incorporation, and defines the powers and obligations of the incorporated body.

247. A **Firm** is the name under which an unincorporated company transacts business.

248. **Capital** or **Stock** is the money contributed and employed to carry on the business of an individual, corporation, company, or firm; it receives different names, as Bank Stock, Railroad Stock, Government Stock, etc.

249. **Certificates of Stock** or **Scrip** are the papers or documents issued by a corporation specifying the number of shares of the joint capital which the holders own.

250. A **Share** is one of the equal parts into which the stock is divided.

251. **Stockholders** are the owners of the shares.

252. The **Nominal, Face,** or **Par Value** of stock is its first cost, or original valuation.

The original valuation of a share varies in different companies. A share of bank, insurance, railroad, or like stock is usually $100.

253. Stock is **At Par** when it sells for its first cost, or original valuation; **Above Par,** or at a premium, when it sells for more than its original cost; and **Below Par,** or at a discount, when it sells for less than its original cost.

254. The **Market** or **Real Value** of stock is what it will bring per share in money.

255. A **Dividend** is a sum paid to stockholders from the profits of the business of the company.

256. An **Assessment** is a sum required of stockholders to meet the losses or expenses of the business of the company.

257. Premium, or advance, and discount on stock, dividends, and assessments, are computed at a certain per cent upon the original or *par value* of the shares of the stock.

258. A **Stock Broker** is a person who buys and sells stocks, either for himself, or as an agent of another.

259. The calculations in stock-jobbing are based upon the following relations:

I. Premium, discount, and brokerage are each a *percentage*, computed upon the *par value* of the stock as the *base*.

II. The market value of stock, or the proceeds of a sale, is the *amount*, or *difference*, according as the sum is greater or less than the par value.

1. In all examples relating to stocks, $100 will be considered a share, unless otherwise stated. When we speak of a stock as being at 105, 110, 95, etc., we mean that one $100 share can be bought for $105, $110, $95, respectively.

2. The rate of brokerage in New York City has been fixed by custom at $\frac{1}{4}$ per cent.

260. A **Bond** is a written instrument securing the payment of a sum of money at or before a specified time.

261. Net Earnings are the moneys left from the profits of a business after paying expenses, losses, and the interest upon the bonds.

The net earnings of a corporation are usually divided among the stockholders, in semi-annual dividends. The income of *capital stock* is therefore fluctuating, being dependent upon the condition of business; while the income arising from *bonds*, whether of government or corporations, is fixed, being a certain rate per cent, annually, of the par value, or face of the bonds.

262. Federal or **United States Bonds** are payable at a fixed date, and are known and quoted in commercial transactions by the rate of interest they bear.

Thus, U. S. 4's, that is, United States Bonds bearing 4 % interest.

Securities fluctuate in value from various causes. On this account the newspapers give daily quotations.

Below are the closing quotations of the principal investment bonds for two successive days in 1891:

BONDS.

	Sept. 30.	Oct. 1.
New 4's, reg.	116	116
New 4's, cou.	117	116
U. S. 2's, r.	99½	99½
P. 6's of 1895	110¾	110¾
D. C. 3.65's, c.	112	112
Can. So. 2d's	98	96½
Cen. P. 1st's	106	106
C. & O. 1st's	101¾	101
D. & R. G. 1st's	115	115
" 4's	79	78½
Erie 2d's	104	103
Ft. W. & D. 1st's	99	100
Iowa C. 1st's	86¼	86
Kan. P. 1st's (Den. div.)	108	107
K. Pac. con.	106	106
Lake Shore 2d's, r.	121	121
M., K. & T. gen. 4's	77¾	77
" 2d's	44	43¾
Mut. U. 6's	104	...
Mob. & Ohio	66	65

BONDS (*Continued*).

	Sept. 30.	Oct. 1.
N. Y., C. & St. Louis 4's	95⅝	93½
N. J. C. g. 5's	109½	110
N. & W. 6's	121	121
No. Pac. 1st's	115⅝	115⅝
" 2d's	112¾	109
" 3d's	108½	107
Northwest deb. 5's	104½	104½
Ore. N. 1st's	106½	105¾
O. S. L. 6's	100¾	100¾
O. & U. N. 5's	75	72
Ore. Im. 1st's	99¾	99¾
Reading 1st's pf. in.	68	68
Rg. g. m. 4's	80¾	80
Rg. 1st's pf. in.	68	66½
R. G. W. 1st's	76	76
Rock Isl. ex. 5's	100½	100½
St. L. & I. M. gen. 5's	89	86½
St. L. & S. F. gen. mtg.	105	105
S. P. cons.	124	124
St. P. Ch. & Pac. 1st's	113½	113½
S. P. C. 1st's	112½	112½
Tex. P. 1st's	83½	83
" 2d's	31½	31
U. Pac. 1st's	107	107
West Sh. 4's	102	101½
Wab. 1st 5's	101½	101

These daily accounts of the state of the market enable purchasers and sellers to decide whether it is to their advantage to purchase or to sell certain securities at any given day.

If the daily reports show a steady rise in values, purchasers are eager to buy, and holders are not anxious to sell.

When the quotations are lower from day to day, the holders wish to sell and the buyers are generally very cautious respecting investment.

EXAMPLES.

263. To find the value of stock when at a premium or discount.

1. What will 32 shares of bank stock cost at 8% premium, brokerage ¼%?

OPERATION.

$$32 \text{ shares} = 32 \times \$100 = \$3200 \text{ par value.}$$
$$\$1 + .08 = \$1.08.$$
$$\$1.08 + .0025 = \$1.0825.$$
$$\$1.0825 \times 3200 = \$3464.$$

SOLUTION. — At $100 a share the par value of 32 shares will be $3200. To find the price of stock, we add the rate of premium to $1; to this result add the brokerage, and we have the cost of $1 = $1.0825. If $1 worth of stock costs $1.0825, $3200 worth will cost 3200 times $1.0825, which is $3464.

RULE. — *Multiply the cost of* $1 *by the number indicating the par value of the stock.*

If the stock in the above example had been at a discount of 8 per cent, $1 worth of stock would have cost $1 − $.08 = $.92 + ¼ per cent brokerage = $.9225, and $3200 worth 3200 times $.9225.

2. If the stock of an insurance company sells at 5% below par, what will 12 shares of the stock cost?

Ans. $1140.

3. What is the market value of 35 shares of New York Central Railroad stock, at 15% below par?

Ans. $2975.

4. What must be paid for 48 shares of Panama Railroad stock, at a premium of 5½%, if the par value is $150 per share, brokerage ¼%? *Ans.* $7632.

5. What will $5364 worth of stock in copper mines cost, at 9% above par, brokerage ⅛%? *Ans.* $5853.465.

6. What is the market value of 175 shares of stock in the Suffolk Bank, at ¾% premium? *Ans.* $17631.25.

7. A man purchased $6275 worth of stock in the Pennsylvania Coal Company, at par, and sold the same at a discount of 12%. What was his loss? *Ans.* $753.

8. What must be paid for 125 shares of stock, at 4¾% premium, the par value being $1000 per share, brokerage ¼%? *Ans.* $131250.

9. I bought 42 shares of Illinois Central Railroad stock, at 14% discount, and sold the same at a premium of 12½%. What did I gain? *Ans.* $1113.

264. To find how much stock can be purchased for a given sum.

1. How many shares of bank stock, at 25% premium, can be bought for $5010, brokerage ¼%?

OPERATION.

$5010 ÷ $1.2525
= $4000
= 40 shares, *Ans.*

SOLUTION. — Since the stock is at a premium of 25%, $1 worth of stock with brokerage will cost $1.25¼, and as many dollars' worth can be bought for $5010 as $1.25¼ is contained times in $5010, which is $4000 worth.

RULE. — *Divide the given sum by the cost of* $1 *of stock, and the quotient will be the nominal amount of stock purchased.*

2. I invested $6187.50, in Ocean Telegraph stock, at 10% discount. How much stock did I purchase?
Ans. $6875.

3. How many shares of railroad stock, at 5% premium, can be purchased for $6300? *Ans.* 60 shares.

4. I sent my agent $53500 to be invested in Illinois Central Railroad stock, which was selling at 7% premium. What amount did he purchase? *Ans.* $50000.

5. I sold 50 shares of stock in a Pittsburgh ferry company, at 8% discount, and received $1150. What is the par value of 1 share? *Ans.* $25.

265. To find what income any investment will produce.

1. What income will be obtained by investing $6840 in stock bearing 6%, and purchased at 95?

OPERATION.

$6840 ÷ $.95 = $7200, stock purchased.
$7200 × .06 = $432, annual income.

SOLUTION. — We divide the investment, $6840, by the cost of $1, and obtain $7200, the stock which the investment will purchase (**264**). And since the stock bears 6% interest, $7200 × .06 = $432, the annual income obtained by the investment.

RULE. — *Find how much stock the investment will purchase, and then compute the income at the given rate upon the par value.*

2. If I invest $867 in 6% bonds at 102, what income will I receive on my investment? *Ans.* $51.

3. What will be my yearly income, if I invest $8428 in 5 per cents, at 98? *Ans.* $430.

4. How much stock at a premium of $4\frac{3}{4}$% can be bought for $10500, brokerage $\frac{1}{4}$%? *Ans.* $10000.

5. If a man invests $4795 in Maryland 5's at 87, brokerage $\frac{1}{2}$%, what will be his yearly income?

Ans. $274.

6. Having $10476 to invest, I find I can purchase N. Y. 6's at $107\frac{1}{2}$, and Erie R.R. 5's at $96\frac{1}{2}$, brokerage $\frac{1}{2}$%, in each instance. How much more will I receive yearly by investing in the former than in the latter?

Ans. $42.

7. A having a farm of 109 acres, which rents for $681.25, sells the same for $125 per acre, and invests the proceeds in Pacific R.R. 6's at $108\frac{3}{4}$, brokerage $\frac{1}{4}$% for purchasing. Will his yearly income be increased or diminished, and how much? *Ans.* Increased $68.75.

266. To find what sum must be invested to obtain a given income.

1. What sum must be invested in Virginia 5% bonds, purchasable at 80, to obtain an income of $600?

OPERATION.

$600 ÷ .05 = $12000, stock required.
$12000 × .80 = $9600, cost or investment.

SOLUTION. — Since $1 of the stock will obtain $.05 income, to obtain $600 will require a par value of $600 ÷ .05 = $12000. Multiplying the par value of the stock by the market price of $1, we have $12000 × .80 = $9600, the cost of the required stock, or the sum to be invested.

RULE. — I. *Divide the given income by the per cent which the stock pays; the quotient will be the par value of the stock required.*

II. *Multiply the par value of the stock by the market value of one dollar of the stock; the product will be the required investment.*

2. If N. Y. 6's are 5% below par, what sum must be invested in this stock to obtain an income of $840?

Ans. $13300.

3. What sum must be invested in 5 per cents at 98¼, brokerage ¼% for buying, to secure an annual income of $1860? *Ans.* $36642.

4. When 5 per cents are quoted at 108¼, what sum must I invest to secure an annual income of $1080, brokerage ¼%? *Ans.* $23436.

5. If I sell $25000 in railroad bonds at 93¾, and invest a sufficient amount of the proceeds in Pacific R.R. 6's, at 109¼, to yield an annual income of $960, and buy a house with the remainder, how much will the house cost me? *Ans.* $5957.50.

267. To find what per cent the income is of the investment, when stock is purchased at a given price.

1. What per cent of my investment will I secure by purchasing N. Y. 7's at 105?

OPERATION.

$.07 \div 1.05 = 6\frac{2}{3}\%$.

SOLUTION. — Since \$1 of the stock will *cost* \$1.05, and *pay* \$.07, the income is $\frac{7}{105} = 6\frac{2}{3}\%$ of the investment.

RULE. — *Divide the annual rate of income which the stock bears by the price of the stock; the quotient will be the rate upon the investment.*

2. What is the rate of income upon money invested in 6% bonds, purchased at 87? *Ans.* $6\frac{26}{29}\%$.

3. What per cent on his money will a man receive annually if he invests in N. Y. 6's at 105? *Ans.* $5\frac{5}{7}\%$.

4. What is the rate of income upon money invested in Missouri 6's at 75? *Ans.* 8%.

5. I purchased Pacific R.R. 6's at $107\frac{3}{4}$, brokerage $\frac{1}{4}\%$. What is the income on the investment? *Ans.* $5\frac{5}{9}\%$.

6. Which is the better investment, Mass. 5's at $98\frac{1}{4}$, or Pacific R.R. 6's at $108\frac{3}{4}$, brokerage $\frac{1}{8}\%$ in each?

7. If stock paying 10% dividends is quoted at $112\frac{1}{2}$, what per cent of income will be realized on an investment in it? *Ans.* $8\frac{8}{9}\%$.

8. Which will yield the better income, 8% bonds at 110, or 5's at 75? *Ans.* 8% bonds at 110, $\frac{20}{33}\%$ better.

9. Which is the more profitable, and how much, to buy 7's at 105, or 6% bonds at 84?

Ans. 6% bonds at 84, $\frac{10}{21}\%$ better.

10. What per cent of income does stock paying 10% dividends yield, if bought at 106? *Ans.* $9\frac{23}{53}\%$.

11. What per cent will stock which pays 5% dividends yield, if bought at 85? *Ans.* $5\frac{15}{17}\%$.

268. To find the price at which stock must be purchased to obtain a given rate upon the investment.

1. At what price must 6% stocks be purchased in order to obtain 8% income on the investment?

OPERATION.

$\$6 \div .08 = \$75.$

SOLUTION. — Since $6, the income of one share of the stock, must be 8% of the sum paid for it, we have $\$6 \div .08 = \$75$, the purchase price of one share of the stock, usually expressed simply as 75.

RULE. — *Divide the annual income which the stock bears by the rate required on the investment; the quotient will be the price of the stock.*

2. What must I pay for Missouri 6's that my investment may yield 9% annually? *Ans.* 66⅔%.

3. What must I pay for a 6% stock so that the investment will yield 5%? *Ans.* 120.

4. What must I pay for 5 per cents, that my investment may yield 8%? *Ans.* 62½.

5. At what price must Vermont 6% bonds be purchased that the person investing may secure 6¼% upon his money? *Ans.* 96.

6. At what price must I buy stock that pays 10% dividends, so as to realize 7½% on the investment? *Ans.* 133⅓.

7. At what price must I buy 7% stock so as to realize 6% on my investment? *Ans.* 116⅔.

8. At what price must stock, of the par value of $50 a share, and that pays 6% dividends, be bought, to yield an income of 7½%? *Ans.* 40.

9. At what price must 4% stock be bought to pay as good an income as 8% stock bought at par? *Ans.* 50.

TRADE DISCOUNT.

269. A **Discount** is a deduction from the face of bills, the list price of goods, or from the amount of debt, without regard to *time*, and is usually expressed by the term "*per cent off.*"

Thus, a discount of 25% means a deduction of 25% from the asking price. A discount of 30% and 10% does not mean 40% off, but that 30% is first to be deducted, leaving 70% of the price, then 10% of 70% of the price, equal to 7%, is deducted from the remainder, leaving 63%; so that the total discount is but 37%. 3 *tens* and 5% off means *three* successive discounts of 10% and 5% from the remainder, etc.

In some kinds of business fixed price lists are printed, and when a rise or fall in prices occurs the rate of discount is changed.

270. The **List Price** is called the fixed price, and the discount is called **Trade Discount.**

271. The **Net Price** is the list price, less the discount, or the price *received* for the goods.

EXAMPLES.

1. What is the net cost of a bill of goods amounting to $975, bought on 4 mo., at 10% discount, and 5% off for cash? *Ans.* $833.62½.

Find the *net cost* and *discount* of the following bills:

2. Bought for $750, on 3 mo., at 20% discount, and 4% off for cash. *Ans.* $576; $174.

3. Bought for $365.75, on 4 mo., at 20%, 10%, and 5% off for cash. *Ans.* $250.17; $115.58.

4. Bought for $260, on 90 da., at 2 tens and 3% off for cash. *Ans.* $204.28; $55.72.

5. What is the difference on a bill of $650, between a discount of 30%, and a discount of 25% and 5% off? *Ans.* $8.125.

PROFIT AND LOSS.

272. Profit and Loss are commercial terms, used to express the gain or loss in business transactions, which is usually reckoned at a certain per cent on the prime or first cost of articles.

The operations in Profit and Loss involve the same principles as those of Percentage. The cost corresponds to the base; the per cent of profit or loss to the rate per cent; the profit or loss to the percentage, and the selling price to the amount or difference.

EXAMPLES.

273. To find the amount of profit or loss, when the cost and the gain or loss per cent are given.

1. A man bought a horse for $135, and afterwards sold him for 20% more than he gave. How much did he gain?

OPERATION.

$135 × .20 = $27, *Ans.*

Or, $\frac{20}{100} = \frac{1}{5}$; $135 × $\frac{1}{5}$ = $27.

SOLUTION. — Since $1 gains 20 cents, or 20%, $135 will gain $135 × .20 = $27.

RULE. — *Multiply the cost by the rate per cent expressed decimally.* Or,

Reduce the per cent to a common fraction, and take that fractional part of the cost.

2. A grocer bought a hogshead of sugar for $84.80, and sold it at 12½% profit. What was his gain?

3. A miller bought 500 bushels of wheat at $1.15 a bushel, and he sold the flour at 16⅔% advance on the cost of the wheat. What was his gain? *Ans.* $95.83⅓.

4. I bought 76 cords of wood at $3.62½ a cord, and sold it so as to gain 26%. What did I make?

5. A hatter bought 40 hats at $1.75 apiece, and sold them at a loss of 14⅔%. What was his whole loss?

6. A sloop, freighted with 3840 bu. of corn, encountering a storm, threw 37½% of her cargo overboard. What was the loss, at 62½ cents a bushel? *Ans.* $900 loss.

7. A grocer bought 3 barrels of sugar, each containing 230 pounds, at $8\frac{1}{4}$ cents a pound, and sold it at $18\frac{2}{11}\%$ profit. What was his whole gain, and what the selling price per pound?

Ans. Gain, \$10.35; price per lb., $9\frac{3}{4}$ cents.

8. A man bought a carriage for \$160, and after paying 10% for repairs, sold it at $12\frac{1}{2}\%$ profit. What was his gain?

9. What is the profit on 6 doz. pairs of gloves, bought at \$12 a doz. and sold at a gain of 50% a pair?

10. I bought two houses for \$2500 each, and sold one at an advance of $33\frac{1}{3}\%$ and the other at a loss of $33\frac{1}{3}\%$. What did I gain or lose on both?

11. A merchant pays \$6840 for a stock of spring goods, and sells them at an advance of $26\frac{1}{2}\%$ on the purchase price. After deducting \$375 for expenses, what is his gain?

274. To find the gain or loss per cent when the cost and selling price are given.

1. Wool bought at 32 cents a pound, was sold for 40 cents a pound. What per cent was gained?

OPERATION.

$40¢ - 32¢ = 8¢$; $8 \div 32 = \frac{8}{32} = .25$, *Ans.*

SOLUTION. — Since the gain on 32 cents is $40 - 32 = 8$ cents, the whole gain is $\frac{8}{32} = \frac{1}{4}$ of the purchase money; and $\frac{1}{4}$ reduced to a decimal is 25 *hundredths*, equal to 25 per cent.

RULE. — I. *Find the difference between the cost and selling price which will be the gain or loss.*

II. *Make the gain or loss the numerator, and the cost the denominator; reduce to a decimal, and the result will be the per cent.*

2. A man bought a pair of horses for $275, and sold them for $330. What per cent did he gain? *Ans.* 20%.

3. If a merchant buys cloth at $.60 a yard, and sells it for $.75 a yard, what per cent does he gain?

4. A speculator bought 108 barrels of flour at $4.62½ a barrel, and sold it so as to gain $114.88½. What per cent profit did he make? *Ans.* 23%.

5. Sugar bought at 8 cents a pound, was sold for 9½ cents a pound. What per cent was gained?

6. A drover bought 150 head of cattle for $42 per head, and sold them for $5400. What was his loss per cent? *Ans.* 14⅔%.

7. If I sell for $15 what cost me $25, what per cent do I lose? *Ans.* 40%.

8. Paper bought at $2 per ream, was sold at 25 cents a quire. What was the gain? *Ans.* 150%.

9. If I sell ½ of an article for ¾ of the cost of the article, what per cent is gained? *Ans.* 50%.

10. If ⅜ of an article is sold for what ½ of the article cost, what is the loss per cent? *Ans.* 37½%.

11. If I sell 3 pecks of clover-seed for what 1 bushel cost me, what per cent do I gain? *Ans.* 33⅓%.

12. A, having a debt against B, agreed to take $.87½ on the dollar. What per cent did A lose?

13. A grocer bought 7 cwt. 20 lb. of sugar, at 7 cents a pound, and sold 3 cwt. 42 lb. at 8 cents, and the remainder at 8½ cents. What was his gain per cent?

Ans. 18$\frac{1}{28}$%.

14. A grain dealer bought corn at $.55 a bushel, which he sold at $.66, and wheat for $1.10, which he sold for $1.37½. Upon which did he make the greater per cent? *Ans.* 5% upon the wheat.

15. If I buy a piano for $650 and sell it for $400, what per cent do I lose?

275. To find the selling price, when the cost and the gain or loss per cent are given.

1. A horse was bought for $136. For how much must he be sold to gain 25%?

OPERATION.

$1 + .25 = $1.25.
$1.25 × 136 = $170, *Ans.*

SOLUTION. — Since $1 of cost sells for $1.25, $136 of cost will sell for 136 times $1.25, which equals $170, the selling price.

RULE. — *Multiply $1 increased by the gain or diminished by the loss per cent by the number denoting the cost.* Or,

Take such a part of the cost as is equal to $\frac{100}{100}$ increased or diminished by the gain or loss per cent.

2. If $12\frac{1}{2}$ hundred-weight of sugar cost $140, for how much must it be sold per pound to gain 25%?

3. I bought a hogshead of molasses for 30 cents a gallon, and paid $16\frac{2}{3}$% on the prime cost for freight and cartage. What must I sell it for, per gallon, to gain $33\frac{1}{3}$% on the whole cost? *Ans.* $.46$\frac{2}{3}$.

4. For what price must I sell coffee that cost $10\frac{1}{2}$ cents a pound, to gain $17\frac{1}{3}$%?

5. If I am compelled to sell damaged goods at a loss of 15%, how should I mark goods that cost me $.62$\frac{1}{2}$? $1.20? 3.87\frac{1}{2}$? *Ans.* $.53$\frac{1}{8}$; $1.02; 3.29\frac{3}{8}$.

6. A man, wishing to raise some money, offers his house and lot, which cost him $3240, for 18% less than cost. What is the price for which he offers it?

7. C bought a farm of 120 acres, at $28 an acre, paid $480 for fencing, and then sold it for $12\frac{1}{2}$% advance on the whole cost. What was his whole gain, and what did he receive an acre? *Ans.* $480 gain; $36 an acre.

8. A merchant bought 15 pieces of broadcloth, each piece containing $23\frac{1}{3}$ yards, for $840, and sold it so as to gain $18\frac{3}{4}$%. What did he receive a yard?

276. To find the cost, when the selling price and the gain or loss per cent are given.

1. A merchant sold cloth for $4.80 a yard, and by so doing made $33\frac{1}{3}\%$. How much did it cost?

OPERATION.

$1.00 + .33\frac{1}{3} = 1.33\frac{1}{3}$; $\$4.80 \div 1.33\frac{1}{3} = \$3.60$, *Ans.*

SOLUTION. — Since the gain is $33\frac{1}{3}\%$ of the cost, the selling price must be $100\% + 33\frac{1}{3}\% = 133\frac{1}{3}\%$ of the cost. If $4.80 is $1.33\frac{1}{3}$ of the cost, the cost must be $4.80 \div 1.33\frac{1}{3}$, which is $3.60.

If the rate per cent is loss, subtract it from 100 per cent, instead of adding it.

RULE. — *Divide the selling price by* 100% *increased by the gain or diminished by the loss per cent, expressed decimally, or in the form of a common fraction, and the quotient will be the cost.*

2. By selling sugar at 8 cents a pound, a merchant lost 20%. What did the sugar cost him? *Ans.* 10 cents.

3. I sold flour for $\$6.12\frac{1}{2}$ per barrel, and lost $12\frac{1}{2}\%$. What was the cost? *Ans.* $7.00.

4. A grocer, by selling tea at $.96 a pound, gains 28%. What did it cost him? *Ans.* $.75.

5. A quantity of flour was sold for $1881, which was $18\frac{3}{4}\%$ more than it cost. What did it cost?

6. I sold 25 barrels of apples for $69.75, and made 24%. What did they cost me per barrel?

7. A man sold $9\frac{1}{3}$ cwt. of sugar at $\$8\frac{1}{4}$ per cwt., and thereby lost 12%. What was the whole cost?

8. Having used a carriage six months, I sold it for $96, which was 20% below cost. What would I have received had I sold it for 15% above cost? *Ans.* $138.

9. A grocer sold 4 barrels of sugar for $24 each; on 2 barrels he gained 20%, and on the other 2 he lost 20%. Did he gain or lose on the whole? *Ans.* Lost $4.

INSURANCE.

277. **Insurance** is security guaranteed by one party to another, for a stipulated sum, against damage or risk. It is of two kinds: insurance on property and life insurance.

Property insurance includes Fire Insurance, Marine Insurance, and Live Stock Insurance.

Fire Insurance is indemnity for loss of property by fire. *Marine* and *Inland Insurance* are indemnity for the loss of a vessel, or cargo, by casualties of navigation on the ocean or on inland waters. *Stock Insurance* is indemnity for the loss of cattle, horses, etc.

278. The **Insurer** or **Underwriter** is the party taking the risk.

279. The **Insured** or **Assured** is the party protected by the insurance.

280. The **Policy** is the written contract between the parties.

281. The **Premium** is the sum paid by the *insured* to the *insurer*, and in insuring property is estimated at a certain rate per cent of the amount insured, which rate varies according to the degree of hazard, or class of risk.

1. As a security against fraud, most insurance companies take risks at not more than two thirds the full value of the property insured.

2. Insurance business is generally conducted by Joint Stock Companies or Mutual Companies.

3. A Stock Insurance Company is one in which the capital is owned by individuals called stockholders. They alone share the profits, and are liable for the losses.

4. A Mutual Insurance Company is one in which each person insured is entitled to a share in the profits of the concern, and is liable for the losses. The annual dividends returned by such companies (in case of profits) reduces the premium or cost of insurance; and the assessments made (in case of loss) increase the cost to the policy holder.

EXAMPLES.

282. To find the premium or cost of insurance when the rate of insurance and the amount insured are given.

1. What must I pay annually for insuring my house to the amount of $3250, at $1\frac{1}{4}$ per cent premium?

OPERATION.

$\$3250 \times .01\frac{1}{4}$ or $.0125 = \$40.625$.

Or, $1\frac{1}{4}$ per ct. $= \frac{5}{400} = \frac{1}{80}$;

$\$3250 \times \frac{1}{80} = \$40.62\frac{1}{2}$.

SOLUTION.—We multiply the amount insured, $3250, by the rate, $1\frac{1}{4}$ per cent, and the result, $40.625, is the premium.

2. A man insured his life for $10,000, the rate being $21.40 per 1000. The dividend reduces the cost of the premium an average of 30 %. How much is the average annual cost of his insurance?

OPERATION.

$\$21.40 \times 10 = \$214$, premium.

30 % of $\$214 = \$64.20$, dividend.

$\$214 - \$64.20 = \$149.80$, annual cost of insurance, *Ans.*

SOLUTION. — If $1000 worth of insurance costs $21.40, $10,000 worth will cost 10 times $21.40, which is $214, the premium. Since the dividend reduces this premium 30 %, the cost of insurance will be 30 % less than $214, which is $149.80.

RULE. — I. *Multiply the amount insured by the rate per cent, and the product will be the premium.* Or,

II. *To find the net cost of insurance, deduct the annual dividend from the premium.*

3. What is the premium on a policy for $750, at 4%? *Ans.* $30.

4. What premium must be paid for $4572.80 insurance, at $2\frac{1}{2}$ %? *Ans.* $114.32.

5. A house and furniture, valued at $5700, are insured at $1\frac{3}{4}\%$. What is the premium? *Ans.* $99.75.

6. A vessel and cargo, valued at $28400, are insured at $3\frac{1}{2}\%$. What is the premium? *Ans.* $994.

7. A woolen factory and contents, valued at $55800, are insured at $2\frac{1}{5}\%$. If destroyed by fire, what would be the actual loss of the company? *Ans.* $54237.60.

8. What must be paid to insure a steamboat and cargo from Pittsburg to New Orleans, valued at $47500, at $\frac{3}{4}$ of 1%? *Ans.* $356.25.

9. Mr. Bowtell has a house, insured for $8000, and the furniture for $4000, at $2\frac{3}{8}\%$. What premium must he pay? *Ans.* $285.

10. A cargo of 4000 bushels of wheat, worth $1.20 a bushel, is insured at $\frac{3}{4}$ of $1\frac{1}{2}\%$ on $\frac{2}{3}$ of its value. If the cargo is lost, how much will the owner of the wheat lose? *Ans.* $1636.

11. What will be the cost of insuring a quantity of wheat valued at $7500 at $\frac{4}{5}\%$?

12. What will it cost to insure a factory valued at $21000, at $\frac{4}{5}\%$, and the machinery valued at $15400, at $\frac{5}{8}\%$? *Ans.* $264.25.

13. A man insured his life for $25000, the rate being $25 per 1000. The dividend reduced the cost of the premium an average of 10 %. How much is the average annual cost of his insurance? *Ans.* $562.50.

14. If the man died 12 years after he was insured, how much would the amount received by his family exceed the total cost of insurance, no account being taken of interest? *Ans.* $18250.

15. If he died 4 years after he was insured, how much would the amount received by his family exceed the total cost of insurance, the interest on the money amounting to $337.50? *Ans.* $22412.50.

TAXES.

283. A **Tax** is a sum of money assessed on the person or property of an individual, for public purposes.

284. When a tax is assessed on *property*, it is a fixed sum apportioned at a certain *per cent* on the estimated value.

When assessed on the *person*, it is apportioned *equally* among the male citizens liable to assessment, and is called a *poll tax*. Each person so assessed is called a *poll*.

Non-resident tax-payers are not subject to a poll tax.

285. **Property** is of two kinds, real estate and personal property.

286. **Real Estate** consists of *immovable* property, such as lands, houses, etc.

287. **Personal Property** consists of *movable* property, such as money, notes, furniture, cattle, tools, etc.

288. An **Assessment Roll** is a list or schedule containing the names of all persons liable to taxation in the district or company to be assessed, and the valuation of each person's taxable property.

289. **Assessors** are the officers appointed to determine the taxable value of property, prepare the assessment rolls, and apportion the taxes.

If the assessment includes a poll tax, then a complete list of taxable polls must also be made out.

EXAMPLES.

1. A tax of $3165 is to be assessed on a certain town; the valuation of the taxable property, as shown by the assessment roll, is $600000, and there are 220 polls to be assessed 75 cents each; what will be the tax on a dollar, and how much will be A's tax, whose property is valued at $3750, and who pays for 3 polls?

OPERATION.

$.75 × 220 = $165, amount assessed on the polls.
$3165 − $165 = $3000, amount to be assessed on the property.
$3000 ÷ $600000 = .005, tax on $1.
$3750 × .005 = $18.75, A's tax on property.
$.75 × 3 = $2.25, A's tax on 3 polls.
$18.75 + $2.25 = $21, amount of A's tax.

RULE. — I. *Find the amount of poll tax, if any, and subtract this sum from the whole amount of tax to be assessed.*

II. *Divide the sum to be raised on property by the whole amount of taxable property, and the quotient will be the per cent, or the tax on* $1.

III. *Multiply each man's taxable property by the per cent, or the tax on* $1, *and to the product add his poll tax, if any; the result will be the whole amount of his tax.*

Having found the tax on $1, or the per cent, which in the preceding example we find to be 5 mills, or ½ per cent, the operation of assessing taxes may be greatly facilitated by finding the tax on $2, $3, etc., to $10, and then on $20, $30, etc., to $100, and arranging the numbers as in the following table:

TABLE.

PROP.	TAX.	PROP.	TAX.	PROP.	TAX.	PROP.	TAX.
$1 gives	$.005	$10	$.05	$100	$ .50	$1000	$ 5.00
2 "	.01	20	.10	200	1.00	2000	10.00
3 "	.015	30	.15	300	1.50	3000	15.00
4 "	.02	40	.20	400	2.00	4000	20.00
5 "	.025	50	.25	500	2.50	5000	25.00
6 "	.03	60	.30	600	3.00	6000	30.00
7 "	.035	70	.35	700	3.50	7000	35.00
8 "	.04	80	.40	800	4.00	8000	40.00
9 "	.045	90	.45	900	4.50	9000	45.00

2. According to the conditions of the last example, how much would be a person's tax whose property was assessed at $3845, and who paid for 2 polls?

Finding the amount from the table,

The tax on	$3000	is	$15.00
" " "	800	is	4.00
" " "	40	is	.20
" " "	5	is	.025
" " "	2 polls	is	1.50
	Total tax is		$20.725

3. How much would be W's tax, who was assessed for 1 poll, and on property valued at $5390? *Ans.* $27.70.

4. A tax of $13662 is to be assessed on a certain village; the property is valued at $1400000, and there are 2981 polls, to be taxed $2.00. What is the assessment on a dollar? What is C's tax, his property being assessed at $12450, and he paying for 2 polls?

Ans. $.005½ on $1; $72.47½, C's tax.

5. What is the tax of a non-resident, having property in the same village valued at $5375? *Ans.* $29.5625.

6. A mining corporation, consisting of 30 persons, is taxed $4384; its property is assessed for $188000, and each poll is assessed $2.00. What per cent is their tax, and how much must he pay whose share is assessed for $2500, and who pays for 1 poll?

Ans. $2\frac{3}{10}\%$; $59.50.

7. The number of polls in a certain school district is 225, and the taxable property is $1246093.75; it is proposed to build a union school house at an expense of $10000; if the poll tax is $1.25 a poll, and the cost of collecting is 2½ per cent, what will be the tax on a dollar, and how much will be E's tax, who pays for 1 poll, and has property to the amount of $11500?

DUTIES OR CUSTOMS.

290. Duties, or **Customs,** are taxes levied on imported goods, for the support of government and the protection of home industry.

A *Tariff* is a schedule showing the rates of duties fixed by law on all kinds of imported merchandise.

291. A **Customhouse** is an office established by government for the transaction of business relating to duties.

292. Tonnage is a tax levied upon a vessel, independent of its cargo, for the privilege of coming into a port of entry.

293. Revenue is the income to government from duties and tonnage.

294. Duties are of two kinds, **Ad Valorem** and **Specific.**

Ad Valorem Duty is a sum computed on the cost of goods in the country from which they were imported. *Specific Duty* is a sum computed on the weight or measure of goods, without regard to their cost.

295. An **Invoice** is a bill of goods showing the quantity and price of each kind.

296. In collecting customs, it is the design of the government to tax only so much of the merchandise as will be available to the importer in the market. The goods are weighed, measured, gauged, or inspected, in order to ascertain the actual quantity and value received in port; and an allowance is made in every case of waste, loss, or damage.

297. Tare is an allowance for the weight of the package or covering that contains the goods. It is ascertained by actually weighing one or more of the empty boxes, casks, or coverings. In common articles of importation, it is sometimes computed at a certain per cent previously ascertained by frequent trials.

Breakage is an allowance on liquors imported in bottles.

298. Gross Weight or **Value** is the weight or value of goods before any allowance has been made.

299. Net Weight or **Value** is the weight or value after all allowances have been deducted.

EXAMPLES.

1. What is the duty, at 24%, on 50 gross of London ale, invoiced at $1.20 per dozen, 2½% being allowed for breakage?

OPERATION.

$1.20 × 12 × 50 = $720, gross value.
$720 × .025 = $18, breakage.
$720 − $18 = $702, net value.
$702 × .24 = $168.48, duty.

SOLUTION.—First we find the cost of the ale, at the invoice price, which is $720. From this sum we deduct the allowance for breakage, $18, and compute the duty on the remainder.

RULE.—*Deduct allowances, if necessary, and compute the duty, at the given rate on the net value.*

2. What is the duty, at 19%, on 224 yards of cloth, invoiced at $.95 per yard? *Ans.* $40.43+.

3. What is the duty, at 24%, on 50 barrels of sperm oil, each containing originally 31½ gallons, invoiced at $.54 per gallon, allowing 2% for leakage?

4. What is the duty, at 15%, on 175 bags of merchandise, each containing 115 lb., valued at 15 cents per pound? *Ans.* $452.81¼.

SIMPLE INTEREST.

300. Interest is a sum paid for the use of money.

301. Principal is the sum for the use of which interest is paid.

302. Rate per cent per annum is the sum per cent paid for the use of any principal annually.

303. Amount is the sum of the principal and interest.

304. Simple Interest is the sum paid for the use of the principal only, during the whole time of the loan or credit.

305. Legal Interest is the rate per cent established by law. It varies in different States. **Usury** is a higher rate of interest than is allowed by law.

States and Territories.	Legal.	Special.	States and Territories.	Legal.	Special.
Alabama	8	Limit 8.	Montana	10	No limit.
Arizona	7	No limit.	Nebraska	7	Limit 10.
Arkansas	6	Limit 10.	Nevada	7	No limit.
California	7	No limit.	New Hampshire	6	Limit 6.
Colorado	8	No limit.	New Jersey	6	Limit 6.
Connecticut	6	Limit 6.	New Mexico	6	Limit 12.
Delaware	6	Limit 6.	New York	6	Limit 6.
Dist. of Columbia	6	Limit 10.	North Carolina	6	Limit 8.
Florida	8	No limit.	North Dakota	7	Limit 12.
Georgia	7	Limit 8.	Ohio	6	Limit 8.
Idaho	10	Limit 18.	Oregon	8	Limit 10.
Illinois	5	Limit 8.	Pennsylvania	6	Limit 6.
Indiana	6	Limit 8.	Rhode Island	6	No limit.
Iowa	6	Limit 8.	South Carolina	7	Limit 8.
Kansas	6	Limit 10.	South Dakota	7	Limit 12.
Kentucky	6	Limit 6.	Tennessee	6	Limit 6.
Louisiana	5	Limit 8.	Texas	8	Limit 12.
Maine	6	No limit.	Utah	10	No limit.
Maryland	6	Limit 6.	Vermont	6	Limit 6.
Massachusetts	6	No limit.	Virginia	6	Limit 6.
Michigan	6	Limit 10.	Washington	10	No limit.
Minnesota	7	Limit 10.	West Virginia	6	Limit 6.
Mississippi	6	Limit 10.	Wisconsin	7	Limit 10.
Missouri	6	Limit 8.	Wyoming	12	No limit.

306. In *percentage*, any per cent of any given number is so many hundredths of that number; but in *interest*, any rate per cent is confined to 1 year, and the per cent to be obtained of any given number is *greater* than the rate per cent per annum when the time is *more* than 1 year, and *less* than the rate per cent per annum when the time is *less* than 1 year.

Thus, the interest on any sum, at any rate per cent, for 3 years 6 months, is $3\frac{1}{2}$ times the interest on the same sum for 1 year; and the interest for 3 months is $\frac{1}{4}$ of the interest for 1 year.

EXAMPLES.

307. The principal, rate per cent, and time being given, to find the interest and amount.

1. What are the interest and amount on \$75.19 for 3 years 6 months, at 4% ?

OPERATION.

\$75.19	
.04	
\$3.0076	
$3\frac{1}{2}$	
15038	
90228	
\$10.5266	Int. *Ans*
75.19	
\$85.7166	Amt. *Ans.*

SOLUTION. — The interest on \$75.19 for 1 yr., at 4%, is .04 of the principal, or \$3.0076, and the interest for 3 yr. 6 mo. is $3\frac{1}{2}$ times the interest for 1 yr., \$3.0076, which is \$10.5266.

The amount is equal to the sum of the principal and interest.

RULE. — I. *Multiply the principal by the rate per cent, and the product will be the interest for* 1 *year.*

II. *Multiply this product by the time in years and fractions of a year, and the result will be the required interest.*

III. *Add the principal to this interest, and the result will be the amount.*

2. What is the interest of \$150 for 3 years, at 4% ?

Ans. \$18.

3. What is the interest of $328 for 2 years, at 7%?

4. What is the interest of $125 for 1 year 6 months, at 6%? *Ans.* $11.25.

5. What is the interest of $200 for 3 years 10 months, at 7%? *Ans.* $53.66 +.

6. What is the interest of $76.50 for 2 years 2 months, at 5%? *Ans.* $8.287.

7. What is the interest of $1276.25 for 11 months, at 7%? *Ans.* $81.89 +.

8. What is the interest of $2569.75 for 4 years 6 months, at 6%?

9. What is the interest of $1500.60 for 2 years 4 months, at $6\frac{1}{4}$%? *Ans.* $218.8375.

10. What is the amount of $26.84 for 2 years 6 months, at 5%? *Ans.* $30.195.

11. What is the amount of $450 for 5 years, at 7%?

12. What is the interest of $4562.09 for 3 years 3 months, at 3%? *Ans.* $444.80 +.

13. What is the amount of $3050 for 4 years 8 months, at $5\frac{1}{4}$%? *Ans.* $3797.25 +.

14. What is the interest of $5000 for 9 months, at 8%? *Ans.* $300.

15. If a person borrows $375, at 7%, how much will be due the lender at the end of 2 yr. 6 mo.?

16. What is the interest paid on a loan of $1374.74, at 6%, made January 1, 1892, and called in January 1, 1896? *Ans.* $329.937.

17. If a note of $605.70, given May 20, 1890, on interest at 8%, is taken up May 20, 1893, what amount will then be due if no interest has been paid?

Ans. $751.068.

18. What is the interest of $400 for 5 years, at 6%?

19. What is the interest of $950 for 4 years 6 months, at 8%? *Ans.* $342.

308. To find the interest on any sum, for any time, at any rate per cent.

The analysis of our rule is based upon the following *obvious relations between time and interest.*

I. The interest on any sum for 1 year, at 1%, is .01 of that sum, and is equal to the principal with the decimal point removed two places to the left.

II. A month being $\frac{1}{12}$ of a year, $\frac{1}{12}$ of the interest on any sum for 1 year is the interest for 1 month.

III. The interest on any sum for 3 days is $\frac{3}{30} = \frac{1}{10} = .1$ of the interest for 1 month, and any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum, for 1 month, multiplied by any given time expressed in months and tenths of a month, will produce the required interest.

1. What is the interest on $724.68 for 2 yr. 5 mo. 19 da., at 7% ?

OPERATION.

2 yr. 5 mo. 19 da. = $29.6\frac{1}{3}$ mo.

12)$7.2468

$.6039

$29.6\frac{1}{3}$

2013

36234

54351

12078

$17.89557

7

$125.26899, *Ans.*

SOLUTION. — We remove the decimal point in the given principal two places to the left, and have $7.2468, the interest on the given sum for 1 year at 1% (I.). Dividing this by 12, we have $.6039, the interest for 1 month, at 1% (II.).

Multiplying this quotient by $29.6\frac{1}{3}$, the time expressed in months and decimals of a month (III.), we have $17.89557, the interest on the given sum for the given time, at 1% (IV.). And multiplying this product by 7 (7 times 1%), we have $125.268, the interest on the given principal, for the given time, at the given rate per cent.

RULE. — I. *Remove the decimal point in the given principal two places to the left; the result will be the interest for* 1 *year, at* 1%.

II. *Divide this interest by* 12; *the result will be the interest for* 1 *month, at* 1%.

III. *Multiply this interest by the given time expressed in months and tenths of a month; the result will be the interest for the given time, at* 1%.

IV. *Multiply this interest by the given rate; the product will be the interest required.*

CONTRACTIONS. — After removing the decimal point in the principal two places to the left, the result may be regarded either as the interest on the given principal for 12 months at 1%, or for 1 month at 12%. If we regard it as for 1 month at 12%, and if the given rate is an aliquot part of 12%, the interest on the given principal for 1 month may readily be found by taking such an aliquot part of the interest for 1 month as the given rate is part of 12%. Thus,

To find the interest for 1 month at 6%, remove the decimal point two places to the left, and divide by 2.

To find it at 3%, proceed as before, and divide by 4; at 4%, divide by 3; at 2%, divide by 6, etc.

309. Six per cent method. By referring to the table on page 267 it will be seen that the legal rate of interest in 25 States is 6%. This is a sufficient reason for introducing the following brief method into this work:

At 6% per annum the interest on \$1
For 12 months . . . . . is \$.06.
" 2 months ($\frac{2}{12} = \frac{1}{6}$ of 12 mo.). . . " .01.
" 1 month, or 30 days ($\frac{1}{12}$ of 12 mo.) " \$.00½ = \$.005 ($\frac{1}{2}$ of \$.06).
" 6 days ($\frac{1}{5}$ of 30 days) . . . . . " .001.
" 1 day ($\frac{1}{6}$ of 6 da. = $\frac{1}{60}$ of 30 days) " .000⅙.

The interest of any sum at 6% is half as many hundredths of the principal as there are months in the given time, and one sixth as many thousandths as there are days in the given time.

2. Find the interest on $500 for 2 years 4 months 7 days at 6% and at 8%.

$.12	Int. of $1 for 2 yr.
.02	Int. of $1 for 4 mo.
.001$\frac{1}{6}$	Int. of $1 for 7 da.
$.141$\frac{1}{6}$	Int. of $1 for 2 yr. 4 mo. 7 da. at 6%.

Or,

2 yr. 4 mo. = 28 mo. $\frac{1}{2}$ of 28 = .14

$\frac{1}{6}$ of 7 da. = $1\frac{1}{6}$ da. = .001$\frac{1}{6}$

.141$\frac{1}{6}$

$\$.141\frac{1}{6} \times 500 = \$70.583\frac{1}{3}$, Int. at 6%.

$8\% = \frac{8}{6} = \frac{4}{3}$ of 6%.

$\frac{4}{3}$ of $\$70.583\frac{1}{3} = \$94.11\frac{1}{9}$, Int. at 8%.

SOLUTION. — Since the interest on $1 for 1 yr. is $.06, for 2 yr. it will be $.12. Since the interest for 2 mo. is $.01, for 4 mo. it will be $.02. Since the interest for 6 da. is $.001, and for 1 da. is $.000^1, for 7 da. it is $.001^1. Adding, we find the interest on $1 for 2 yr. 4 mo. 7 da. at 6% to be $.141$\frac{1}{6}$. Therefore the interest on $500 is 500 times that, which is 70.583\frac{1}{3}$. Since 8% is $\frac{4}{3}$ of 6%, the interest at 8% is $\frac{4}{3}$ of 70.583\frac{1}{3}$, which is 94.11\frac{1}{9}$, *Ans.*

RULE. — I. *Find the interest of $1 for the given time at 6%, which will be half as many hundredths of a dollar as there are months, and one sixth as many thousandths as there are days.*

II. *Multiply the principal by the interest of $1 to obtain the interest of the given number of dollars.*

1. To find the interest at any other rate per cent by this method, first find it at 6%, and then increase or diminish the result by as many times itself as the given rate is greater or less than 6%. Thus, for 7% add $\frac{1}{6}$, for 4% subtract $\frac{1}{3}$, etc.

2. The interest of $10 for 6 days, or of $1 for 60 days, is $.01. Therefore, if the principal is less than $10 and the time less than 6 days, or the principal less than $1 and the time less than 60 days, the interest will be less than $.01, and may be disregarded.

3. Since the interest of $1 for 60 days is $.01, the interest of $1 for any number of days is as many cents as 60 is contained times in the number of days. Therefore, if any principal is multiplied by the number of days in any given number of months and days, and the product divided by 60, the result will be the interest in cents. That is, *Multiply the principal by the number of days, divide the product by* 60, *and point off two decimal places in the quotient. The result will be the interest in the same denomination as the principal.*

When the time is short, interest is usually computed for the actual number of days, — 360 days being considered as a year; but when it is desirable to find the *exact interest*, we must consider the year as consisting of 365 days, and a leap year as of 366 days.

3. What is the interest of $600 from April 6, 1893, to May 4, 1893, at 6%?

$\frac{1}{6}$ of $28 = 4\frac{2}{3} = \$.004\frac{2}{3}$, Int. on $1.

$\$.004\frac{2}{3}$

600

$\$2.80$, Int. on $600, *Ans.*

SOLUTION. — The time from April 6 to May 4 is 28 da., and at 6% the interest on $1 for 28 da. is $\$.004\frac{2}{3}$. Hence the interest of $600 is $600 \times \$.004\frac{2}{3} = \$2.80$.

4. Find the *exact interest* of $600 from April 6, 1893, to May 4, 1893, at 6%.

28 da. $= \frac{28}{365}$ yr.

$\frac{28}{365}$ of $\$.06 = \frac{168}{36500} = \$.046$, Int. on $1.

$\$.0046$

600

$\$2.76$, Int. on $600.

Or,

$600

.06

$\$36.00$, Int. $600, 1 yr. at 6%.

$\frac{28}{365}$ of $\$36 = \$2.76$, Int. $600, 28 da., at 6%.

SOLUTION. — The time is 28 days. 28 days are $\frac{28}{365}$ of a year. Since the interest on $1 for 1 year is $.06, for $\frac{28}{365}$ of a year it is $\frac{28}{365}$ of $.06, which is $.0046. Hence the interest of $600 is 600 times $.0046, which is $2.76.

Or, since the interest of $600 for *one year* at 6% is $36, for $\frac{28}{365}$ of a year it is $\frac{28}{365}$ of $36, which is $2.76.

5. What is the interest of $100 for 7 years 7 months, at 6%? *Ans.* $45.50.

6. What is the amount of $47.50 for 4 years 1 month, at 9%? *Ans.* $64.956.

7. What is the amount of $2000 for 3 months, at 7%? *Ans.* $2035.

8. What is the interest of $250 for 1 year 10 months and 15 days, at 6%? *Ans.* $28.12½.

9. What is the interest of $36.75 for 2 years 4 months and 12 days, at 7%? *Ans.* $6.088.

10. What is the interest of $51.10 for 10 months and 3 days, at 4%?

11. What is the interest (6% method) of $200 from June 7, 1892, to Aug. 7, 1892, at 10%?

12. What is the *exact interest* of $200 from June 7, 1892, to Aug. 7, 1892?

NOTE. — Since 1892 is a leap year, we must reckon 363 days to the year.

13. What is the amount of $84.25 for 1 year 5 months and 10 days, at 6¼%?

14. What is the interest of $25 for 3 years 6 months and 20 days, at 6%? *Ans.* $5.33⅓.

15. What is the interest of $112.50 for 3 months and 1 day, at 9½%? *Ans.* $2.70+.

16. What is the interest of $408 for 20 days, at 6%? *Ans.* $1.36.

17. What is the interest of $500 for 22 days, at 7%?

18. What is the amount of $4500 for 10 days, at 10%? *Ans.* $4512.50

19. What is the amount of $1000 for 1 month 5 days at 6¾%? *Ans.* $1006.56¼

20. Find the interest of $973.68 for 7 months 9 days at 1⅕%?

21. A man bought a piece of property for $2870 and agreed to pay for it in 1 year and 6 months, with 6½% interest. What amount did he pay? *Ans.* $3149.82

22. In settling with a merchant, I gave my note for $97.75, due in 11 months, at 5%. What must be paid when the note falls due? *Ans.* $102.23+.

23. What is the amount of $84 for 5 years 5 months and 9 days, at 5%?

ANNUAL INTEREST.

310. Annual Interest is simple interest on the principal and on each year's interest remaining unpaid, but so computed as not to increase the original principal.

It is allowed in the case of promissory notes and other contracts which contain the words, "with interest payable annually," or "with compound interest." In such cases, the interest is not compounded beyond the second year.

EXAMPLES.

1. Find the annual interest and amount of $8000 for 5 yr., at 6% per annum.

OPERATION.

Int. of $8000 for 1 yr. at 6% = $480.
" $8000 for 5 yr. at 6% = $2400.
" $480 for 10 yr. at 6% = $288.
$2400 + $288 = $2688, Annual int.
$8000 + $2688 = $10688, Amount.

SOLUTION.—The interest on $8000 for 1 yr. at 6% is $480, and for 5 yr. is $2400.

The interest for the first year, remaining unpaid, draws interest for 4 yr.; that for the second year, for 3 yr.; that for the third year, for 2 yr.; and that for the fourth year, for 1 yr., the sum of which is equal to the interest of $480 for 4 yr. + 3 yr. + 2 yr. + 1 yr. = 10 yr.; and the interest of $480 at 6% for 10 yr. is $288. Hence the total amount of interest is $2400 + $288, or $2688, and the amount is $10688.

RULE. — *Compute the interest on the principal for the given time and rate, to which add the interest on each year's interest for the time it has remained unpaid.*

To obtain the latter, when the interest has remained unpaid for a number of years, multiply the interest for one year by the product of the number of years and half that number diminished by one.

Thus, if the time is 9 yr., the interest for 1 yr. should be multiplied by $9 \times (9 - 1) \div 2$, or $9 \times 4 = 36$. Since the interest for the first year draws 8 years' interest, that for the second year 7 years' interest, etc., the sum of the series $8+7+6+5+4+3+2+1$ is 36.

2. What is the annual interest of $1500 for 4 yr. at 7%?

3. What will $3500 amount to in 10 yr., annual interest, at 8%?

4. What is the difference between the annual interest and the simple interest of $2500 for 6 yr. at 6%?

5. Find the amount of $575, at 8% annual interest, for 9½ yr.

6. Find the amount of $2300 for 3 yr. 5 mo. 18 da., at 8% annual interest.

7. Find the amount of $1800 for 6 yr., at 6% annual interest.

8. What is the difference between the annual interest and the simple interest of $1800 for 5 yr. 8 mo., at 5%?

9. Find the annual interest and the amount of $10000 for 3 yr. 6 mo., at 8%.

10. Find the amount of $3420 for 6 yr. 6 mo., at 6% annual interest.

11. Find the annual interest and the amount of $850 for 4 yr. 2 mo., at 7%.

12. What is the difference between the annual interest and the simple interest of $5600 for 4 yr. 3 mo. 15 da., at 6%?

13. What will $2425 amount to in 5 yr., at 8% annual interest?

14. What is the annual interest of $10000 for 3 yr. 5 mo. 10 da., at 6%? What is the simple interest? What is the difference between the annual interest and the simple interest?

15. What is the difference between the annual interest and the simple interest of $600 for 1 yr., at 5%? For 2 yr.? For 3 yr.?

16. What is the amount of $13425 put at annual interest for 5 yr. 6 mo., at 4%?

PARTIAL PAYMENTS OR INDORSEMENTS.

311. A **Partial Payment** is payment in part of a note, bond, or other obligation.

When the amount of a payment is written on the back of the obligation, it becomes a receipt, and is called an *Indorsement.*

1. *Promissory Notes* are written promises to pay certain sums of money on demand, or at specified times, as "30 days after date," 2 months after date, etc. They constitute one of the most important forms of *commercial paper.* Other forms, as checks, drafts, etc., will be treated of under other heads.

2. The *Maker* or *Drawer* of the note is the person who draws it up and signs it.

3. The *Payee* is the person to whom or to whose order the money is paid.

4. An *Indorser* is a person who signs his name on the back of the note. By such an indorsement he makes himself responsible for the payment of the note.

5. The *Face* of a note is the sum of money made payable by the note.

6. A *Negotiable Note* is one made payable to bearer, or to any person's order. When so made it can be sold or transferred.

EXAMPLES.

$2000. SPRINGFIELD, MASS., Jan. 4, 1890.

1. For value received I promise to pay James Parish, or order, two thousand dollars, one year after date, with interest. GEORGE JONES.

On this note were indorsed the following payments:

Feb. 19, 1891 . . . . . $400
June 29, 1892 . . . . . $1000
Nov. 14, 1892 . . . . . $520

What remained due Dec. 24, 1893?

OPERATION.

Principal on interest from Jan. 4, 1890	$2000
Interest to Feb. 19, 1 yr. 1 mo. 15 da.	135
Amount	$2135
Payment Feb. 19, 1891	400
Remainder for a new principal	$1735
Interest from Feb. 19, 1891, to June 29, 1892, 1 yr. 4 mo. 10 da.	141.69
Amount	$1876.69
Payment June 29, 1892	1000
Remainder for a new principal	$876.69
Interest from June 29, 1892, to Nov. 14, 1892, 4 mo. 15 da.	19.725
Amount	$896.415
Payment Nov. 14, 1892	520
Remainder for a new principal	$376.415
Interest from Nov. 14, 1892, to Dec. 24, 1893, 1 yr. 1 mo. 10 da.	25.09
Remainder due Dec. 24, 1893	$401.505+

$475.50. NEW YORK, May 1, 1889.

2. For value received, we jointly and severally promise to pay Mason & Bro., or order, four hundred seventy-five dollars fifty cents, nine months after date, with interest.

JONES, SMITH & Co.

The following indorsements were made on this note:

Dec. 25, 1889,	received	$50
July 10, 1890,	"	15.75
Sept. 1, 1891,	"	25.50
June 14, 1892,	"	104

How much was due April 15, 1893?

OPERATION.

Principal on interest from May 1, 1889		$475.50
Interest to Dec. 25, 1889, 7 mo. 24 da.		21.63
Amount		$497.13
Payment Dec. 25, 1889		50
Remainder for a new principal		$447.13
Interest from Dec. 25, 1889, to June 14, 1892, 2 yr. 5 mo. 19 da.		77.29
Amount		$524.42
Payment July 10, 1890, less than interest then due	$15.75	
Payment Sept. 1, 1891	25.50	
Their sum less than interest then due	$41.25	
Payment June 14, 1892	104	
Their sum exceeds the interest then due		$145.25
Remainder for a new principal		$379.17
Interest from June 14, 1892, to April 15, 1893, 10 mo. 1 da.		22.19
Balance due April 15, 1893		$401.36+

These examples have been worked according to the method prescribed by the Supreme Court of the U. S., and are sufficient to illustrate the following rule:

UNITED STATES RULE.

I. *Find the amount of the given principal to the time of the first payment, and if this payment equals or exceeds the interest then due, subtract it from the amount obtained, and treat the remainder as a new principal.*

II. *But if the interest is greater than any payment, find the amount on the same principal to a time when the sum of the payments equals or exceeds the interest then due; subtracting the sum of the payments from that amount, the remainder will form a new principal, on which interest is to be computed as before.*

$514.96. SAN FRANCISCO, June 20, 1888.

3. Three years after date we promise to pay Ross & Wade, or order, five hundred fourteen and $\frac{96}{100}$ dollars, for value received, with 10% interest.

WILDER & BRO.

On this note were indorsed the following payments: Nov. 12, 1888, $105.50; March 20, 1890, $200; July 10, 1890, $75.60. How much remains due on the note at the time of its maturity? *Ans.* $242.12+.

$3000. CHARLESTON, May 7, 1889.

4. For value received, I promise to pay George Babcock three thousand dollars, on demand, with 7% interest.

JOHN MAY.

On this note were indorsed the following payments:

Sept. 10, 1889,	received	$25
Jan. 1, 1890,	"	500
Oct. 25, 1890,	"	75
April 4, 1891,	"	1500

How much was due Feb. 20, 1892? *Ans.* $1344.35+.

912\frac{75}{100}$. NEW ORLEANS, Aug. 3, 1889.

5. One year after date I promise to pay George Bailey, or order, nine hundred twelve $\frac{75}{100}$ dollars, with 5% interest, for value received.

JAMES POWELL.

The note was not paid when due, but was settled Sept. 15, 1892, one payment of $250 having been made Jan. 1, 1891, and another of $316.75, May 4, 1892. How much was due at the time of settlement?

Ans. $467.53+

312. The following method of computation is often used by merchants in the settlement of notes and of interest accounts running a year or less:

MERCANTILE RULE.

I. *Find the amount of the principal from the date of the note to the time of settlement.*

II. *Find the amount of each payment from the time it was made to the time of settlement.*

III. *Subtract the sum of the amounts of the payments from the amount of the principal, and the remainder will be the sum due.*

An accurate application of this rule requires that exact interest be computed by the rule for days. (See page 273, Ex. 4.)

1. On a note for $600 at 7%, dated Feb. 15, 1893, were the following indorsements: March 25, 1893, $150; June 1, 1893, $75; Oct. 10, 1893, $100. What was due Dec. 31, 1893?

OPERATION.

Am't of	$600	from	Feb.	15	to Dec.	31,	319 da.,	$636.71
"	" $150	"	Mar.	25	" "	281	" $158.08	
"	" $75	"	June	1	" "	213	" 78.06	
"	" $100	"	Oct.	10	" "	82	" 101.57	337.71
					Balance due Dec. 31, 1874,			$299.00

2. A note for $950, dated Jan. 25, 1892, payable in 9 mo., at 7% interest, had the following indorsements: March 2, 1892, $225; May 5, 1892, $174.19; June 29, 1892, $187.50; Aug. 1, 1892, $79.15. What was the balance due at the time of its maturity? *Ans.* $312.46.

3. Payments were made on a debt of $1750, dated April 5, 1891, as follows: May 10, 1891, $190; July 1, 1891, $230; Aug. 5, 1891, $645; Oct. 1, 1891, $372. What was due Dec. 31, 1891, interest at 6%? *Ans.* $355.16.

PROBLEMS IN INTEREST.

313. In all examples in interest there are five parts to be considered, the Principal, the Rate, the Time, the Interest, and the Amount.

These parts bear such a relation to one another, that any three of them being given (provided one of the three be the time or rate), the other two may readily be found. Hence there are many problems in Simple Interest, but the five following cases are the ones that occur most frequently in business transactions.

* I. The principal, rate, and time being given, to find the interest and amount.

II. The time, rate, and interest being given, to find the principal and amount.

III. The time, rate, and amount being given, to find the principal and interest.

IV. The principal, time, and interest being given, to find the rate and amount.

V. The principal, interest, and rate being given, to find the time and amount.

EXAMPLES.

314. The time, rate per cent, and interest being given, to find the principal and amount.

1. What principal in 2 years, at 6%, will gain $31.80 interest, and what will be the amount?

OPERATION.

$.12, interest of $1 in 2 years, at 6 per cent.

$31.80 ÷ $.12 = $265, Prin.

$265 + $31.80 = $296.80, Amt.

SOLUTION. — Since $1, in 2 years, at 6%, will gain $.12 interest, the principal that will gain $31.80, at the same rate and time, must be as many dollars as $.12 is contained times in $31.80; dividing, we obtain $265, the required principal. Adding this principal to the interest, we obtain $296.80, the *Amt.*

* This case has been explained in § 307.

RULE. — I. *Divide the given interest by the interest of* $1 *for the given time and rate, and the quotient will be the principal.*

II. *Add the principal and interest to obtain the amount.*

2. What principal, at 6%, will gain $28.12½ in 6 years 3 months? *Ans.* $75.

3. What sum, put at interest for 4 months 18 days, at 4%, will gain $9.20? *Ans.* $600.

4. What sum of money, invested at 7%, will pay me an annual income of $1260? *Ans.* $18000.

5. What sum must be invested in real estate, yielding 10% profit in rents, to produce an income of $3370? *Ans.* $33700.

315. The time, rate per cent, and amount being given, to find the principal and interest.

1. What principal in 2 years 6 months, at 7%, will amount to $88.125, and what will be the interest?

OPERATION.

$1.175 Amt. of $1 in 2 years 6 months at 7%.

$88.125 ÷ $1.175 = $75, Prin.

$88.125 − $75 = $13.125, Int.

SOLUTION. — Since $1, in 2 years 6 months, at 7%, will amount to $1.175, the principal that will amount to $88.125, at the same rate and time, must be as many dollars as $1.175 is contained times in $88.125; dividing, we obtain $75, the required principal. Subtracting this principal from the amount, we obtain the interest.

RULE. — I. *Divide the given amount by the amount of* $1 *for the given time and rate, and the quotient will be the principal required.*

II. *Subtract the principal from the amount to obtain the interest.*

2. What principal, at 6%, will amount to $655.20 in 8 months? *Ans.* $630.

3. What principal, at 5%, will amount to $106.855 in 5 years 5 months and 9 days? *Ans.* $84.

4. What sum, put at interest at 5½%, for 8 years 5 months, will amount to $1897.545? *Ans.* $1297.09+.

5. What sum, at 7%, will amount to $221.075 in 3 years 4 months? *Ans.* $179.25.

316. The principal, time, and interest being given, to find the rate per cent and amount.

1. I lent $450 for 3 years, and received for interest $67.50. What was the rate per cent?

OPERATION.

$ 4.50
3

$13.50, Int. of $450 for 3 yr. at 1 per cent.
$67.50 ÷ $13.50 = 5%, Rate.
$450 + $67.50 = $517.50, Amt.

SOLUTION. — Since at 1% $450, in 3 years, will gain $13.50 interest, the rate per cent at which the same principal, in the same time, will gain $67.50, must be equal to the number of times $13.50 is contained in $67.50; dividing, we obtain 5, the required rate per cent.

Adding the principal and interest, we obtain $517.50, the amount.

RULE. — I. *Divide the given interest by the interest on the principal for the given time at 1%, and the quotient will be the rate per cent required.*

II. *Add the principal and interest to obtain the amount.*

2. If I pay $45 interest for the use of $500 for 3 years, what is the rate per cent? *Ans.* 3.

3. The interest of $180 for 1 yr. 2 mo. 6 da. is $12.78. What is the rate per cent? *Ans.* 6.

4. A man invests $2000 in bank stock, and receives semi-annual dividend of $75. What is the rate per cent?

317. The principal, interest, and rate per cent being given, to find the time and amount.

1. In what time will $360 gain $86.40 interest, at 6%, and what will this principal amount to in this time at the given rate?

OPERATION.

$ 360
.06

$21.60, Int. of $360 in 1 yr. at 6 per cent.
$86.40 ÷ $21.60 = 4 yr., Time.
$360 + $86.40 = $446.40, Amt.

SOLUTION. — Since in 1 year $360, at 6%, will gain $21.60, the number of years in which the same principal, at the same rate, will gain $86.40, will be as many as $21.60 is contained times in $86.40; dividing, we obtain 4 years, the required time. Adding the principal and interest, we obtain $446.40, the amount.

RULE. — I. *Divide the given interest by the interest on the principal for* 1 *year, and the quotient will be the time required in years and decimals.*

II. *Add the principal and interest to obtain the amount.*

The decimal part of the quotient, if any, may be reduced to months and days.

2. The interest of $325 at 6% is $58.50. What is the time? *Ans.* 3 years.

3. B loaned $1600 at 6% until it amounted to $2000. What was the time? *Ans.* 4 years 2 months.

4. How long must $204 be on interest at 7%, to amount to $217.09? *Ans.* 11 months.

5. Engaging in business, I borrowed $750 of a friend at 6%, and kept it until it amounted to $942. How long did I retain it? *Ans.* 4 years 3 months 6 days.

6. How long will it take $200 to double itself at 6% simple interest? *Ans.* 16 years 8 months.

7. In what time will $675 double itself at 5%?

The time in years in which any sum will double itself may be found by dividing 100 by the rate per cent.

COMPOUND INTEREST.

318. Compound Interest is interest on both principal and interest, when the interest is not paid when due.

The simple interest may be added to the principal annually, semi-annually, or quarterly, as the parties may agree; but the taking of compound interest is not *legal*.

EXAMPLES.

1. What is the compound interest of $200, for 3 years, at 6%?

OPERATION.

	$200	Principal for 1st year.
$200 × .06 =	12	Interest for 1st year.
	$212	Principal for 2d year.
$212 × .06 =	12.72	Interest for 2d year.
	$224.72	Principal for 3d year.
$224.72 × .06 =	13.483	Interest for 3d year.
	$238.203	Amount for 3 years.
	200.000	Given principal.
	$38.203	Compound interest.

SECOND OPERATION.

1.06 × 1.06 × 1.06 = 1.191		
$200 × 1.191 =	$238.20	Amount for 3 years.
	200.00	Principal.
	$ 38.20	Compound interest.

SOLUTION. — The amount for the first year will be 106% of the principal; for the second year 106% of this, and for the third year 106% of the last result. Multiplying 106% × 106% × 106%, we have $119\frac{1}{10}\% = 1.191$, the third amount expressed decimally. Multiplying $200 by this amount, we have $238.20, and subtracting the principal from the amount, we find the compound interest to be $38.20.

RULE. — I. *Find the amount of the given principal at the given rate for one year, and make it the principal for the second year.*

II. *Find the amount of this new principal, and make it the principal for the third year, and so continue to do for the given number of years.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.* Or,

I. *Find the amount expressed in per cent for the given time.*

II. *Take this per cent of the principal.*

III. *Subtract the given principal from the amount thus obtained, and the remainder will be the compound interest.*

1. When the interest is payable semi-annually or quarterly, find the amount of the given principal for the first interval, and make it the principal for the second interval, proceeding in all respects as when the interest is payable yearly.

2. When the time contains years, months, and days, find the amount for the years, upon which compute the interest for the months and days, and add it to the last amount, before subtracting.

2. What is the compound interest of $500 for 2 years, at 7%? *Ans.* $72.45.

3. What is the amount of $312 for 3 years, at 6%, compound interest? *Ans.* $371.59+.

4. What is the compound interest of $250 for 2 years, payable semi-annually, at 6%? *Ans.* $31.37+.

5. What will $450 amount to in 1 year, at 7%, compound interest, payable quarterly? *Ans.* $482.33.

6. What is the compound interest of $236 for 4 years 7 months and 6 days, at 6%? *Ans.* $72.66+.

7. What is the amount of $700 for 3 years 9 months and 24 days, at 7% compound interest?

Ans. $906.55+.

8. What is the compound interest of $640 for 4 years, at 5%? *Ans.* $137.924.

9. What will $250 amount to in 3 years, at 7% compound interest? *Ans.* $306.26.

10. Find the compound interest of $376 for 3 years 8 months 15 days, at 6%. *Ans.* $90.85.

A more expeditious method of computing compound interest is by means of the following table:

TABLE.

Showing the amount of $1, *or* £1, *at* 3, 4, 5, 6, *and* 7 *per cent, compound interest, for any number of years, from* 1 *to* 20.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030,000	1.040,000	1.050,000	1.060,000	1.07,000
2	1.060,900	1.081,600	1.102,500	1.123,600	1.14,490
3	1.092,727	1.124,864	1.157,625	1.191,016	1.22,504
4	1.125,509	1.169,859	1.215,506	1.262,477	1.31,079
5	1.159,274	1.216,653	1.276,282	1.338,226	1.40,255
6	1.194,052	1.265,319	1.340,096	1.418,519	1.50,073
7	1.229,874	1.315,932	1.407,100	1.503,630	1.60,578
8	1.266,770	1.368,569	1.477,455	1.593,848	1.71,818
9	1.304,773	1.423,312	1.551,328	1.689,479	1.83,845
10	1.343,916	1.480,244	1.628,895	1.790,848	1.96,715
11	1.384,234	1.539,454	1.710,339	1.898,299	2.10,485
12	1.425,761	1.601,032	1.795,856	2.012,196	2.25,219
13	1.468,534	1.665,074	1.885,649	2.132,928	2.40,984
14	1.512,590	1.731,676	1.979,932	2.260,904	2.57,853
15	1.557,967	1.800,944	2.078,928	2.396,558	2.75,903
16	1.604,706	1.872,981	2.182,875	2.540,352	2.95,216
17	1.652,848	1.947,900	2.292,018	2.692,773	3.15,881
18	1.702,433	2.025,817	2.406,619	2.854,339	3.37,293
19	1.753,506	2.106,849	2.526,950	3.025,600	3.61,652
20	1.806,111	2.191,123	2.653,298	3.207,135	3.86,968

11. What is the amount of $800 for 6 years, at 7%?

OPERATION.

From the table $1.50073 = Amount of $1 at 7% for the time.
800 Principal.
$1200.58400, *Ans.*

12. What is the compound interest of $120 for 15 years, at 5%? *Ans.* $129.47+.

13. What is the amount of $.10 for 20 years, at 7%? *Ans.* $.38696.

PROBLEMS IN COMPOUND INTEREST.

319. The problems in compound interest are the same as those in simple interest, with the exception that the interest is compounded. There are five parts to be considered, the principal, rate, time, compound interest, and amount.

I. The principal, rate, and time being given, to find the compound interest and amount. (307.)

II. The time, rate, and compound interest being given, to find the principal and amount. (314.)

III. The time, rate, and amount being given, to find the principal and compound interest. (315.)

IV. The principal, time, and compound interest being given, to find the rate and amount. (316.)

V. The principal, compound interest, and rate being given, to find the time and amount. (317.)

The operations will be greatly facilitated by consulting the Compound Interest Table (p. 288).

1. What are the compound interest and amount of $200 for 3 years, at 5% ? *Ans.* Amt. $231.525.

2. What principal, at 6% compound interest, will gain $6180 in 2 years, and what will be the amount?

3. What principal, at 6% compound interest, will amount to $2382.032 in 3 years, and what will be the interest? *Ans.* Prin. $2000; Comp. Int. $382.032.

4. If $84.8428 compound interest is paid for the use of a principal of $ 700 for 3 years, what is the rate per cent? *Ans.* 4%.

5. In what time will $500 gain $72.45, at 7% compound interest?

SOLUTION. — Since $500 gains $72.45 in a certain time, $1, in the same time at the same rate, will gain $\frac{1}{500}$ of $72.45, which is $.1449. Consulting the table to find in what time $1 will gain $.1449, at 7%, we find it to be 2 years.

6. In what time will $300 gain $122.13, compound interest at 5% ? *Ans.* 7 years.

TRUE DISCOUNT.

320. True Discount is an abatement or allowance made for the payment of a debt before it is due.

321. The **Present Worth** of a debt, payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the given debt when it becomes due.

The True Discount is the difference between the whole debt and the Present Worth.

EXAMPLES.

1. A owes B $321, payable in 1 year. What is the present worth of the debt, the use of money being worth 7%? What is the true discount?

OPERATION.

Amount of $1 = $1.07) $321 ($300, Present value.
321

$321 Given sum or debt.
300 Present worth.

$21 Discount.

SOLUTION.—The amount of $1 for 1 year is $1.07; therefore the present worth of every $1.07 of the given debt is $1; and the present worth of $321 will be as many dollars as $1.07 is contained times in $321. $321 ÷ $1.07 = $300, *Ans.*

RULE.—I. *Divide the given sum or debt by the amount of* $1 *at the given rate, for the given time, and the quotient will be the present worth of the debt.*

II. *Subtract the present worth from the given sum or debt, and the remainder will be the true discount.*

The terms *present worth, discount,* and *debt,* correspond respectively to *principal, interest,* and *amount.* Hence, when the time, rate per cent, and debt (amount) are given, the present worth (principal) may be found by (315); and the true discount (interest), by subtracting the principal from the amount.

2. What is the present worth of $180, payable in 3 years 4 months, at 6% ? *Ans.* $150.

3. What is the present worth of a note for $1315.389, due in 2 years 6 months, at 7% ? *Ans.* $1119.48.

4. What is the present worth of a note for $866.038, due in 3 years 6 months and 6 days, when money is worth 8% ? What is the discount?

Ans. $190.15+, discount.

5. What is the present worth of a debt for $1005, on which $475 is to be paid in 10 months, and the remainder in 1 year 3 months, the rate of interest being 6% ?

When payments are to be made at different times without interest, find the present worth of each payment separately and add the results.

6. I hold a note against C for $529.925, due Sept. 1, 1893. What must I discount for the payment of it to-day, Feb. 7, 1893, money being worth 6% ?

Ans. $17.425.

7. A man was offered $3675 in cash for his house, or $4235 in 3 years, without interest. He accepted the latter offer. How much did he lose, money being worth 7% ? *Ans.* $175.

8. A man, having a span of horses for sale, offered them for $480 cash in hand, or a note of $550 due in 1 year 8 months, without interest. The buyer accepted the latter offer. Did the seller gain or lose thereby, and how much, interest being 6% ? *Ans.* Seller gained $20.

9. What must be discounted for the present payment of a debt of $2637.72, of which $517.50 is to be paid in 6 months, $793.75 in 10 months, and the remainder in 1 year 6 months, the use of money being worth 7% ?

Ans. $187.29+.

10. What is the difference between the interest and true discount of $130, due 10 months hence, at 10% ?

Ans. $.83$\frac{1}{3}$.

BANK DISCOUNT.

322. A **Bank** is a corporation chartered by law for the purpose of receiving and loaning money.

323. A **Promissory Note** is a written or printed engagement to pay a certain sum, either on demand, or at a specified time.

324. **Bank Discount** is an allowance made to a bank for the payment of a note before it becomes due.

If a banker is satisfied that a note is valid, and has ample security, he will cash it; that is, advance the sum due less the Bank Discount, which is simple interest upon the amount due for three days more than the time, at the legal rate.

325. The **Face** of a note is the sum made payable by the note.

326. **Days of Grace** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

327. The **Maturity** of a note is the expiration of the days of grace; a note is *due* at maturity.

The term of discount is the time from the discount of a note to its maturity.

328. Notes sometimes contain a promise of interest, which is reckoned from the date of the note, unless some other time is specified.

329. The transaction of borrowing money at banks is usually conducted in accordance with the following custom: the borrower presents a note, either made or indorsed by himself, payable at a specified time, and receives for it a sum equal to the face, less the interest for the time the note has to run. The amount thus withheld by the bank is in consideration of advancing money on the note prior to its maturity.

330. The **Proceeds** of a note is the sum received for it when discounted, and is equal to the face of the note less the discount.

EXAMPLES.

331. Given the face of a note, to find the proceeds and bank discount.

1. What are the proceeds and bank discount of a note for $2000, due in 2 mo. 15 da. at 6% ?

OPERATION.

Time = 2 mo. 18 da.
$.013 = Int. on $1 for 2 mo. 18 da.
$.013 × 2000 = $26, Bank Discount.
$2000 − $26 = $1974, Proceeds.

SOLUTION. — The time + 3 da. = 78 da. The interest of $1 for 78 da. = $.013. The interest of $2000 = 2000 × $.013 = $26, bank discount. The face of the note = $2000 − bank discount, $26, = the proceeds $1974, *Ans.*

RULE. — I. *Compute the interest on the face of the note for three days more than the specified time; the result will be the discount.*

II. *Subtract the discount from the face of the note, and the remainder will be the proceeds.*

2. What is the discount and what the proceeds of a note for $450, at 60 days, discounted at a bank at 6% ?

Ans. Discount, $4.725; proceeds, $445.275.

3. What are the proceeds of a note for $368, at 90 days, discounted at the Bank of New York ? *Ans.* $362.30.

4. What will I receive on my note for $475.50, at 60 days, if discounted at the Crescent City Bank, New Orleans ? *Ans.* $471.33+.

5. What are the proceeds of a note for $10000, at 90 days, discounted at the Philadelphia Bank ? *Ans.* $9845.

6. I paid, in cash, $240 for a lot of merchandise. I sold it the same day, receiving a note for $250, at 60 days, which I got discounted at the Hartford Bank. What did I make by this speculation?

7. A note for $360.76, drawn at 90 days, is discounted at the Vermont Bank. Find the proceeds.

Ans. $355.168.

8. Wishing to borrow $530 of a western bank which is discounting paper at 8%, I give my note for $536.75, payable in 60 days. How much do I need to make up the required amount? *Ans.* $.7645.

1. To indicate the maturity of a note or draft, a vertical line (|) is often used, with the day at which the note is nominally due on the left, and the date of maturity on the right; thus, Jan. $^{7}|_{10}$.

2. When a note is *on interest*, payable at a future specified time, the *amount* is the face of the note, or the sum made payable, and must be made the basis of discount.

Find the maturity, term of discount, and proceeds of the following notes:

$500. BOSTON, Jan. 4, 1889.

9. Three months after date, I promise to pay to the order of John Brown & Co. five hundred dollars, at the Suffolk Bank, value received. JAMES BARKER.

Discounted March 2.

Ans. Due, April $^{4}|_{7}$.
Term of discount, 36 da.
Proceeds, $497.

$750. ST. LOUIS, June 12, 1891.

10. Six months after date, for value received, I promise to pay Thomas Lee, or order, seven hundred fifty dollars, with interest. BYRON QUINBY.

Discounted at a broker's, Nov. 15, at 10%.

Ans. Due, Dec. $^{12}|_{15}$.
Term of discount, 30 da.
Proceeds, $766.434+.

332. Given the proceeds of a note, to find the face.

1. I wish to borrow $400 at a bank. For what sum must I draw my note, payable in 60 days, so that when discounted at 6% I shall receive the desired amount?

OPERATION.

$1.0000
.0105 = discount on $1 for 63 da.

$ .9895 = proceeds of $1.
$400 ÷ $.9895 = $404.244 = face of the required note.

SOLUTION.—$400 is the proceeds of a certain note, the face of which we are required to find. We first obtain the proceeds of $1 for 63 da., which is $.9895, and then divide the given proceeds, $400, by this sum; for, as many times as the proceeds of $1 is contained in the given proceeds, so many dollars must be the face of the required note.

RULE.—*Divide the proceeds by the proceeds of* $1 *for the time and rate mentioned, and the quotient will be the face of the note.*

2. What is the face of a note at 60 days, which yields $680 when discounted at a bank at 7%?

Ans. $688.433.

3. What is the face of a note at 90 days, of which the proceeds are $1000 when discounted at a Louisiana bank? *Ans.* $1013.085.

4. Wishing to borrow $500 at a bank, for what sum must my note be drawn, at 30 days, to obtain the required amount, discount being at 7%? *Ans.* $503.22.

5. James Hopkins buys merchandise of me in New York, at cash price, to the amount of $1256. Not having money, he gives his note in payment, drawn at 6 months. What must be the face of the note?

Ans. $1295.51.

6. The proceeds of a note drawn at 2 mo. 15 da., rate 6%, are $987. What is the face of the note? *Ans.* $1000.

SAVINGS-BANK ACCOUNTS.

333. A **Savings-Bank** is designed chiefly to accommodate depositors of small sums of money.

Interest is allowed semi-annually on all sums that have been on deposit for a certain time, if not drawn out before the regular day of paying interest — generally on the 1st of January and of July.

Savings-banks generally allow interest only from the commencement of each quarter; but in some banks money deposited previous to the 1st day of any month draws interest from that date to the day of declaring interest dividends, provided it has not been previously withdrawn.

EXAMPLES.

1. A person had on deposit Jan. 1, 1874, $150. His subsequent deposits were, Feb. 3, $35; March 29, $20; April 10, $43; May 15, $26. His drafts during the same time were, Jan. 15, $50; Feb. 27, $15; April 19, $45. What interest was due July 1st, at 6%?

OPERATION.

Date of Int. payments.	Balance 1st of month.	Smallest bal. during mo.	Interest for 1 month.	Smallest bal. dur'g q'rter.	Interest for 1 quarter.
JAN. 1	$150				
Feb. 1	100	$100	$.50		
Mar. 1	120	100	.50		
APR. 1	140	120	.60	$100	$1.50
May 1	138	138	.69		
June 1	164	138	.69		
JULY 1	164	164	.82	138	2.07
			$3.80		$3.57

Balance due with interest by *monthly* periods . . . $167.80.
Balance due with interest by *quarterly* periods . . $167.57.

SOLUTION. — At the end of January, the balance due is $100, which having been on deposit for the month, draws interest for 1 mo.; at the end of February the balance is $120; but the *smallest balance* during the month is $100; hence interest is allowed only on that sum. The same principle applies to the other balances. If only *quarterly periods* of interest are allowed, the interest is calculated at the end of each quarter on the smallest balance during the quarter, or, in this case, on $100, April 1, and $138, July 1.

RULE. — *At the end of each term compute the interest for the term on the smallest balance on deposit at any time during the quarter; and at the end of each period of six months add to the balance of principal the whole amount of interest due, and the sum will be the principal at the commencement of the next six months.*

2. Find the balance due July 1, on the following account: *Deposits,* Jan. 15, $175; April 10, $60; May 31, $110. *Drafts,* March 5, $75; May 1, $35; June 10, $50. Interest at 6%, from the 1st day of each month.

Ans. $188.43, balance July 1.

3. A person deposits in a savings-bank the following sums: Jan. 1, $350; Feb. 5, $150; March 15, $75; May 10, $30; June 15, $100. During the same time he draws, Jan. 15, $150; Feb. 10, $200; March 31, $50; June 1, $75. What interest at 6%, payable from the 1st of each month, must be added to the account July 1?

4. Balance the following, Jan. 1, 1875: Balance due to Margaret Brown, July 1, 1874, $275. Deposits received as follows: Aug. 1, $125; Sept. 15, $57; Oct. 10, $350. Drafts paid: July 15, $100; Sept. 1, $150; Nov. 15, $68; Dec. 15, $125. Interest at 6%, from the first of each quarter, July 1 and Oct. 1. *Ans.* $369.36.

5. How much was due Jan. 1, 1876, on the following account, allowing interest, computed from the 1st of each quarter, Jan. 1 to July 1, at 6% per annum?

Dr. GREENWICH SAVINGS BANK, *in acct. with* MARY WILLIAMS. *Cr.*

1874.				1874.			
Jan. 1	To Cash	$136	00	Sept. 15	By Check	$75	00
Mar. 17	" "	25	00	1875.			
Aug. 1	" "	87	50	Jan. 20	" "	37	50
1875.				Mar. 3	" "	50	00
June 11	" "	150	00				
Nov. 17	" "	72	00				

EXCHANGE.

334. Exchange is a method of remitting money from one place to another, or of making payments by written orders.

335. A **Bill of Exchange** is a written request or order upon one person to pay a certain sum to another person, or to his order, at a specified time.

336. A **Sight Draft** or **Bill** is one requiring payment to be made "at sight," which means at the time of its presentation to the person ordered to pay. In **Time Drafts,** the time specified is usually a certain number of days "after sight."

There are always three parties, and usually four, to a transaction in exchange.

337. The **Drawer** or **Maker** is the person who signs the order or bill. The **Drawee** is the person to whom the order is addressed.

338. The **Payee** is the person to whom the money is ordered to be paid.

339. The **Buyer** or **Remitter** is the person who purchases the bill. He may be himself the *payee*, or the bill may be drawn in favor of any other person.

340. The **Indorsement** of a bill is the *writing* upon its back, by which the *payee* relinquishes his title, and transfers the payment to another. The payee may indorse *in blank* by writing his name only, which makes the bill payable to the *bearer*, and consequently transferable like a bank note; or he may

accompany his signature by a special order to pay to another person, who in his turn may transfer the title in like manner. *Indorsers* become separately responsible for the amount of the bill, in case the *drawee* fails to make payment. A bill made payable to the *bearer* is transferable without indorsement.

341. The **Acceptance** of a bill is the promise which the *drawee* makes when the bill is presented to him to pay it at maturity; this obligation is usually acknowledged by writing the word "Accepted," with his signature, across the face of the bill.

Three days of grace are usually allowed for the payment of a bill of exchange after the time specified has expired. But in New York State no grace is allowed on sight drafts.

From these definitions, the use of a bill of exchange in monetary transactions is readily perceived. If a man wishes to make a remittance to a creditor, agent, or any other person residing at a distance, instead of transporting specie, which is attended with expense and risk, or sending bank notes, which are liable to be uncurrent at a distance from the banks that issue them, he remits a bill of exchange, purchased at a bank or elsewhere, and made payable to the proper person in or near the place where he resides.

Thus a man by paying Boston funds in Boston, may put New York funds into the hands of his New York agent.

342. The **Course of Exchange** is the variation of the cost of *sight bills* from their par value, as affected by the relative conditions of trade and commercial credit at the two places between which exchange is made. It may be either at a *premium* or *discount*,

and is rated at a certain per cent on the face of the bill. Bills payable a specified time *after sight* are subject to *discount*, like notes of hand, for the term of credit given. Hence their value in the money market is affected by both the course of exchange and the discount for time.

343. FORMS OF DRAFTS AND BILLS.

A SIGHT DRAFT.

$500. NEW YORK, March 1, 1893.

At sight, pay to the order of Harold Brown, five hundred dollars, value received, and charge to the acc't of

HENRY M. LANE.

To CLARKE, MORRIS & Co.,
Cincinnati, O.

Other drafts have the same form as the above, except that instead of the words "at sight," "—— days after sight," or "—— days after date," are used. When the time is *after sight*, it means after acceptance.

SET OF EXCHANGE.

£700. NEW YORK, Feb. 1, 1892.

At sight of this FIRST of Exchange (Second and Third of the same tenor and date unpaid), pay to the order of Samuel Monmouth, *Seven Hundred Pounds Sterling*, for value received, and charge the same to the account of

H. B. CLAFLIN Co.

ROBINS, BARCLAY & Co.,
London.

The above is the form of the *first bill;* the *second* requires only the change of "FIRST" into "SECOND," and instead of "Second and Third of the same tenor," etc., "First and Third." The Third Bill varies similarly.

DOMESTIC OR INLAND EXCHANGE.

344. Domestic or **Inland Exchange** relates to remittances made between different places in the same country.

EXAMPLES.

345. To find the cost of a draft.

$500. SYRACUSE, May 7, 1889.

1. At sight, pay to James Clark, or order, five hundred dollars, value received, and charge the same to our account. M. SMITH & Co.

To MESSRS. BROWN & FOSTER, Baltimore.

What is the cost of the above draft, the rate of exchange being $1\frac{1}{2}\%$ premium?

OPERATION.

$\$500 \times 1.015 = \$507.50$, *Ans.*

SOLUTION. — Since exchange is at $1\frac{1}{2}\%$ premium, each dollar of the draft will cost $1.015; and to find the whole cost of the draft, we multiply its face, $500, by 1.015, and obtain $507.50, the required result.

$480. BOSTON, June 12, 1892.

2. Thirty days after sight, pay to John Otis, or bearer, four hundred eighty dollars, value received, and charge the same to account of AMOS TRENCHARD.

To JOHN STILES & Co., New York.

What is the cost of the above draft, exchange being at a premium of 3%?

OPERATION.

$1.0000
.0055 = discount for 33 days.
$ .9945 = proceeds of $1.
.03 = rate of exchange.
$1.0245 = cost of $1 of the draft.
$480 × 1.0245 = $491.76, *Ans.*

SOLUTION. — Since time is allowed, the draft must suffer discount in the sale. The discount of $1, at the legal rate in Boston, for the specified time, allowing grace, is $.0055 which, subtracted from $1, gives

$.9945, the cost of $1 of the draft, provided sight exchange were at *par;* but sight exchange being at *premium*, we add the rate, .03, to .9945, and obtain $1.0245, the *actual* cost of $1. Then, multiplying $480 by 1.0245, we obtain $491.76.

RULE. — I. *For sight drafts. — Multiply the face of the draft by* 1 *plus the rate when exchange is at a premium, and by* 1 *minus the rate when exchange is at a discount.*

II. *For time drafts. — Find the proceeds of* $1 *at bank discount for the specified time, at the legal rate where the draft is purchased; then add the rate of exchange when at a premium, or subtract it when at a discount, and multiply the face of the draft by this result.*

3. A merchant in Cincinnati wishes to remit $1000 by draft to his agent in New York. What will the bill cost, exchange being at 3% premium? *Ans.* $1030.

4. What will be the cost in Rochester of a draft on Albany for $400, payable at sight, exchange being at $\frac{3}{4}$% premium? *Ans.* $403.

5. A merchant in St. Louis orders goods from New York, to the amount of $530, which amount he remits by draft, exchange being at $2\frac{3}{4}$% premium. If he pays $20 for transportation, what will the goods cost him in St. Louis? *Ans.* $564.575.

6. What will be the cost, in Detroit, of a draft on Boston for $800, payable 60 days after sight, exchange being at a premium of 2%?

7. A man in Philadelphia purchased a draft on Chicago for $420, payable 30 days after sight. What did it cost him, the rate of exchange being $1\frac{1}{2}$% discount?

8. A merchant in Portland receives from his agent 320 barrels of flour, purchased in Chicago at $10 per barrel; in payment for which he remits a draft on Chicago, at $2\frac{1}{2}$% discount. The transportation of his flour cost $312. What must he sell it for per barrel to gain $400?

346. To find the face of a draft which a given sum will purchase.

1. A man in Indiana paid $369.72 for a draft on Boston, drawn at 30 days. What was the face of the draft, exchange being at $3\frac{1}{4}\%$ premium?

OPERATION.

$369.72 ÷ $1.027 = $360, *Ans.*

SOLUTION. — We find, **345**, that a draft for $1 will cost $1.027; hence the draft that will cost $369.72 must be for as many dollars as $1.027 is contained times in $369.72; dividing, we obtain $360, the required result.

RULE. — *Divide the given cost by the cost of a draft for* $1, *at the given rate of exchange; the quotient will be the face of the required draft.*

2. What draft may be purchased for $243.60, exchange being at $1\frac{1}{2}\%$ premium? *Ans.* $240.

3. An agent in Pittsburg holding $282.66, due his employer in New Haven, is directed to make the remittance by draft, drawn at 60 days. What will be the face of the draft, exchange being at 2% premium?

Ans. $280.

4. What draft may be purchased for $79.20, exchange being at 1% discount? *Ans.* $80.

5. A Philadelphia manufacturer is informed by his agent in Buffalo that $3600 is due him on the sale of some property. He instructs the agent to remit by a draft payable in 60 days after sight, exchange being at $\frac{3}{4}\%$ premium. The agent, by mistake, remits a sight draft, which, when received in Philadelphia, is *accepted*, and paid after the expiration of the three days of grace. If the manufacturer immediately puts this money at interest at the legal rate, will he gain or lose by the blunder of his agent? *Ans.* He will lose $1.90.

FOREIGN EXCHANGE.

347. Foreign Exchange relates to remittances made between different countries.

The drafts or bills of exchange are expressed in the money of the country in which they are made payable.

Exchanges with Europe are effected chiefly through prominent financial centers, as London, Paris, Amsterdam, Antwerp, Berlin, Frankfort, Hamburg, Bremen, etc.

Sterling Exchange consists of bills on any part of Great Britain.

348. Quotations are the published rates at which bills of exchange, stocks, bonds, etc., are bought and sold in the money market.

The rate of exchange varies with the amount of business and the direction of money remittances. In computing exchange, it is the custom in some cases to state the value of the United States monetary unit in units and fractions of a unit of foreign currency, and in other cases to express the value of the foreign monetary unit in dollars and fractions of a dollar United States money.

Thus, quotations of exchange on London give the value of £1 sterling in *dollars* and *cents;* on Paris, Antwerp, and Geneva, the value of $1 in *francs;* on Amsterdam, the value of 1 *guilder* or *florin* in *cents;* on Hamburg, Frankfort, and Berlin, the value of 4 mark in *cents.*

349. A **Set of Exchange** is a bill drawn in triplicate, named FIRST, SECOND, and THIRD of exchange, each copy being valid until the amount of the bill is paid. These copies are sent by different mails, to provide against miscarriage. When one is paid, the others are *void.*

EXAMPLES.

350. To find the cost of a foreign bill of exchange.

1. Find the cost in New York, of the following bill on London, at 3 da. sight, exchange quoted at $4.87½.

£ 500. NEW YORK, April 1, 1896.

At sight of this First of Exchange (Second and Third unpaid), pay to the order of John Walker & Co., Five Hundred Pounds sterling, value received, and charge the same to the account of BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co.,
London, England.

OPERATION.

$\$4.875 \times 500 = \$2437.50$, *cost* of the bill, *Ans.*

SOLUTION. — Since £1 costs $4.875, £500 will cost 500 times $4.875 = $2437.50, *Ans.*

2. Find the cost of a bill on Paris, for 495 francs, at 5.15.

OPERATION.

$495 \div 5.15 = \$96.12$, *Ans.*

SOLUTION. — Since 5.15 francs cost $1, 495 francs will cost as many dollars as 5.15 francs are contained times in 495 francs, or $96.12.

3. Find the cost of a bill of exchange on Berlin, for 1750 marks, quoted at 96¼.

OPERATION.

$\$.9625 \div 4 \times 1750 = \$421.09$, *cost* of the bill.

SOLUTION. — Since $.9625 is the value of 4 mark, the value of 1 mark is ¼ of $.9625, and the value of 1750 mark is 1750 times the quotient, which is $421.09, *Ans.*

RULE. — *Multiply the face of the bill of exchange by the value of the foreign monetary unit in United States money.* Or,

Divide the face of the bill of exchange by the value of $1 *in the foreign monetary unit expressed decimally.*

4. Find the cost of a bill on Liverpool, for £600 15*s.*, at $4.86½.

5. Find the cost of a bill on Geneva, Switzerland, for 5460 francs, at 5.21¼.

6. Find the cost of a bill on Hamburg, for 2560 mark, at 95⅝.

351. To find the face of a bill of exchange.

1. What will be the face of a bill on London, that can be bought for $5488.26, exchange selling at $4.85?

OPERATION.

$5488.26 ÷ $4.85 = 1131.6, or £1131.6 = £1131 12*s.*

SOLUTION. — Since £1 = $4.85, $5488.26 will equal as many pounds as $4.85 is contained times in $5488.26, which is 1131.6 times, or £1113.6 = £1131 12*s.*

2. What will be the face of a bill on Paris, bought for $325, exchange at 5.17?

OPERATION.

5.17 francs × 325 = 1680.25 francs.

SOLUTION. — Since $1 = 5.17 francs, $325 = 325 × 5.17 francs, = 1680.25 francs, *Ans.*

3. What will be the face of a bill on Hamburg, bought for $4000, exchange quoted at 96?

OPERATION.

$.96 ÷ 4 = $.24.

$4000 ÷ $.24 = 16666⅔ mark.

SOLUTION. — Since 4 mark = $.96, 1 mark = ¼ of $.96 = $.24, and $4000 = as many mark as $.24 is contained times in $4000 = 16666⅔ times, or 16666⅔ mark.

RULE. — *Divide the cost of exchange by the value of the foreign monetary unit in United States money.* Or,

Multiply the cost of exchange by the value of $1 *in the foreign monetary unit expressed decimally.*

4. Find the face of a bill on Manchester, England, bought for $7500, exchange at 4.86.

5. Find the face of a bill on Frankfort, bought for $395.75, exchange at 95⅛.

6. Find the face of a bill on Geneva, Switzerland, bought for $4856, exchange at 5.22¼.

7. Find the face of a bill on Amsterdam, bought for $3750.67, exchange at 42½.

8. Find the face of a bill on Berlin, bought for $4000, exchange being 93¾.

PAYMENTS AND ACCOUNTS.

352. Equation of Payments is the process of finding the mean or equitable time of payment of several sums, due at different times without interest.

353. The **Term of Credit** is the time to elapse before a debt becomes due.

354. The **Average Term of Credit** is the time to elapse before several debts, due at different times, may all be paid at once, without loss to debtor or creditor.

355. The **Equated Time** is the date at which the several debts may be canceled by one payment.

356. An **Account** is a statement or record of mercantile transactions in business form.

357. The **Items** of an account may be sums due at the date of the transaction, or on credit for a specified time.

An account may have both a debit and a credit side, the former marked *Dr.*, the latter *Cr.* Suppose A and B have dealings in which there is an interchange of money or property; A keeps the account, heading it with B's name; the *Dr.* side of the account shows what B has received from A, the *Cr.* side shows what he has parted with to A.

358. The **Balance** of account is the difference of the two sides, and may be in favor of either party.

1. If, in the transactions, one party has received nothing from the other, the balance is simply the whole amount, and the account has but one side. Bills of purchase are of this class.

2. Book accounts bear interest after the expiration of the term of credit, and notes after they become due.

359. To **Average an Account** is to find the mean or equitable time of payment of the balance.

360. A **Focal Date** is a date to which all the others are compared in averaging an account.

EXAMPLES.

361. To find the equated time when all the terms of credit begin at the same date.

1. On the first day of January I find that I owe Mr. Smith 8 dollars, to be paid in 5 months, 10 dollars to be paid in 2 months, and 12 dollars to be paid in 10 months. At what time may I pay the whole amount?

OPERATION.

$8 \times 5 = \$40$
$10 \times 2 = 20$
$12 \times 10 = 120$

$\$30$ $\quad$ $\$180 \div \$30 = 6$ mo., average time of credit.

Jan. $1 + 6$ mo. = July 1, equated time of payment.

SOLUTION. — The whole amount to be paid, as seen above, is $30; and we are to find how long it shall be withheld, or what term of credit it shall have, as an equivalent for the various terms of credit on the different items. Now the value of credit on any sum is measured by the product of the money and time. The credit on $8 for 5 mo. = the credit on $40 for 1 mo. In the same manner, we have the credit on $10 for 2 mo. = the credit on $20 for 1 mo.; and the credit on $12 for 10 mo. = the credit on $120 for 1 mo. The value of the several terms of credit equals a credit of 1 month on $180; and this equals a credit of 6 months on $30, since

$$30 \times 6 = 180 \times 1.$$

RULE. — I. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient will be the average term of credit.*

II. *Add the average term of credit to the date at which all the credits begin, and the result will be the equated time of payment.*

1. The periods of time used as multipliers must all be of the same denomination, and the quotient will be of the same denomination as the terms of credit; if these are months, and there is a remainder after the division, continue the division to days by reduction, always taking the nearest unit in the last result.

2. The several rules in equation of payments are based upon the principle of bank discount; for they imply that the *discount* of a sum paid before it is due equals the *interest* of the same amount paid after it is due.

2. On the 25th of September a trader bought merchandise, as follows: $700 on 20 days' credit; $400 on 30 days' credit; $700 on 40 days' credit. What was the average term of credit, and what was the equated time of payment?

3. On July 1 a merchant gave notes, as follows: the first for $250, due in 4 months; the second for $750, due in 2 months; the third for $500, due in 7 months. At what time may they all be paid in one sum?

4. A farmer bought a cow, and agreed to pay $1 on Monday, $2 on Tuesday, $3 on Wednesday, and so on for a week; desirous afterward to avoid the Sunday payment, he offered to pay the whole at one time. On what day of the week would this payment come?

362. To find the equated time when the terms of credit begin at different dates, and the account has only one side.

1. When does the amount of the following bill become due, by averaging?

J. C. SMITH,

1892. To C. E. BORDEN, Dr.

June 1.	To cash	$450
" 12.	" Mdse. on 4 mo.	500
Aug. 16.	" Mdse.	250

FIRST OPERATION.

Due.	da.	Items.	Prod.
June 1	0	$450	
Oct. 12	133	500	$66500
Aug. 16	76	250	19000
		$1200	$85500

$85500 ÷ $1200 = 71 da.

Ans. { 71 da. after June 1, or Aug. 11.

SECOND OPERATION.

Due.	da.	Items.	Prod.
June 1	133	$450	$59850
Oct. 12	0	500	
Aug. 16	57	250	14250
		$1200	$74100

$74100 ÷ $1200 = 62 da.

Ans. { 62 da. before Oct. 12, or Aug. 11.

SOLUTION. — By reference to the example, it will be seen that the items are due June 1, Oct. 12, and Aug. 16, as shown in the two operations. In the first operation we use the *earliest* date, June 1, as a focal date, and find the difference in days between this date and each of the others, regard being taken of the number of days in calendar months. From June 1 to Oct. 12 is 133 da.; from June 1 to Aug. 16 is 76 da. Hence the first item has no credit from June 1, the second item has 133 days' credit from June 1, and the third item has 76 days' credit from June 1, as appears in the column marked da. After this we proceed precisely as in **361**, and find the average credit, 71 da., and the equated time, Aug. 11.

In the second operation, the *latest* date, Oct. 12, is taken for a focal date; the work is explained thus: Suppose the account to be settled Oct. 12. At that time the first item has been due 133 days, and must therefore draw interest for this time. But interest on \$450 for 133 days = the interest on \$59850 for 1 da. The second item draws no interest, because it falls due Oct. 12. The third item must draw interest 57 days. But interest on \$250 for 57 days = the interest on \$14250 for 1 day. Taking the sum of the products, we find the whole amount of interest due on the account, at Oct. 12, equals the interest on \$74100 for 1 day; and this, by division, is found to be equal to the interest on \$1200 for 62 days, which time is the average term of interest. Hence the account would be settled Oct. 12, by paying \$1200 with interest on the same for 62 days. This shows that \$1200 has been due 62 days; that is, it falls due Aug. 11, *without interest.*

RULE. — I. *Find the time at which each item becomes due, by adding to the date of each transaction the term of credit, if any is specified, and write these dates in a column.*

II. *Assume either the earliest or the latest date for a focal date, find the difference in days between the focal date and each of the other dates, and write the results in a second column.*

III. *Write the items of the account in a third column, and multiply each sum by the corresponding number of days in the preceding column, writing the products in a final column.*

IV. *Divide the sum of the products by the sum of the items. The quotient will be the average term of credit when the earliest date is the focal date, or the average term of interest when the latest date is the focal date; in either case always reckon from the focal date toward the other dates, to find the equated time of payment.*

2. JOHN BROWN,

1892.				To JAMES GREIGG,		Dr.
Jan. 1.	To	50	yd.	Broadcloth,	@ $3.00 . . .	$150
" 16.	"	2000	"	Calico,	" .10 . . .	200
Feb. 4.	"	75	"	Carpeting,	" 1.33⅓ . .	100
March 3.	"	400	"	Oil Cloth,	" .40 . . .	160

If James Greigg wishes to settle the above bill by giving his note, from what date should the note draw interest?

3. JAMES RUSSEL,

1892.		To WYNKOOP & BRO., Dr.
March 1.	To Cash	$300
April 4.	" Mdse.	240
June 18.	" " on 2 mo.	100
Aug. 8.	" Cash	400

What is the equated time of payment of the above account? *Ans.* May 26.

4. JOHN OTIS,

1892.				To JAMES LADD,		Dr.
June 1.	To	500	bu.	Wheat,	@ $1.20	$600
" 12.	"	200	"	"	" 1.50	300
" 15.	"	640	"	"	" 1.30	832
" 25.	"	760	"	"	" 1.00	760
" 30.	"	500	"	"	" 1.50	750

What is the equated time of payment of the whole amount of the above bill? *Ans.* June 18.

5. THOMAS WHITING,

1892. To ISRAEL PALMER, Dr.

Jan.	1.	To	60 bbl. Flour,	@ $7.00	$420
"	28.	"	90 bu. Wheat,	" 1.50	135
March	15.	"	300 bbl. Flour,	" 6.00	1800

If credit of 3 months is given to each item, when will the above account become due? *Ans.* May 30.

363. To find the equated time when the terms of credit begin at different times, and the account has both a debit and a credit side.

1. Average the following account:

Dr. DAVID WARE. *Cr.*

1891.					1891.				
June	1	To Mdse. . . .	$400	00	July	4	By Mdse. . .	$200	00
"	16	" Draft, 3 mo..	800	00	Aug.	20	" Cash . . .	150	00
Oct.	20	" Cash . . .	250	00	Sept.	20	" " . . .	500	00

Dr. OPERATION. *Cr.*

	Due.	da.	Items.	Prod.	Due.	da.	Items.	Prod.
Focal date.	June 1	141	$400	$56400	July 4	108	$200	$21600
	Sept. 19	31	800	24800	Aug. 20	61	150	9150
	Oct. 20	0	250		Sept. 20	30	500	15000
			$1450	$81200			$850	$45750
			850	45750				
	Balances. . .		$600	$35450				

$35450 ÷ $600 = 59 da., average term of interest.

Oct. 20 − 59 da. = Aug. 22, balance due.

SOLUTION. — In the operation we have written the dates, showing when the items become due on either side of the account, adding 3 days' grace to the time allowed to the draft. The latest date, Oct. 20, is assumed as the focal date for both sides, and the two columns marked da. show the difference in days between each date and the focal date. The products are obtained as in the last case, and a balance is struck between the items charged and the products. These balances, being on the *Dr.* side, show that David Ware, on the day of the focal date, Oct. 20, owes $600 with in-

terest on $35450 for 1 day. By division, this interest is found to be equal to the interest on $600 for 59 days. The balance, $600, therefore, has been due 59 days. Reckoning back from Oct. 20, we find the date when the balance fell due, Aug. 22.

RULE. — I. *Find the time when each item of the account is due; and write the dates, in two columns, on the sides of the account to which they respectively belong.*

II. *Use either the earliest or the latest of these dates as the focal date for both sides, and find the products as in the last case.*

III. *Divide the balance of the products by the balance of the account; the quotient will be the interval of time, which must be reckoned from the focal date* TOWARD *the other dates when both balances are on the same side of the account, but* FROM *the other dates when the balances are on opposite sides of the account.*

2. What is the balance of the following account, and when is it due?

Dr. JOHN WILSON. *Cr.*

1891.					1891.				
Jan.	1	To Mdse..	$448	00	Jan.	20	By Amt. brought forward.	$560	00
Feb.	4	" Cash.	364	00	Feb.	16	" 1 Carriage	264	00
"	20	" " ..	232	00	"	25	" Cash	900	00

Ans. { Balance, $680. Due March 13.

3. If the following account is settled by giving a note, what will be the face of the note, and what its date?

Dr. ISAAC FOSTER. *Cr.*

1891.						1891.				
Jan.	1	To Mdse. on 3 mo.	$145	86		May	11	By Cash.....	$11	00
"	12	" " " 5 "	37	48		July	12	" "	15	00
June	3	" " " 3 "	12	25		Oct.	12	" "	82	00
Aug.	4	" " " 2 "	66	48						

Ans. { $154.07, face of note. Mar. 26, 1891, date.

PROMISCUOUS EXAMPLES.

1. A merchant bought sugar in New York at $\$.06\frac{1}{4}$ per pound. The wastage by transportation and retailing was 5%, and the interest on the first cost to the time of sale was 2%. How much must he ask per pound to gain 25%? *Ans.* $\$.08\frac{1}{8}+$.

2. A man purchased 2 lots of land for $\$200$ each, and sold one at 40% more than cost, and the other at 20% less. What was his gain? *Ans.* $\$40$.

3. I sold goods on 6 months' credit to the amount of $\$425$, which was $\$25$ more than the goods cost. What was the true profit, money being worth 6%?

Ans. $\$12.62+$.

4. I bought cotton cloth at 13 cents a yard, on 8 months' credit, and sold it the same day at 12 cents *cash*. What per cent did I gain or lose, money being worth 6%?

5. A farmer sold a pair of horses for $\$150$ each; on one he gained 25%, on the other he lost 25%. Did he gain or lose on both, and how much? *Ans.* Lost $\$20$.

6. A man invested $\frac{2}{5}$ of all his fortune in the coal trade, and at the end of 2 years 8 months sold out his entire interest for $\$3100$, which was a yearly gain of 9% on the money invested. How much was he worth when he commenced trade? *Ans.* $\$3750$.

7. In how many years will a man, paying interest at 7% on a debt for land, pay the face of the debt in interest? *Ans.* $14\frac{2}{7}$ years.

8. Two persons engaged in trade: A furnished $\frac{5}{8}$ of the capital, and B $\frac{3}{8}$; and at the end of 3 years 4 months they found they had made a clear profit of $\$5000$, which was $12\frac{1}{2}\%$ per annum on the money invested. How much capital did each furnish? *Ans.* A, $\$7500$; B, $\$4500$.

9. I bought $500 worth of dry goods, and $800 worth of groceries; on the dry goods I lost 20%, but on the groceries I gained 15%. Did I gain or lose on the whole investment, and how much? *Ans.* Gained $20.

10. What amount of accounts must an attorney collect in order to pay over $1100, and retain 8⅓% for collecting? *Ans.* $1200.

11. A merchant sold goods to the amount of $667, to be paid in 8 months; the same goods cost him $600 one year previous to their sale. Money being worth 6%, what was his true gain? *Ans.* $5.346+.

12. A nurseryman sold trees at $18 per hundred, and cleared ⅛ of his receipts. What per cent profit did he make? *Ans.* 50%.

13. If ⅘ of an article is sold for what ⅝ of it cost, what is the gain per cent? *Ans.* 40⅝%.

14. A lumber merchant sells a lot of lumber, which he has had on hand 6 months, on 10 months' credit, at an advance of 30% on the first cost. If he is paying 5% interest on capital, what is the per cent of his profits? *Ans.* 21⅞%.

15. A person, owning ⅝ of a piece of property, sold 20% of his share. What part did he then own?

Ans. ½.

16. A speculator, having money in the bank, drew 60% of it, and expended 30% of 50% of this for 728 bushels of wheat, at $1.12½ per bushel. How much was left in the bank? *Ans.* $3640.

17. A's money is 28% more than B's. How many per cent is B's less than A's? *Ans.* 21⅞%.

18. I pay $12 for an insurance of $800; what is the rate of premium? *Ans.* 1½%.

19. What is the amount of $350.50 for 2 yr. 10 mo., at 7 per cent? *Ans.* $420.01+.

20. On the first day of January, 1892, a man gave 3 notes, the first for $500 payable in 30 days; the second for $400 payable in 60 days; the third for $600 payable in 90 days. What was the average term of credit, and what the equated time of payment?

Ans. Term of credit 62 da.; March 3, 1892.

21. What will be the duty at 15% on 1200 lb. of tapioca, invoiced at 5½ cents per pound? *Ans.* $9.90.

22. How much should be discounted for the present payment of a note for $429.986, due in 1 yr. 6 mo. 1 da., money being worth 5½%? *Ans.* $32.826.

23. A draft on Philadelphia cost £125 in Birmingham, Eng., exchange selling for 4.855; what was the face of the draft? *Ans.* $606.875.

24. In a certain district the school was supported by a rate bill; the teacher's wages amounted to $200, the fuel and other expenses to $75.57; the public money received was $98, and the whole number of days' attendance was 3946. A sent 2 pupils 118 days each. How much was his rate bill? *Ans.* $10.62.

25. A man, receiving a legacy of $48000, invested one half in 5% stock at 95½%, and the other half in 6% stock at 112%, paying brokerage at ½%. What annual income did he secure from his legacy? *Ans.* $2530.

26. What per cent of his money will a man obtain by investing in 6% stock at 108%? *Ans.* 5⅝%.

27. What must be paid in Philadelphia for a draft on St. Paul, drawn at 90 days, for $4800, the course of exchange being 101⅜%? *Ans.* $4791.60.

28. A miller having 720 barrels of flour, sold 288 barrels. What per cent of his stock remained unsold?

Ans. 60.

29. An agent sold my house and lot for $8600. What was his commission at 2¼%? *Ans.* $193.50.

30. A speculator received $3290 as the net proceeds of a sale, after allowing a commission of 6%. What was the value of the property? *Ans.* $3500.

31. A wool grower sold 3150 head of sheep, and had 30% of his original flock left. How many sheep had he at first?

32. A young man having received a fortune, deposited 80% of it in a bank. He afterward drew 20% of his deposit, and then had $5760 in bank. What was his entire fortune?

33. In a certain county, containing 25482 taxable inhabitants, a tax of $103294.60 is assessed for town, county, and state purposes; a part of this sum is raised by a tax of 30 cents on each poll; the entire valuation of property on the assessment roll is $38260000. What per cent is the tax, and how much will a person's tax be who pays for 3 polls, and whose property is valued at $9470? *Ans. to last,* $24.575.

34. A had $6000 in a bank. He drew out 25% of it, then 30% of the remainder, and afterward deposited 10% of what he had drawn. How much had he then in bank? *Ans.* $3435.

35. What sum will produce $12½ interest in 10 days, at 10%? *Ans.* $4500.

36. My agent bought for me 120 shares of N. Y. Central railroad stock, paying 80¾%, and charging brokerage at ¼%. What did the stock cost me? *Ans.* $9750.

37. What will $250 amount to in 3 years at 7% compound interest? *Ans.* $306.26.

38. How many shares of stock must be sold at 4% discount, brokerage ½%, to realize $4775? *Ans.* 50.

39. If a merchant wishes to draw $5000 at a bank, for what sum must he give his note at 90 days, discounting at 6%? *Ans.* $5078.72.

RATIO.

364. Ratio is the relation between two numbers of the same unit value expressed by the quotient of the first divided by the second.

Thus, the ratio of 8 to 4 is $8 \div 4 = 2$; the ratio of 6 to 9, $6 \div 9 = \frac{2}{3}$, etc.

365. The **Sign** of ratio is the colon (:).

Thus, the ratio of 4 to 5 may be expressed 4 : 5.

366. Ratio may also be expressed in the form of a fraction.

Thus, the ratio of 4 to 9 may be expressed by $\frac{4}{9}$, or by $.44\frac{4}{9} = 44\frac{4}{9}\%$; of 9 to 4 by $\frac{9}{4} = 2\frac{1}{4} = 2.25 = 225\%$.

367. The **Terms** of a ratio are the two numbers compared.

368. The **Antecedent** is the first term, or the dividend.

369. The **Consequent** is the second term, or the divisor.

370. The two terms together form a **Couplet.**

371. The **Value** of a ratio is the *quotient* of the antecedent divided by the consequent, and is an abstract number.

Thus, in the ratio \$24 : \$6, \$24 is the antecedent, \$6 is the consequent, and 4, the quotient of \$24 ÷ \$6, is the *value* of the ratio.

372. A **Direct Ratio** arises from dividing the antecedent by the consequent.

373. An **Inverse** or **Reciprocal Ratio** is obtained by dividing the consequent by the antecedent. The reciprocal of a ratio equals 1 divided by the ratio.

When the numerator and denominator of a fraction are interchanged the fraction is said to be inverted; and in the same way, when the antecedent and consequent are interchanged the ratio is called an inverse ratio.

Thus, the *direct* ratio of 5 to 15 is $\frac{5}{15} = \frac{1}{3}$; and the *inverse* ratio of 5 to 15 is $\frac{15}{5} = 3$.

374. A **Simple Ratio** is the ratio between two terms; as, 3 : 12.

375. A **Compound Ratio** is the ratio of the corresponding terms of two or more simple ratios.

Thus, 3 : 9 :: 5 : 15.

376. When the multiplication is performed in a compound ratio, the result is a simple ratio.

Thus, the compound ratio formed from the simple ratios 8 : 4 and 9 : 12 may be expressed:

$$(8 : 4) \times (9 : 12), \text{ or}$$

$$8 \times 9 : 4 \times 12 = 72 : 48, \text{ or}$$

$$\frac{8}{4} \times \frac{9}{12} = \frac{8}{2} = 3 : 2.$$

377. In comparing numbers with each other, they must be of the *same kind*. If the terms of a ratio are denominate numbers, they must be reduced to the same unit value.

378. The ratio of two *fractions* is obtained by dividing the first by the second; or by reducing them to a common denominator, when they will be to each other as their numerators.

Thus, the ratio of $\frac{3}{10} : \frac{2}{5}$ is $\frac{3}{10} \div \frac{2}{5} = \frac{15}{30} = \frac{1}{2}$, which is the same as the ratio of the numerator 3 to the numerator 6 of the equivalent fractions $\frac{3}{10}$ and $\frac{6}{10}$.

Since the antecedent is a dividend and the consequent a divisor, any change in either or both terms will be governed by the general principles of division (**86**). We have only to substitute the terms *antecedent*, *consequent*, and *ratio*, for *dividend*, *divisor*, and *quotient*, and these principles become simple.

PRINCIPLES. — I. *Multiplying the antecedent multiplies the ratio; dividing the antecedent divides the ratio.*

II. *Multiplying the consequent divides the ratio; dividing the consequent multiplies the ratio.*

III. *Multiplying or dividing both antecedent and consequent by the same number does not alter the ratio.*

These principles may be embraced in one law.

GENERAL LAW. — *A change in the* ANTECEDENT *produces a* LIKE *change in the ratio; but a change in the* CONSEQUENT *produces an* OPPOSITE *change in the ratio.*

379. Since the *ratio* of two numbers is equal to the antecedent divided by the consequent, it follows that,

1. The *antecedent* is equal to the consequent multiplied by the ratio; and that,

2. The *consequent* is equal to the antecedent divided by the ratio.

EXAMPLES.

1. What is the ratio of 9 to 3? *Ans.* 3.
2. What is the ratio of 20 to 5? *Ans.* 4.
3. What is the ratio of 36 to 4? *Ans.* 9.
4. What is the ratio of 7 to 49? *Ans.* $\frac{1}{7}$.
5. What is the ratio of 16 to 88? *Ans.* $\frac{2}{11}$.
6. What is the ratio of 6 to $8\frac{1}{2}$? *Ans.* $\frac{12}{17}$.
7. What is the ratio of $6\frac{1}{2}$ to 78? *Ans.* $\frac{1}{12}$.
8. What is the ratio of 16 to 66? *Ans.* $\frac{8}{33}$.

9. What is the ratio of $\frac{3}{4}$ to $\frac{3}{5}$? *Ans.* $1\frac{1}{4}$.

10. What is the ratio of $\frac{5}{8}$ to $\frac{4}{16}$? *Ans.* $2\frac{1}{2}$.

11. What is the ratio of $3\frac{1}{3}$ to $16\frac{2}{3}$? *Ans.* $\frac{1}{5}$.

12. What is the ratio of 3 gal. to 2 qt. 1 pt.? *Ans.* $4\frac{4}{5}$.

13. What is the ratio of 6.3*s.* to 8*s.* 6*d.*? *Ans.* $\frac{63}{85}$.

14. What is the ratio of 5.6 to .56? *Ans.* 10.

15. What is the ratio of 19 lb. 5 oz. 8 pwt. to 25 lb. 11 oz. 4 pwt.? *Ans.* $\frac{3}{4}$.

16. What is the inverse ratio of 12 to 16? *Ans.* $1\frac{1}{3}$.

17. What is the inverse ratio of $\frac{2}{7}$ to $\frac{4}{9}$? *Ans.* $1\frac{5}{8}$.

18. What is the inverse ratio of $5\frac{3}{4}$ to $17\frac{1}{4}$? *Ans.* 3.

19. If the consequent is 16 and the ratio $2\frac{1}{4}$, what is the antecedent? *Ans.* $36\frac{1}{4}$.

20. If the antecedent is 14.5 and the ratio 3, what is the consequent? *Ans.* $4\frac{5}{6}$.

21. If the consequent is $\frac{7}{8}$ and the ratio $\frac{3}{4}$, what is the antecedent? *Ans.* $\frac{21}{32}$.

22. If the antecedent is $\frac{3}{5}$ and the ratio $\frac{1}{6}$, what is the consequent? *Ans.* $3\frac{3}{5}$.

23. What is the ratio of $8\frac{4}{7}$ to 60?

24. What is the ratio of $\frac{7}{18}$ to 26?

25. What is the ratio of $7\frac{1}{8}$ to $2\frac{1}{2}$? *Ans.* $2\frac{17}{20}$.

26. What is the ratio of $\frac{1}{6}$ to $\frac{7}{10}$? *Ans.* $\frac{5}{21}$.

27. Find the reciprocal of the ratio of 42 to 28.

Ans. $\frac{2}{3}$.

28. Find the reciprocal of the ratio of 3 qt. to 43 gal.

29. If the antecedent is 15 and the ratio $\frac{4}{5}$, what is the consequent? *Ans.* $18\frac{3}{4}$.

30. If the consequent is $3\frac{1}{4}$ and the ratio 7, what is the antecedent? *Ans.* $22\frac{3}{4}$.

31. If the antecedent is $\frac{1}{2}$ of $\frac{5}{8}$ and the consequent .75, what is the ratio?

32. If the consequent is \$6.12$\frac{1}{2}$ and the ratio 25, what is the antecedent? *Ans.* \$153.125.

PROPORTION.

380. Proportion is an equality of ratios.

Thus, the ratios 8:4 and 12:6 each being equal to 2, form a proportion.

381. Proportion is indicated in two ways:

1. By a double colon placed between the two ratios.

2. By the sign of equality placed between the two ratios.

Thus, $2:5::4:10$ $2:5=4:10$.

382. Since each ratio consists of two terms, every proportion must consist of at least *four terms*.

Each ratio is called a *couplet* and each term a *proportional*.

383. The **Antecedents** of a proportion are the first and third terms, that is, the antecedents of its ratios. The **Consequents** of a proportion are the second and fourth terms, or the consequents of its ratios.

384. The **Extremes** are the first and fourth terms. The **Means** are the second and third terms.

385. Three numbers may be in proportion when the first is to the second as the second is to the third.

Thus, the numbers 3, 9, and 27 are in proportion since $3:9::9:27$, the ratio of each couplet being $\frac{1}{3}$.

In such a proportion the second term is said to be a *mean proportional* between the other two.

386. In every proportion the product of the extremes is equal to the product of the means.

Thus, in the proportion $3 : 5 :: 6 : 10$ we have $3 \times 10 = 5 \times 6$.

387. Four numbers that are proportional in the direct order are proportional by *inversion*, and also by *alternation*, or by inverting the means.

Thus, the proportion $2 : 3 :: 6 : 9$, by *inversion* becomes $3 : 2 :: 9 : 6$, and by *alternation* $2 : 6 :: 3 : 9$.

388. From the preceding principles and illustrations, it follows that any three terms of a proportion being given, the fourth may readily be found by the following rule:

RULE. — I. *Divide the product of the extremes by one of the means, and the quotient will be the other mean.* Or,

II. *Divide the product of the means by one of the extremes, and the quotient will be the other extreme.*

EXAMPLES.

Find the term not given in each of the following proportions:

1. $48 : 20 :: (\ ) : 50$. *Ans.* 120.
2. $42 : 70 :: 3 : (\ )$.
3. $(\ ) : 300 :: 20 : 100$. *Ans.* 6.
4. $1 : (\ ) :: 7 : 84$. *Ans.* 12.
5. 48 yd. : ( ) :: \$67.25 : \$201.75. *Ans.* 144 yd.
6. 3 lb. 12 oz. : ( ) :: \$3.50 : \$10.50. *Ans.* 11 lb. 4 oz.
7. ( ) : \$38.25 :: 8 bu. 2 pk. : 76 bu. 2 pk. *Ans.* \$4.25.
8. $4\frac{1}{4} : 38\frac{1}{4} :: (\ ) : 76\frac{1}{2}$. *Ans.* $8\frac{1}{2}$.

SIMPLE PROPORTION.

389. Simple Proportion is an equality of two simple ratios, and consists of four terms, any three of which being given, the fourth may readily be found.

When three terms of a proportion are given, the method of finding the fourth is called the *Rule of Three.*

390. Every question in simple proportion involves the principle of *cause and effect.*

391. Causes may be regarded as *action*, of whatever kind, the producer, the consumer, men, animals, time, distance, weight, goods bought or sold, money at interest, etc.

392. Effects may be regarded as whatever is accomplished by action of any kind, the thing produced or consumed, money paid, etc.

393. Causes and effects are of two kinds — simple and compound.

394. A **Simple Cause,** or **Effect,** contains but one element; as goods purchased or sold, and the money paid or received for them.

395. A **Compound Cause,** or **Effect,** is the product of two or more elements; as men at work taken in connection with time, and the result produced by them taken in connection with dimensions, length and breadth, etc.

396. Causes and effects that admit of computation, that is, involve the *idea of quantity*, may be represented by numbers, which will have the same relation to each other as the things they represent. And since it is a principle of philosophy that *like causes*

produce like effects, and that *effects are always in proportion to their causes*, we have the following proportions:

1st Cause : 2d Cause :: 1st Effect : 2d Effect,
Or, 1st Effect : 2d Effect :: 1st Cause : 2d Cause.

The two causes, or effects, forming one couplet, must be *like* numbers, and of the same denomination.

Considering all the terms of the proportion as *abstract numbers*, we may say that

1st Cause : 1st Effect :: 2d Cause : 2d Effect;

which will produce the same numerical result.

But as ratio is the result of comparing two numbers or things of the *same kind* (**377**), the first form is regarded as more natural and philosophical.

397. Simple causes and simple effects give rise to simple ratios; compound causes and compound effects to compound ratios.

EXAMPLES.

398. 1. If 5 tons of coal cost \$ 30, what will 3 tons cost?

The required term will be denoted by a ( ), and designated "blank."

STATEMENT.

tons.		tons.		\$		\$
5	:	3	::	30	:	( )
1st cause.		2d cause.		1st effect.		2d effect.

OPERATION.

$$5 \times ( ) = 3 \times 30.$$

$$( ) = \frac{3 \times 30}{5} = \$18, \textit{Ans.}$$

SOLUTION. — In this example an *effect* is required, and 5 tons must have the same ratio to 3 tons, as \$30, the cost of 5 tons, to (blank) dollars, the cost of 3 tons.

Since the product of the extremes is equal to the product of the means (**386**), and the product of the means divided by one of the extremes will give the other, (blank) dollars will be equal to the product of 3 × 30 divided by 5, which is \$18.

2. If 15 barrels of flour cost $ 90, how many barrels can be bought for $ 30?

STATEMENT.

bar.	bar.	$	$
15	: ( )	:: 90	: 30
1st cause.	2d cause.	1st effect.	2d effect.

OPERATION.

$$\begin{array}{r|l} \not\$ & 5 \\ \not 9\not 0 & \not 3\not 0 \\ (\ ) & \not 15^{5} \end{array}$$

Or, $\frac{\not 15 \times \not 30}{\not 90} = 5$, *Ans.*

( ) = 5 bar., *Ans.* $\not\$$

SOLUTION. — In this example a *cause* is required, and the statement may be read thus: If 15 barrels cost $ 90, how many or (blank) barrels will cost $ 30? The product of the extremes, 30 × 15, divided by the given mean, 90, will give the required term, 5, as shown in the operation.

RULE. — I. *Arrange the terms in the statement so that the causes shall compose one couplet, and the effects the other, putting ( ) in the place of the required term.*

II. *If the required term is an extreme, divide the product of the means by the given extreme; if the required term is a mean, divide the product of the extremes by the given mean.*

1. If the terms of any couplet are of different denominations, they must be reduced to the same unit value.

2. If the odd term is a compound number, it must be reduced to its lowest unit.

3. If the divisor and dividend contain one or more factors common to both, they should be canceled. If any of the terms of a proportion contain mixed numbers, they should first be changed to improper fractions, or the fractional part to a decimal.

4. When the vertical line is used, the divisor and the required term are written on the left, and the terms of the dividend on the right.

399. There is another method of solving questions in simple proportion, without making the statement, which may be used, by those who prefer it to the one already given.

SECOND METHOD.

Every question which properly belongs to simple proportion must contain *four* numbers, at least *three of which* must be given (**389**).

Of the three given numbers, one must always be of the same denomination as the required number.

The remaining two will be *like* numbers, and bear the same relation to each other that the third does to the required number; in other words, the ratio of the third to the required number will be the same as the ratio of the other two numbers.

Regarding the third or odd term as the antecedent of the second couplet of a proportion, we find the consequent or required term by dividing the antecedent by the ratio (**379**).

By comparing the two *like* numbers, in any given question, with the third, we may readily determine whether the answer, or required term, will be *greater* or *less* than the third term.

If *greater*, then the ratio will be *less* than 1, and the two *like* numbers may be arranged in the form of a proper fraction as a divisor; if the answer, or required term, is to be *less* than the third term, then the ratio will be *greater* than 1, and the two *like* numbers may be arranged in the form of an improper fraction, as a divisor.

1. If 4 cords of wood cost \$12, what will 20 cords cost?

OPERATION.

$$4 : 20 :: 12 : (\ ).$$

$$12 \div \frac{4}{20} = \frac{\overset{3}{\cancel{12}} \times 20}{\cancel{4}} = \$60.$$

$$\text{Or,}\quad \frac{20 \times \overset{3}{\cancel{12}}}{\cancel{4}} = \$60.$$

SOLUTION.—It will be readily seen in this example, that 4 cords and 20 cords are the *like* terms, and that \$12 is the third term, and of the same denomination as the answer or required term.

If 4 cords cost \$12, will 20 cords cost more, or less, than 4 cords? Evidently more: then the answer or required term will be *greater* than the third term, and the ratio *less* than 1. The ratio of 4 cords to 20 cords is $\frac{4}{20}$, or $\frac{1}{5}$; hence the

ratio of $12 to the answer must be $\frac{1}{3}$, and the answer will be 5 times $12, which is $60.

Or, after having stated the proportion 4 : 20 :: 12 : ( ), we find the product of the means and divide by the extreme.

2. If 12 yards of cloth cost $48, what will 4 yards cost?

OPERATION.

12 : 4 : : 48 : ( ).

$$48 \div \frac{12}{4} = \overset{4}{\cancel{48}} \times \frac{4}{\cancel{12}} = \$16.$$

Or,

$$\frac{4 \times \overset{4}{\cancel{48}}}{\cancel{12}} = \$16.$$

SOLUTION. — In this example we see that 12 yards and 4 yards are the *like* terms, and $48 the third term, and of the same denomination as the required answer.

If 12 yards cost $48, will 4 yards cost more or less than 12 yards? Less: then the ratio will be *greater* than 1, and the divisor an improper fraction. The ratio of 12 yards to 4 yards is 3, hence the ratio of $48 to the answer is 3, and the answer will be $\frac{1}{3}$ of $48, which is $16. Or, dividing the product of the means by the extreme, we have $4 \times 48 \div 12 = \$16$.

RULE. — I. *With the two given numbers, which are of the same name or kind, form a ratio greater or less than* 1, *according as the answer is to be less or greater than the third given number.*

II. *Divide the third number by this ratio, and the quotient will be the required number or answer.*

1. Mixed numbers should first be reduced to improper fractions, and the ratio of the fractions found according to (378).
2. Reductions and cancellation may be applied as in the first method.

The examples may be solved by either method.

3. If 48 cords of wood cost $120, what will 20 cords cost? *Ans.* $50.

4. If 6 bushels of corn cost $4.75, what will 75 bushels cost? *Ans.* 59.37\frac{1}{2}$.

5. If 12 horses consume 42 bushels of oats in 3 weeks, how many bushels will 20 horses consume in the same time?

6. If 8 yards of cloth cost $\$3\frac{1}{2}$, how many yards can be bought for $\$50$? *Ans.* $114\frac{2}{7}$ yd.

7. If 7 pounds of sugar cost 75 cents, how many pounds can be bought for $\$9$? *Ans.* 84 lb.

8. What will 11 lb. 4 oz. of tea cost, if 3 lb. 12 oz. cost $\$3.50$? *Ans.* $\$10.50$.

9. If a staff 3 ft. 8 in. long casts a shadow 1 ft. 6 in., what is the height of a steeple that casts a shadow 75 feet at the same time? *Ans.* 183 ft. 4 in.

10. At $\$2.75$ for 14 pounds of sugar, what will be the cost of 100 pounds? *Ans.* $\$19.64\frac{2}{7}$.

11. How many bushels of wheat can be bought for $\$51.06$, if 12 bushels can be bought for $\$13.32$?

12. What will be the cost of $28\frac{1}{2}$ gallons of molasses, if 15 hogsheads cost $\$236.25$? *Ans.* $\$7.12\frac{1}{2}$.

13. If 7 barrels of flour are sufficient for a family 6 months, how many barrels will they require for 11 months?

14. At the rate of 9 yards for £5 12*s.*, how many yards of cloth can be bought for £44 16*s.*? *Ans.* 72 yd.

15. An insolvent debtor fails for $\$7560$, of which he is able to pay only $\$3100$; how much will A receive, whose claim is $\$756$? *Ans.* $\$310$.

16. If 2 pounds of sugar cost 25 cents, and 8 pounds of sugar are worth 5 pounds of coffee, what will 100 pounds of coffee cost? *Ans.* $\$20$.

17. If the moon moves 13° 10′ 35″ in 1 day, in what time will it perform one revolution?

18. If $8\frac{3}{4}$ bushels of corn cost $\$4.20$, what will be the cost of $13\frac{1}{2}$ bushels at the same rate? *Ans.* $\$6.48$.

19. If $1\frac{3}{4}$ yards of cloth cost $6\frac{1}{4}$ pence, how many yards can be bought for £10 6*s.* 8*d.*? *Ans.* $694\frac{4}{5}$ yd.

20. If $12\frac{1}{2}$ cwt. of iron cost $\$42\frac{1}{4}$, what will $48\frac{3}{8}$ cwt. cost?

21. What quantity of tobacco can be bought for $317.23, if $8\frac{3}{5}$ lb. cost $\$1\frac{3}{4}$? *Ans.* 15 cwt. 22.7+ lb.

22. If $15\frac{5}{8}$ bushels of clover seed cost $\$156\frac{1}{4}$, how much can be bought for $\$95.75$? *Ans.* 9 bu. 2 pk. $2\frac{3}{5}$ qt.

23. If $\frac{5}{6}$ of a barrel of oil costs $\$\frac{5}{77}$, how much will $\frac{7}{8}$ of a barrel cost? *Ans.* $\$\frac{2}{11}$.

24. If a piece of land of a certain length and 4 rods in breadth contains $\frac{3}{4}$ of an acre, how much would there be if it were $11\frac{3}{5}$ rods wide? *Ans.* 2 A. 28 sq. rods.

25. If 13 cwt. of iron cost $\$42\frac{1}{4}$, what will 12 cwt. cost?

26. A grocer has a false balance, by which 1 pound will weigh but 12 oz. What is the real value of a barrel of sugar that sells for $\$28$? *Ans.* $\$21$.

27. A butcher in selling meat sells $14\frac{11}{16}$ oz. for a pound. Of how much does he cheat a customer who buys of him to the amount of $\$30$? *Ans.* $\$2.46+$.

28. If a man clears $\$750$ by his business in 1 yr. 6 mo., how much would he gain in 3 yr. 9 mo. at the same rate?

29. If a certain business yields $\$350$ net profits in 10 mo., in what time would the same business yield $\$1050$ profits?

30. B and C have each a farm; B's farm is worth $\$25$ an acre, and C's $\$30\frac{1}{2}$. If, in trading, B values his land at $\$28$ an acre, what value should C put upon his?

Ans. $\$34.16$.

31. If I borrow $\$500$, and keep it 1 yr. 4 mo., for how long a time should I lend $\$240$ as an equivalent for the favor? *Ans.* 2 yr. 9 mo. 10 da.

32. How many days will 12 men require to do a piece of work that 95 men can do in $7\frac{1}{2}$ days?

33. When 10 barrels of flour cost $\$112.50$, what will be the cost of 476 barrels?

COMPOUND PROPORTION.

400. Compound Proportion embraces that class of questions in which the causes, or the effects, or both, are compound.

The required term may be a cause, or a single element of a cause; or it may be an effect, or a single element of an effect.

EXAMPLES.

1. If 16 horses consume 128 bu. of oats in 50 days, how many bushels will 5 horses consume in 90 days?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
$\left\{\begin{matrix}16\\50\end{matrix}\right.$	:	$\left\{\begin{matrix}5\\90\end{matrix}\right.$	::	128	:	( )

Or, $16 \times 50 : 5 \times 90 :: 128 : ( )$

OPERATION.

$$\frac{\cancel{5} \times \cancel{90}^{9} \times \cancel{128}^{8}}{\cancel{16} \times \cancel{50}_{\cancel{5}}} = 72 \text{ bu.}$$

SOLUTION. — In this example the required term is the second effect; and the question may be read: If 16 horses in 50 days consume 128 bushels of oats, 5 horses in 90 days will consume how many, or (blank) bushels? Dividing the product of the means by the extremes, we have 72 bu.

These examples are most readily performed by cancellation.

2. If $480 gains $84 interest in 30 months, what sum will gain $21 in 15 months?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
$\left\{\begin{matrix}480\\30\end{matrix}\right.$	:	$\left\{\begin{matrix}(\ )\\15\end{matrix}\right.$	::	84	:	21

OPERATION.

$$\frac{\cancel{480}^{120} \times \cancel{30}^{2} \times \cancel{21}}{\cancel{84} \times \cancel{15}_{\cancel{4}}} = \$240, \textit{ Ans.}$$

SOLUTION. — The required term in this example is an element of the second cause; and the question may be read, If $480 in 30 months gains $84, what principal in 15 months will gain $21? Dividing the product of the extremes by the means, we have $240.

3. If 7 men dig a ditch 60 feet long, 8 feet wide, and 6 feet deep, in 12 days, what length of ditch can 21 men dig in $2\frac{2}{3}$ days, if it is 3 feet wide and 8 feet deep?

STATEMENT.

$$\left\{\begin{matrix}7\\12\end{matrix}\right. : \left\{\begin{matrix}21\\2\frac{2}{3}\end{matrix}\right. :: \left\{\begin{matrix}60\\8\\6\end{matrix}\right. : \left\{\begin{matrix}(\ )\\3\\8\end{matrix}\right.$$

Or, $7 \times 12 : 21 \times \frac{8}{3} :: 60 \times 8 \times 6 : (\ ) \times 3 \times 8.$

OPERATION.

$$\frac{\cancel{7} \quad 5 \quad 2}{\cancel{21} \times \cancel{8} \times \cancel{60} \times \cancel{8} \times \cancel{6}} = 80 \text{ ft., } \textit{Ans.}$$
$$\cancel{7} \times \cancel{12} \times \cancel{3} \times \cancel{8} \times \cancel{3}$$

SOLUTION. — In this example the required term is the *length* of the ditch, and is an element of the second effect. The question, as stated, will read thus: If 7 men, in 12 days, dig a ditch 50 feet long, 8 feet wide, and 6 feet deep, 21 men, in $2\frac{2}{3}$ days, will dig a ditch how many, or (blank) feet long, 3 feet wide, and 8 feet deep? Dividing the product of the means by the extreme, we have 80 ft.

RULE. — I. *Of the given terms, select those which constitute the causes, and those which constitute the effects, and arrange them in couplets, putting* ( ) *in place of the required term.*

II. *Then, if the blank term* ( ) *occurs in either of the extremes, make the product of the means a dividend, and the product of the extremes a divisor; but if the blank term occurs in either mean, make the product of the extremes a dividend, and the product of the means a divisor.*

The causes must be exactly alike in the *number* and *kind* of their terms; the same is true of the effects. The same preparation of the terms by reduction is to be observed as in simple proportion.

4. If $320 will pay the board of 4 persons for 8 weeks, for how many weeks will $800 pay the board of 15 persons?

5. If a man walks 192 miles in 6 days, walking 8 h. a day, how far can he walk in 18 days, walking 6 h. a day?

Examples in Compound Proportion may also be solved by the Second Method in Simple Proportion (**399**).

6. If 18 men can build 42 rods of wall in 16 days, how many men can build 28 rods in 8 days?

OPERATION.

$$\left.\begin{matrix} 42 : 28 \\ 8 : 16 \end{matrix}\right\} :: 18 : (\ ).$$

$$18 \div \frac{42}{28} \times \frac{8}{16} = \overset{6}{\cancel{18}} \times \frac{\overset{2}{\cancel{28}}}{\underset{14}{\cancel{42}}} \times \frac{\overset{2}{\cancel{16}}}{\cancel{8}} = 24 \text{ men.}$$

Or,

$$\frac{\overset{2}{\cancel{28}} \times \overset{2}{\cancel{16}} \times \overset{6}{\cancel{18}}}{\underset{14}{\cancel{42}} \times \cancel{8}} = 24 \text{ men.}$$

SOLUTION. — In this example all the terms appear in couplets, except one, which is 18 men, and that is of the same kind as the required answer.

Since compound proportion is made up of two or more simple proportions, if this third or odd term is divided by the compound ratio, or by the simple ratio of each couplet successively, the product will be the required term.

By comparing the terms of each couplet with the third term we may readily determine whether the answer, or term sought, will be greater or less than the third term; if *greater*, then the ratio will be *less* than 1, and the divisor a proper fraction; if *less*, the ratio will be *more* than 1, and the divisor an improper fraction.

First we will compare the terms composing the first couplet, 42 rods and 28 rods, with the third term, 18 men. If 42 rods require 18 men, how many men will 28 rods require? Less men: hence the ratio is *greater* than 1, and the divisor an improper fraction, $\frac{42}{28}$. Next, if 16 days require 18 men, how many men will 8 days require? More men: hence the ratio is *less* than 1, and the divisor a proper fraction, $\frac{8}{16}$. Regarding the third term as the antecedent of a couplet, the consequent being the term sought, if we divide this third term by the simple ratios, or by their product, we shall have

the required term or answer, thus: $18 \div \frac{42}{28} \times \frac{8}{16} = 24$, as shown in the operation.

Or, finding the product of the means and dividing by the extremes, we have $\frac{28 \times 16 \times 18}{42 \times 8} = 24$.

7. If 5 compositors, in 16 days, of 14 hours each, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters in a line, in how many days, of 7 hours each, will 10 compositors compose a volume to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

OPERATION.

Days. Comp. Hours. Sheets. Pages. Lines. Letters.

$$16 \div \frac{10}{5} \times \frac{7}{14} \times \frac{20}{40} \times \frac{24}{16} \times \frac{50}{60} \times \frac{40}{50} = 32 \text{ days.}$$

BY CANCELLATION.

	16
$^{2}\not{10}$	$\not{5}$
$\not{7}$	$\not{14}^{2}$
$\not{20}$	$\not{40}$
$^{3}\not{24}$	$\not{16}^{2}$
$\not{50}$	$\not{60}^{3}$
$\not{40}$	$\not{50}$

32 days, *Ans.*

SOLUTION.—The required term or answer is to be in *days;* and we see that all the terms appear in pairs or couplets, except the 16 days, which is of the same kind as the answer sought.

We will proceed to compare the terms of each couplet with the 16 days. First, if 5 compositors require 16 days, how many days will 10 compositors require? Less days; hence, the divisor is the improper fraction $\frac{10}{5}$, and we have $16 \div \frac{10}{5}$. Next, if 14 hours a day require 16 days, how many days will 7 hours a day require? More days; hence the divisor is the proper fraction $\frac{7}{14}$, and we have $16 \div \frac{10}{5} \times \frac{7}{14}$. Next, if 20 sheets require 16 days, how many days will 40 sheets require? more days; hence the divisor is the proper fraction $\frac{20}{40}$, and we have $16 \div \frac{10}{5} \times \frac{7}{14} \times \frac{20}{40}$. Pursuing the same method with the other couplets, we obtain the result as shown in the operation.

RULE.—I. *Of the terms composing each couplet form a ratio greater or less than* 1, *in the same manner as if the answer depended on those two and the third or odd term.*

II. *Divide the third or odd term by these ratios successively and the quotient will be the answer sought.*

By the *odd* term is meant the one that is of the same kind as the answer.

8. If 16 horses consume 128 bushels of oats in 50 days, how many bushels will 5 horses consume in 90 days?

9. If a man travels 120 miles in 3 days when the days are 12 hours long, how many days of 10 hours each will he require to travel 360 miles? *Ans.* $10\frac{4}{5}$ days.

10. If 6 laborers dig a ditch 34 yards long in 10 days, how many yards can 20 laborers dig in 15 days?

Ans. 170 yd.

11. If 450 tiles, each 12 inches square, will pave a cellar, how many tiles that are 9 inches long and 8 inches wide will pave the same? *Ans.* 900.

12. If it requires 1200 yards of cloth $\frac{5}{4}$ yd. wide to clothe 500 men, how many yards which are $\frac{7}{8}$ yd. wide will it take to clothe 960 men? *Ans.* $3291\frac{3}{7}$ yd.

13. If 8 men mow 36 acres of grass in 9 days, of 9 hours each, how many men will be required to mow 48 acres in 12 days, working 12 hours each day?

Ans. 6 men.

14. If 4 men, in $2\frac{1}{2}$ days, mow $6\frac{2}{3}$ acres of grass by working $8\frac{1}{4}$ hours a day, how many acres will 15 men mow in $3\frac{3}{4}$ days by working 9 hours a day?

Ans. $40\frac{10}{11}$ acres.

15. If, by traveling 6 hours a day at the rate of $4\frac{1}{2}$ miles an hour, a man performs a journey of 540 miles in 20 days, in how many days, traveling 9 hours a day at the rate of $4\frac{2}{3}$ miles an hour, will he travel 600 miles?

Ans. $14\frac{4}{7}$ days.

16. If $2\frac{1}{2}$ yards of cloth, $1\frac{2}{5}$ yards wide, cost $\$3.37\frac{1}{2}$, what will be the cost of $36\frac{1}{2}$ yards, $1\frac{1}{2}$ yards wide?

Ans. \$ 52.79 +.

17. If 5 men reap 52.2 acres in 6 days, how many men will reap 417.6 acres in 12 days? *Ans.* 20 men

PARTITIVE PROPORTION.

401. Partitive Proportion is the process by which a number is divided into proportional parts.

EXAMPLES.

1. Divide $1305 into parts proportional to the numbers 2, 3, and 4.

$2 + 3 + 4 = 9$
$\frac{2}{9}$ of $1305 = $290
$\frac{3}{9}$ of $1305 = $435
$\frac{4}{9}$ of $1305 = $580

SOLUTION. — The problem is to take three fractions of $1305 which shall be in the proportion of 2, 3, 4. Any fractions with a common denominator, and with 2, 3, and 4 as respective numerators, would be in the proper proportion; but since the sum of the three parts of $1305, when found, must equal the *whole* of $1305, the sum of the three fractions must equal a *whole unit*, and since the sum of the numerators is $2 + 3 + 4 = 9$, the common denominator must be 9, since $\frac{9}{9} = 1$. Therefore the fractions must be $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$. $\frac{2}{9}$ of $1305 = $290; $\frac{3}{9}$ of $1305 = $435; $\frac{4}{9}$ of $1305 = $580.

2. Divide 5200 into parts proportional to $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{6}$.

$\frac{1}{2} = \frac{15}{30}$; $\frac{1}{5} = \frac{6}{30}$; $\frac{1}{6} = \frac{5}{30}$
$15 + 6 + 5 = 26$
$\frac{15}{26}$ of $5200 = 3000$
$\frac{6}{26}$ of $5200 = 1200$ } *Ans.*
$\frac{5}{26}$ of $5200 = 1000$

SOLUTION. — When fractions have a common denominator, they have the ratios of their numerators (**378**). Reducing $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{6}$ to common denominators, we have $\frac{15}{30}$, $\frac{6}{30}$, and $\frac{5}{30}$. The problem now is to divide 5200 into parts proportional to 15, 6, 5. Proceeding, as in Example 1, we have $15 + 6 + 5 = 26$. $\frac{15}{26}$ of $5200 = 3000$; $\frac{6}{26}$ of $5200 = 1200$; $\frac{5}{26}$ of $5200 = 1000$.

RULE. — I. *Find the sum of the numbers in proportion for a common denominator. Form fractions with this sum as denominator, and the proportional numbers as respective numerators. Take these fractional parts of the number given.*

II. *When the numbers in proportion are fractions, reduce them to a common denominator. The numerators*

may then be regarded as whole numbers in the given proportion, with which proceed as before.

3. Divide 105 into parts proportional to 5, 7, 9.

Ans. 25, 35, 45.

4. Divide 126 into six parts proportional to 1, 2, 3, 4, 5, 6. *Ans.* 6, 12, 18, 24, 30, 36.

5. Divide 666 into three parts proportional to $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$. *Ans.* 333, 222, 111.

6. Three men A, B, and C, enter into partnership. For every \$3 of capital A puts in, B puts in \$4, and C \$1. Their whole capital amounts to \$20480. How much money does each put in?

Ans. A, \$7680; B, \$10240; C, \$2560.

7. A man bequeathed \$63000 worth of property to three heirs as follows: To his son $\frac{1}{3}$ as much as to his wife, and to his daughter $\frac{1}{2}$ as much as to his son. How much did each receive?

Ans. Wife, \$42000; Son, \$14000; Dau., \$7000.

8. A man bought three houses for \$112500. The first cost twice as much as the second, and the third three times as much as the first. How much did he pay for each? *Ans.* 1st, \$25000; 2d, \$12500; 3d, \$75000.

9. Three men, A, B, and C, owned a vessel worth \$52000. B owned $\frac{2}{3}$ as much as A, and C $\frac{1}{2}$ as much as A. How much did each own?

Ans. A, \$24000; B, \$16000; C, \$12000.

10. A farmer bought a farm consisting of house, barn, implements, horses, and cows for \$5300. The cows cost $\frac{1}{20}$ as much as the barn; the horses $\frac{1}{10}$ as much as the barn; the implements $\frac{1}{4}$ as much; and the house $1\frac{1}{4}$ times as much. How much did he pay for each?

11. Divide 115200 into parts proportional to 6, 12, 18, 24, 30.

PARTNERSHIP.

402. **Partnership** is the association of two or more persons under a certain name, for the transaction of business.

403. The **Partners** are the individuals associated.

404. A **Firm, Company,** or **House** is any particular partnership association.

405. **Capital,** or **Stock,** is the money or property invested by the partners, called also **Investment,** or **Joint Stock.**

406. The **Resources** of a firm are the amounts due the firm, together with the property of all kinds belonging to it, called also **Assets,** or **Effects.**

407. The **Liabilities** of a firm are its debts.

The *Net Capital* is the excess of resources over liabilities.

408. A **Dividend** is the profit to be divided. An **Assessment** is a tax to meet losses sustained.

409. **Gains** and **Losses** are shared in proportion to the sums invested, and the periods of investment.

410. Partnership may be **simple** or **compound.** In *simple partnership* the capital of each partner is invested for the same time. In *compound partnership* the time for which the capital of each partner is invested must be taken into account.

SIMPLE PARTNERSHIP.

EXAMPLES.

411. To find each partner's share of the profit or loss, when the capital of each is employed for equal periods of time.

1. A and B engage in trade; A furnishes $300, and B $400 of the capital; they gain $182. What is each one's share of the profit?

OPERATION.

$300 + $400 = $700, whole stock.
$\frac{300}{700} = \frac{3}{7}$, A's share of stock.
$\frac{400}{700} = \frac{4}{7}$, B's share of stock.
$182 × $\frac{3}{7}$ = $78, A's share of gain.
$182 × $\frac{4}{7}$ = $104, B's share of gain.

SOLUTION. — Since the whole capital employed is $300 + $400 = $700, it is evident that A furnishes $\frac{300}{700} = \frac{3}{7}$ of the capital, and B $\frac{400}{700} = \frac{4}{7}$ of the capital. And since each man's share of the profit or loss will have the same ratio to the whole profit or loss that his share of the stock has to the whole stock, A will have $\frac{3}{7}$ of the entire profit, and B $\frac{4}{7}$, as shown in the operation.

We may also regard the whole capital as the *first cause*, and each man's share of it as the *second cause*, the whole gain or loss as the *first effect*, and each man's share of it as the *second effect*, and solve by proportion, thus:

1st cause. 2d cause. 1st effect. 2d effect.
$700 : $300 :: $182 : ( )
$700 : $400 :: $182 : ( )

7 | 700 | 300³ | 182²⁶ | ( ) = $78, A's profit.

7 | 700 | 400⁴ | 182²⁶ | ( ) = $104, B's profit.

RULE. — *Multiply the whole profit or loss by the ratio of each man's share of the capital to the whole capital.* Or,

The whole capital is to each man's share of the capital as the whole profit or loss is to each man's share of the profit or loss.

2. Three men trade in company; A furnishes $8000, B $12000, and C $20000 of the capital; their gain is $1680. What is each man's share?

Ans. A's, $336; B's, $504; C's, $840.

3. Three persons purchased a house for $2800, of which A paid $1200, B $1000, and C $600; they rented it for $224 a year. How much of the rent should each receive?

4. A man failed in business for $20000, and his available means amounted to only $13654. How much will two of his creditors respectively receive, to one of whom he owes $3060, and to the other $1530?

Ans. $2089.062; $1044.531.

5. Four men hired a coach for $13, to convey them to their respective homes, which were at distances from the place of starting as follows: A's 16 miles, B's 24 miles, C's 28 miles, and D's 36 miles. What ought each to pay?

Ans. { A, $2; C, $3.50; B, $3; D, $4.50.

6. A captain, mate, and 12 sailors took a prize of $2240, of which the captain took 14 shares, the mate 6 shares, and each sailor 1 share. What did each receive?

7. A cargo of corn, valued at $3475.60 was entirely lost; $\frac{1}{8}$ of it belonged to A, $\frac{1}{4}$ of it to B, and the remainder to C. How much was the loss of each, there being an insurance of $2512?

Ans. $120.45, A's; $240.90, B's; $602.25, C's.

8. Three persons engaged in the lumber trade; two of the persons furnished the capital, and the third managed the business; they gained $2571.24, of which C received $6 as often as D $4, and E had $\frac{1}{3}$ as much as the other two for taking care of the business. How much was each one's share of the gain?

Ans. $1285.62, C's; $857.08, D's; $428.54, E's.

COMPOUND PARTNERSHIP.

EXAMPLES.

412. To find each partner's share of the profit or loss when the capital of each is employed for unequal periods of time.

It is evident that the respective shares of profit and loss will depend upon two conditions, viz.: *the amount of capital* invested by each, and the *time* it is employed.

1. Two persons form a partnership; A puts in $450 for 7 months, and B $300 for 9 months; they lose $156. How much is each man's share of the loss?

OPERATION.

$450 × 7 = $3150, A's capital for 1 mo.
$300 × 9 = $2700, B's " " "
$5850, entire " " "

$\frac{3150}{5850} = \frac{7}{13}$, A's share of the entire capital.
$\frac{2700}{5850} = \frac{6}{13}$, B's " " " "
$156 × $\frac{7}{13}$ = $84, A's loss.
$156 × $\frac{6}{13}$ = $72, B's "

SOLUTION. — The use of $450 capital for 7 months is the same as the use of 7 times $450, or $3150 for 1 month; and of $300 for 9 months the same as of 9 times $300, or $2700 for 1 month. The entire capital for 1 month is equivalent to $3150 + $2700 = $5850. If the loss, $156, is divided between the two partners, as in **411**, the results will be the loss of each as shown in the operation. Examples of this kind may also be solved by proportion, the *causes* being compounded of *capital* and *time;* thus,

$5850 : $3150 :: $156 : ( )
$5850 : $2700 :: $156 : ( )

450

5850 | 3150⁷
( ) | 156¹²
( ) = $84, A's loss.

450

5850 | 2700⁶
( ) | 156¹²
( ) = $72, B's loss.

RULE. — *Multiply each man's capital by the time it is employed in trade, and add the products. Then multiply the entire profit or loss by the ratio of each product to the sum of the products, and the results will be the respective shares of profit or loss of each partner.* Or,

Multiply each man's capital by the time it is employed in trade, and regard each product as his capital, and the sum of the products as the entire capital, and solve by proportion.

2. Three persons traded together; B put in $250 for 6 months, C $275 for 8 months, and D $450 for 4 months; they gained $825. What was each man's share of the gain?

3. Two merchants formed a partnership for 18 months. A at first put in $1000, and at the end of 8 months he put in $600 more; B at first put in $1500, but at the end of 4 months he drew out $300; at the expiration of the time they found that they had gained $1394.64. What was each man's share of the gain?

Ans. A's, $715.20; B's, $679.44.

4. Three men took a field of grain to harvest and thrash for $\frac{1}{4}$ of the crop; A furnished 4 hands 5 days, B 3 hands 6 days, and C 6 hands 4 days; the whole crop amounted to 372 bushels. What was each man's share?

5. William Gallup began trade January 1, 1890, with a capital of $3000, and, succeeding in business, took in M. H. Decker as a partner on the first day of the following March, with a capital of $2000; four months later, they admitted J. Newman as third partner, who put in $1800 capital; they continued their partnership until April 1, 1892, when they found that $4388.80 had been gained since January 1, 1890. What was each one's share? *Ans.* G.'s, $2106; D.'s, $1300; N.'s, $982.80.

AVERAGE.

413. Average is the method of finding the *mean* or *average* price of a mixture of two or more ingredients of different values or qualities.

An average or mean of any number, is a sum or quantity intermediate to a number of different sums or quantities, obtained by adding them together and dividing by the number of quantities added.

414. The **Mean Price** or **Quality** is the average price or quality of the ingredients, or the price or quality of a unit of the mixture.

415. Averaging consists of two distinct processes, as follows:

1. The process of finding the mean or average value of two or more things of different given values.
2. The process of finding the proportional quantities of articles at given prices or values, to be used to make a combination of a given average value.*

416. The **Simples** are the unmixed ingredients.

EXAMPLES.

417. To find the average value of things of different given values.

1. A miller mixes 40 bushels of rye worth 80 cents a bushel, and 25 bushels of corn worth 70 cents a bushel, with 15 bushels of wheat worth $1.50 a bushel. What is the value of a bushel of the mixture?

*This process will be explained in the Higher Arithmetic under "Alligation Alternate."

OPERATION.

$.80 × 40 = $32.00
$.70 × 25 = 17.50
$1.50 × 15 = 22.50
80) $72.00
$.90, *Ans.*

SOLUTION. — Since 40 bushels of rye at 80 cents a bushel are worth $32, and 25 bushels of corn at 70 cents a bushel are worth $17.50, and 15 bushels of wheat at $1.50 a bushel are worth $22.50, therefore the entire mixture, consisting of 80 bushels, is worth $72, and 1 bushel is worth $\frac{1}{80}$ of $72, or $72 ÷ 80 = $.90, *Ans.*

RULE. — *Divide the entire cost or value of the ingredients by the sum of the simples.*

2. A student received the following returns from an examination: arithmetic 98%; history 95%; geography 100%; grammar 100%; drawing 82%. What was his average per cent?

SOLUTION. — 98% + 95% + 100% + 100% + 82% = 475% ÷ 5 = 95%, *Ans.*

3. If a grocer mixes 8 lb. of tea worth $.60 a lb. with 6 lb. at $.70 a lb., 2 lb. at $1.10, and 4 lb. at $1.20, what is 1 lb. of the mixture worth? *Ans.* $.80.

4. A grocer mixed 10 pounds of sugar at 4 cents with 12 pounds at 4½ cents and 16 pounds at 5½ cents, and sold the mixture at 6 cents per pound. Did he gain or lose by the sale, and how much? *Ans.* He gained 46 cents.

5. On a certain day the thermometer registered the following averages: from 6 to 9 A.M., 64°; from 9 to 12, 74°; from 12 to 3 P.M., 84°; and from 3 to 6, 70°. What was the mean temperature of the day?

6. A drover bought 84 sheep at $5 a head, 86 at $4.75, and 130 at $5.50. At what average price per head must he sell them to gain 20%?

7. How fine is a mixture of 5 pwt. of gold 16 carats fine, 2 pwt. 18 carats fine, 6 pwt. 20 carats fine, and 1 pwt. pure gold (24 carats)? *Ans.* 18$\frac{4}{7}$ carats fine.

INVOLUTION.

418. A **Power** is the product arising from multiplying a number by itself, or repeating it several times as a factor.

Thus, in $2 \times 2 \times 2 = 8$, the product, 8, is a power of 2.

419. The **Exponent** of a power is the number denoting how many times the factor is used to produce the power, and is written above and a little to the right of the factor.

Thus, $2 \times 2 \times 2$ is written 2^3, in which 3 is the exponent.

Exponents likewise give names to the powers, as will be seen in the following illustrations:

$3 = 3^1 = 3$, the first power of 3;
$3 \times 3 = 3^2 = 9$, the second power of 3;
$3 \times 3 \times 3 = 3^3 = 27$, the third power of 3.

The sum of the exponents of two powers of the same number is equal to the exponent of the product of those powers. Thus, $2^2 \times 2^3 = 2^5$; for $2^2 = 2 \times 2$, and $2^3 = 2 \times 2 \times 2$; hence, $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

420. The **Square** of a number is its second power. The **Cube** of a number is its third power.

421. **Involution** is the process of raising a number to a given power.

A **Perfect Power** is a number that can be exactly produced by the involution of some number as a root.

Thus, 25 and 32 are perfect powers, since $25 = 5 \times 5$, and $32 = 2 \times 2 \times 2 \times 2 \times 2$.

EXAMPLES.

422. To find any power of a number.

1. What is the cube of 15?

OPERATION.

$15 \times 15 \times 15 = 3375$, *Ans.*

SOLUTION. — We multiply 15 by 15, and the product by 15, and obtain 3375, which is the 3d power or cube of 15, since 15 has been taken 3 times as a factor.

RULE. — *Multiply the number by itself as many times, less* 1, *as there are units in the exponent of the required power.*

2. What is the square of 25? *Ans.* 625.
3. What is the square of 135? *Ans.* 18225.
4. What is the cube of 72? *Ans.* 373248.
5. What is the 4th power of 24? *Ans.* 331776.
6. Raise 7.2 to the third power. *Ans.* 373.248.
7. Involve 1.06 to the 4th power. *Ans.* 1.26247696.
8. Involve .12 to the 5th power. *Ans.* .0000248832.
9. Involve 1.0002 to the 2d power. *Ans.* 1.00040004.
10. What is the cube of $\frac{2}{5}$?

OPERATION.

$$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{2 \times 2 \times 2}{5 \times 5 \times 5} = \frac{2^3}{5^3} = \frac{8}{125}$$

SOLUTION. — We multiply $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ by finding the product of the numerators $2 \times 2 \times 2$ and of the denominators $5 \times 5 \times 5$, or by cubing both the numerator and denominator.

RULE. — *A common fraction may be raised to any power, by raising each of its terms, separately, to the required power.*

11. What is the square of $\frac{3}{8}$? *Ans.* $\frac{9}{64}$.
12. What is the square of $\frac{5}{8}$?
13. What is the cube of $1\frac{3}{14}$? *Ans.* $2\frac{197}{2744}$.
14. Raise $24\frac{3}{4}$ to the 2d power. *Ans.* $612\frac{9}{16}$.

423. To find the square of a number in terms of its tens and units.

1. Find the square of 23 in terms of its tens and units.

OPERATION.

$$\begin{array}{rl} 23 = & 20+3 \\ 23 = & 20+3 \\ \hline 69 = & 20\times 3+3^2 \\ 46 \ = & 20^2+20\times 3 \\ \hline 529 = & 20^2+2(20\times 3)+3^2 \end{array}$$

SOLUTION. — $23 = 20+3$. Multiplying $20+3$ by 3 and indicating the operation, we have 20×3 and 3×3 or 3^2. Multiplying 20×3 by 20, we have 20×20 or 20^2 and 20×3. Adding the partial products, the result is 20^2 + 2 times $20\times 3+3^2$ which is equal to 529.

RULE. — *To find the square of a number consisting of tens and units, to the square of the tens add twice the product of the tens by the units and the square of the units.*

When a number is separated into *any* two parts, its square is always equal to the square of the first part + twice the product of the first by the second + the square of the second part. Thus, $23 = 12+11$ and $23^2 = 12^2+2(12\times 11)+11^2 = 529$.

In the same way find the square of:

2. 13.	5. 42.	8. 71.	11. 101.
3. 25.	6. 57.	9. 79.	12. 119.
4. 39.	7. 65.	10. 83.	13. 235.

424. To find the cube of a number in terms of its tens and units.

14. Find the cube of 23 in terms of its tens and units.

OPERATION.

$$\begin{array}{rl} 23 = & 20+3 \\ 23 = & 20+3 \\ \hline 69 = & 20\times 3+3^2 \\ 46 \ = & 20^2+20\times 3 \\ \hline 529 = & 20^2+2(20\times 3)+3^2 \\ 23 = & 20+3 \\ \hline 1587 = & 20^2\times 3+2(20\times 3^2)+3^3 \\ 1058 \ = & 20^3+2(20^2\times 3)+(20\times 3^2) \\ \hline 12167 = & 20^3+3(20^2\times 3)+3(20\times 3^2)+3^3 \end{array}$$

SOLUTION. — The cube of $23 = 23 \times 23 \times 23$ or $23^2 \times 23$. We proceed to find the square as before which is $20^2 + 2(20 \times 3) + 3^2$. Multiplying this first by 3 and then by 20 and adding these partial products, the result is $20^3 + 3(20^2 \times 3) + 3(20 \times 3^2) + 3^3$.

RULE. — *To find the cube of a number consisting of tens and units, to the cube of the tens add three times the product of the square of the tens by the units, three times the product of the tens by the square of the units, and the cube of the units.*

The cube of a number divided into any two parts is equal to the cube of the first part + 3 times the square of the first part by the second part, + 3 times the first part by the square of the second part + the cube of the second part. Thus, $23 = 12 + 11$, and $23^3 = 12^3 + 3(12^2 \times 11) + 3(12 \times 11^2) + 11^3 = 12167$.

In the same way find the cube of

15. 17.	18. 49.	21. 89.	24. 135.
16. 25.	19. 67.	22. 95.	25. 225.
17. 34.	20. 78.	23. 125.	26. 319.

APPLICATIONS OF INVOLUTION.

425. The following principles of physics afford application for the rules of involution: —

PRINCIPLES. — I. *The intensity of light varies inversely as the square of the distance from the source of illumination.*

II. *The intensity of sound varies inversely as the square of the distance.*

III. *The heating effect of a small radiant mass upon a distant object, varies inversely as the square of the distance.*

IV. *The force of attraction or repulsion exerted between two magnetic poles is inversely proportional to the square of the distance between them.*

V. *Gravitation varies inversely as the square of the distance between the centers of gravity.*

EXAMPLES.

1. From a vessel on the ocean, a light in a certain lighthouse could be seen dimly at a distance of 20 miles. How much brighter would the light appear, when the vessel was 5 miles from the lighthouse?

Ans. 16 times as bright.

2. Having two lamps, one of 4 candle power and one of 16 candle power, if the former is 10 ft. distant, how far away must I place the latter to give me the same amount of light? *Ans.* 20 ft.

3. A bell heard by A on a certain street is heard by B 6 times as far distant. How loud will it sound to B as compared to A? *Ans.* $\frac{1}{36}$ as loud.

4. If the earth were removed to $\frac{1}{4}$ its present distance from the sun, how much more intense would be the heat received by it? *Ans.* 16 times.

5. A body 4000 miles from the center of the earth (or at the earth's surface) weighs 900 pounds. What would it weigh 12000 miles from the center of the earth? *Ans.* 100 pounds.

6. Two magnetic poles $\frac{1}{4}$ inch apart have an attraction for each other whose force would lift a pound weight. What weight would they lift if they were an inch apart?

Ans. $\frac{1}{16}$ lb.

7. A pistol shot is heard by A. B hears it $\frac{1}{25}$ as loud. How much further is B from the pistol than A?

8. The lights on a certain tower appear very bright to A. They appear $\frac{1}{4}$ as bright to B. Other things being equal, how much further is B from the light than A?

9. A wax taper is held a certain distance from a flame. It requires an amount of heat 16 times as great as that now acting upon it to ignite it. How much nearer must I move the taper if I wish to light it?

EVOLUTION.

426. A **Root** is a factor repeated to produce a power.

Thus, in the expression $5 \times 5 \times 5 = 125$, 5 is the root from which the power, 125, is produced.

427. **Evolution** is the process of extracting the root of a number considered as a power, and is the reverse of Involution.

428. The **Radical Sign** is the character, $\surd$, which, placed before a number, denotes that its root is to be extracted.

429. The **Index** of the root is the figure placed above the radical sign, to denote what root is to be taken. When no index is written, the index 2 is always understood.

430. A **Surd** is the indicated root of an imperfect power.

431. Roots are named from the corresponding powers, as will be seen in the following illustrations:

The square root of 9 is 3, written $\surd 9 = 3$.

The cube root of 27 is 3, written $\sqrt[3]{27} = 3$.

The fourth root of 81 is 3, written $\sqrt[4]{81} = 3$.

Any number whatever may be considered a power whose root is to be extracted; but only the perfect powers can have exact roots.

EVOLUTION BY FACTORING.

EXAMPLES.

432. To find any root of a number by factoring.

1. Find the cube root of 1728.

OPERATION.

3	1728
3	576
3	192
2	64
2	32
2	16
2	8
2	4
	2

SOLUTION. — A number that is a perfect cube is composed of *three* equal factors, and one of them is the cube root of that number.

The prime factors of 1728 are 3, 3, 3, 2, 2, 2, 2, 2, 2; hence $1728 = (3 \times 2 \times 2) \times (3 \times 2 \times 2) \times (3 \times 2 \times 2)$; therefore the cube root of 1728 is $(3 \times 2 \times 2)$, or 12.

RULE. — *Resolve the given number into its prime factors; then, to produce the square root, take one of every two equal factors; to produce the cube root take one of every three equal factors; and so on.*

2. Find the square root of 64, 256, 576, 6561.
3. Find the cube root of 729, 2744, 9261, 3375.
4. Find the square root of 225, 256, 289.
5. Find the cube root of 4913, 8000, 24389.
6. Find the fourth root of 81, 256, 625, 1296.
7. Find the fifth root of 243, 1024, 3125.
8. Find the sixth root of 729, 4096, 15625.
9. Find the seventh root of 2187, 16384, 78125.
10. Find the square root of 8836, 2704, 1444.
11. Find the cube root of 2197, 42875, 1728.
12. Find the square root of 529, 5184, 192721, 1521, 6889.
13. Find the cube root of 6859, 2744, 1331.
14. Find the square root of 1089, 14161, 156025.

SQUARE ROOT.

433. The **Square Root** of a number is one of the two equal factors that produce the number.

Thus, the square root of 49 is 7, for $7 \times 7 = 49$.

In extracting the square root, the first thing to be determined is the relative number of places in a given number and its square root. The law governing this relation is exhibited in the following:

Roots.	Squares.	Roots.	Squares.
1	1	1	1
9	81	10	1,00
99	98,01	100	1,00,00
999	99,80,01	1000	1,00,00,00

From these examples we perceive

1. That a root consisting of 1 place may have 1 or 2 places in the square.

2. That in all cases the addition of 1 place to the root adds 2 places to the square. Hence,

If we point off a number into two-figure periods, commencing at the right hand, the number of full periods and the left hand full or partial period will indicate the number of places in the square root; the highest period corresponding to the highest figure of the root.

To ascertain the relations of the several figures of the root to the periods of the number, observe that if any number, as 2345, is decomposed, the squares of the left hand parts will be related in local value as follows:

$2000^2 = 4\ 00\ 00\ 00$ $\qquad$ $2340^2 = 5\ 47\ 56\ 00$

$2300^2 = 5\ 29\ 00\ 00$ $\qquad$ $2345^2 = 5\ 49\ 90\ 25$

The square of the first figure of the root is contained wholly in the first period of the power; the square of the first two figures of the root in the first two periods of the power; and so on.

EXAMPLES.

434. 1. What is the length of one side of a square plot containing an area of 5417 sq. ft. ?

OPERATION.

```
  54,17 | 73.6
  49
 ------
140   517
143   429
 ------
146.0  88.00
146.6  87.96
 ------
          4
```

SOLUTION. — Since the given figure is a square, its side will be the square root of its area, which we will proceed to compute. Pointing off the given number, the two periods show that there will be two integral figures, tens and units, in the root. The tens of the root must be extracted from the first or left hand period, 54 hundreds. The greatest square in 54 hundreds is 49 hundreds, the square of 7 tens; we therefore write 7 tens in the root, at the right of the given number.

Since the entire root is to be the side of a square, let us form a square (Fig. I), the side of which is 70 feet long. The area of this square is $70 \times 70 = 4900$ sq. ft., which we subtract from the given number. This is done in the operation by subtracting the square number, 49, from the first period, 54, and to the remainder bringing down the second period, making the entire remainder 517.

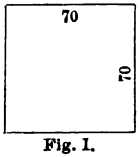


Fig. I.

If we now enlarge our square (Fig. I) by the addition of 517 square feet, in such a manner as to preserve the square form, its size will be that of the required square. To preserve the square form, the addition must be so made as to extend the square equally in two directions; it will therefore be composed of two oblong figures at the sides and a little square at the corner (Fig. II). Now, the *width* of this addition will be the additional length to the side of the square, and consequently *the next figure in the root.* To find *width* we divide square contents, or area, by *length.* But the length of one side of the little square cannot be found till the width of the addition is determined, because it is equal to this width. We will therefore add the lengths of the two oblong figures, and the sum will be sufficiently near the whole length to be used as a trial divisor.

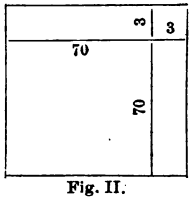


Fig. II.

Each of the oblong figures is equal in length to the side of the square first formed; and their united length is $70 + 70 = 140$ ft.

(Fig. III). This number is obtained in the operation by doubling the 7 and annexing one cipher, the result being written at the left of the dividend. Dividing 517, the area, by 140, the approximate length, we obtain 3, the probable width of the addition, and second figure of the root. Since 3 is also the side of the little square, we can now find the entire length of the addition, or the complete divisor, which is 70 + 70 + 3 = 143 (Fig. III). This number is found in the operation by adding 3 to the trial divisor, and writing the result underneath. Multiplying the complete divisor, 143, by the trial quotient figure, 3, and subtracting the product from the dividend, we obtain another remainder of 88 square feet.

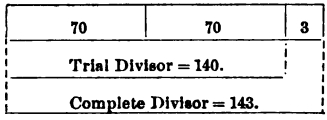


Fig. III.

With this remainder, for the same reason as before, we must proceed to make a new enlargement; and we bring down two decimal ciphers, because the next figure of the root being tenths, its square will be hundredths. The trial divisor to obtain the width of this new enlargement, or the next figure in the root, will be, for the same reason as before, twice 73, the root already found, with one cipher annexed. But since the 7 has already been doubled in the operation, we have only to double the last figure of the complete divisor, 143, and annex a cipher, to obtain the new trial divisor, 146.0. Dividing, we obtain .6 for the trial figure of the root; then proceeding as before, we obtain 146.6 for a complete divisor, 87.96 for a product; and there is still a remainder of .04. Hence, the side of the given square plot is 73.6 feet, nearly.

The above is the geometrical explanation of square root. The following is another clear explanation of the same process:

We find the greatest square in 5400 as before, which is 4900, and place its root at the right. Since the square of a number divided into any two parts is equal to the square of the first part, plus twice the product of the first by the second, plus the square of the second part (**423**), having found the square of the first part, which is 4900, the remainder 517 must be equal to twice the product of the first part 70 by the second part (?) plus the square of the second part. Since we do not know the second part, we take as a trial divisor twice the first, which is 140, and we find the second part to be about 3. Twice the product of the first by the second would be 3 times 140, and the square of the second, 3×3; or, since $3 \times 140 + 3 \times 3 = 3 \times 143$, we take 143 as a complete divisor and multiply it by the quotient figure 3. In the same manner we proceed with the remainder 88, and we find the root to be 73.6+

RULE. — I. *Point off the given number into periods of two figures each, counting from unit's place toward the left and right.*

II. *Find the greatest square number in the left-hand period, and write its root for the first figure in the root; subtract the square number from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *At the left of the dividend write twice the first figure of the root, and annex one cipher, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for a trial figure in the root.*

IV. *Add the trial figure of the root to the trial divisor for a complete divisor; multiply the complete divisor by the trial figure in the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *To the last complete divisor add the last figure of the root, and to the sum annex one cipher, for a new trial divisor, with which proceed as before.*

1. If at any time the product is greater than the dividend, diminish the trial figure of the root, and correct the erroneous work.

2. If a cipher occurs in the root, annex another cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. What is the square root of 406457.2516?

OPERATION.

40,64,57.25,16 | 637.54, *Ans.*

	36	
Trial divisor,	120	464
Complete "	123	369
Trial "	1260	9557
Complete "	1267	8869
Trial "	1274.0	688.25
Complete "	1274.5	637.25
Trial "	1275.00	51.0016
Complete "	1275.04	51.0016

3. The decimal points in the work may be omitted, care being taken to point off in the root according to the number of decimal periods used.

4. The pupil will acquire greater facility, and secure greater accuracy, by keeping units of like order under each other, and each divisor opposite the corresponding dividend, by the use of the lines, as shown in the operation.

3. What is the square root of 576? *Ans.* 24.
4. What is the square root of 6561? *Ans.* 81.
5. What is the square root of 444889? *Ans.* 667.
6. What is the square root of 29855296? *Ans.* 5464.
7. What is the square root of 3486784401? *Ans.* 59049.
8. What is the square root of 54819198225? *Ans.* 234135.

5. The cipher in the trial divisor may be omitted, and its place, after division, may be occupied by the trial root figure, thus forming only *complete divisors*.

9. What is the square root of 2?

	2.	1.41+, *Ans.*
	1	
	100	
24	96	
	400	
281	281	

Extract the square roots of the following numbers:

10. $\sqrt{3}$. *Ans.* 1.7320508+.
11. $\sqrt{5}$. *Ans.* 2.2360679+.
12. $\sqrt{6}$. *Ans.* 2.4494897+.
13. $\sqrt{7}$. *Ans.* 2.6457513+.
14. $\sqrt{8}$. *Ans.* 2.8284271+.
15. $\sqrt{10}$. *Ans.* 3.1622776+.

What is the square root of:

16. .00008836?
17. .0043046721?
18. $\frac{625}{6561}$? *Ans.* $\frac{25}{81}$.
19. $\frac{7056}{9216}$? *Ans.* $\frac{7}{8}$.
20. $\frac{2}{3}$? *Ans.* .816496+.
21. $17\frac{3}{8}$? *Ans.* 4.1683+.

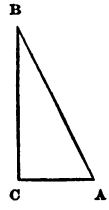
6. The square root of a common fraction may be obtained by extracting the square roots of the numerator and denominator separately, provided the terms are perfect squares; otherwise, the fraction may first be reduced to a decimal.

7. Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

APPLICATIONS OF SQUARE ROOT.

435. An **Angle** is the opening between two lines that meet each other. Thus, the two lines AB and AC meeting, form an angle at A.

436. A **Right-Angled Triangle** is a triangle having one right angle, as at C.



The **Base** is the side on which it stands, as AC.

The **Perpendicular** is the side forming a right angle with the base, as BC.

The **Hypotenuse** is the side opposite the right angle, as AB.

437. **Similar Figures** are figures which have the same form and differ only in size.

438. The following principles, which are demonstrated in geometry, afford applications of square root.

PRINCIPLES. — I. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides; therefore,*

II. *The hypotenuse is equal to the square root of the sum of the squares of the other two sides.*

III. *The areas of two circles are to each other as the squares of their radii, diameters, or circumferences.*

IV. *The base or perpendicular of a right-angled triangle is equal to the square root of the difference of the hypotenuse and that of the other side.*

V. *The ratio of the area of two similar figures, is equal to the square of the ratio of any two like dimensions of them; therefore,*

VI. *The ratio of any two like dimensions of two similar figures is equal to the square root of the ratio of their areas.*

EXAMPLES.

1. The two sides of a right-angled triangle are 3 and 4 feet. What is the length of the hypotenuse?

OPERATION.

$3^2 = 9$, square of one side.
$4^2 = 16$, square of the other side.
25, square of hypotenuse.
$\sqrt{25} = 5$, *Ans.*

SOLUTION. — Squaring the two sides and adding, we find the sum to be 25; and since the sum is equal to the square of the hypotenuse, we extract the square root, and obtain 5 feet, the hypotenuse.

2. A circular skating rink has a diameter of 75 feet. What would be the diameter of a similar rink with $\frac{1}{3}$ the area? *Ans.* 43.3 feet.

3. If an army of 55225 men is drawn up in the form of a square, how many men will there be on a side?

Ans. 235.

4. The diagonals of two similar rectangles are as 6 to 13. How many times does the larger contain the smaller?

Ans. $4\frac{25}{36}$ times.

5. The sides of two square blocks are 5 feet and 10 feet respectively. How do they compare in area?

Ans. 5 foot block is $\frac{1}{4}$ of other.

6. A man has 200 yards of carpeting $1\frac{1}{3}$ yards wide. What is the length of one side of the square room which this carpet will cover? *Ans.* 45 feet.

7. How many rods of fence will be required to inclose 10 acres of land in the form of a square? *Ans.* 160 rods.

8. The end of a May-pole, broken 39 feet from the top, struck the ground 15 feet from the foot. What was the height of the pole? *Ans.* 75 feet.

9. A ladder 40 feet long is so placed in a street, that without being moved at the foot, it will reach a window

on one side 33 feet, and on the other side 21 feet, from the ground. What is the breadth of the street?

Ans. 56.64+ feet.

10. Two men start from one corner of a park one mile square, and travel at the same rate. A goes by the walk around the park, and B takes the diagonal path to the opposite corner, and turns to meet A at the side. How many rods from the corner will the meeting take place?

Ans. 93.7+ rods.

11. The top of a castle is 45 yards high, and the castle is surrounded by a ditch 60 yards wide. What would be the length of a rope that would reach from the outside of the ditch to the top of the castle? *Ans.* 75 yards.

12. A ladder 52 feet long stands close against the side of a building. How many feet must it be drawn out at the bottom, that the top may be lowered 4 feet?

13. A room is 20 feet long, 16 feet wide, and 12 feet high. What is the distance from one of the lower corners to the opposite upper corner? *Ans.* 28.284271+ feet.

14. It requires 63.39 rods of fence to inclose a circular field of 2 acres. What length will be required to inclose 3 acres in circular form? *Ans.* 77.63+ rods.

15. The radius of a certain circle is 5 feet. What will be the radius of another circle containing twice the area of the first? *Ans.* 7.07106+ feet.

16. A certain circular race-track has a diameter of 1500 feet. What would be the diameter of a similar track 4 times as large? *Ans.* 3000 feet.

17. If it costs $ 167.70 to inclose a circular pond containing 17 A. 110 P., what will it cost to inclose another $\frac{1}{5}$ as large? *Ans.* $ 75.

18. If a cistern 6 ft. in diameter holds 80 bbl. of water, what is the diameter of a cistern of the same depth that holds 1200 bbl.?

CUBE ROOT.

439. The **Cube Root** of a number is one of the three equal factors that produce the number.

Thus, the cube root of 27 is 3, since $3 \times 3 \times 3 = 27$.

In extracting the cube root, the first thing to be determined is the relative number of places in a cube and its root. The law governing this relation is exhibited in the following examples:

Roots.	Cubes.	Roots.	Cubes.
1	1	1	1
9	729	10	1,000
99	907,299	100	1,000,000
999	997,002,999	1,000	1,000,000,000

From these examples, we perceive,

1. That a root consisting of 1 place may have from 1 to 3 places in the cube.

2. That in all cases the addition of 1 place to the root adds three places to the cube.

If we point off a number into three-figure periods, commencing at the right hand, the number of full periods and the left hand full or partial period will indicate the number of places in the cube root, the highest period corresponding to the highest figure of the root.

To ascertain the relations of the several figures of the root to the periods of the number, decompose any number, as 5423.

$5000^3 = 125\ 000\ 000\ 000$ $\quad$ $5420^3 = 159\ 220\ 088\ 000$

$5400^3 = 157\ 464\ 000\ 000$ $\quad$ $5423^3 = 159\ 484\ 621\ 967$

The cube of the first figure of the root is contained wholly in the first period of the power: the cube of the first two figures in the first two periods of the power, etc.

EXAMPLES.

40. 1. What is the length of one side of a cubical block containing 413494 solid inches?

OPERATION — COMMENCED.

$$\begin{array}{r|l} 413494 & 74 \\ 343 & \\ \hline 14700 \quad 70494 & \end{array}$$

SOLUTION. — Since the block is a cube, its side will be the cube root of its solid contents, which we will proceed to compute. Pointing off the given number, the two periods show that there will be two figures, tens and units, in the root. The tens of the root must be extracted from the first period, 413 thousands. The greatest cube in 413 thousands is 343 thousands, the cube of 7 tens; we therefore write 7 tens in the root at the right of the given number.

Since the entire root is to be the side of a cube, let us form a cubical block (Fig. I), the side of which is 70 inches in length. The contents of this cube are $70 \times 70 \times 70 = 343000$ solid inches, which we subtract from the given number. This is done in the operation by subtracting the cube number, 343, from the first period, 413, and to the remainder bringing down the second period, making the entire remainder 70494.

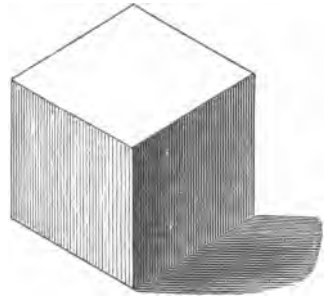


FIG. I.

If we now enlarge our cubical block (Fig. I) by the addition of 70494 solid inches, in such a manner as to preserve the cubical form, its size will be that of the required block. To preserve the cubical form, the addition must be made upon three adjacent sides or faces. The addition will therefore be composed of 3 flat blocks to cover the 3 faces (Fig. II); 3 oblong blocks to fill the vacancies at the edges (Fig. III); and 1 small cubical block to fill the vacancy at the corner (Fig. IV). Now, the *thickness* of this enlargement will be the *additional length* of the *side of the cube*, and, consequently, the *second figure in the root.* To find *thickness,* we may divide solid contents by *surface,* or *area.* But the area of the 3 oblong blocks and little cube cannot be found till the thickness of the addition is determined, because their common breadth

is equal to this thickness. We will, therefore, find the area of the 3 flat blocks, which is sufficiently near the whole area to be used as a *trial divisor*. As these are each equal in length and breadth to the side of the cube whose faces they cover, the whole area of the three is $70 \times 70 \times 3 = 14700$ square inches. This number is obtained in the operation by annexing 2 ciphers to three times the square of 7; the result being written at the left hand of the dividend. Dividing, we obtain 4, the probable thickness of the addition, and second figure of the root. With this assumed figure, we will *complete* our divisor by adding the area of the 4 blocks, before undetermined. The 3 oblong blocks are each 70 inches long; and the little cube, being equal in each of its dimensions to the thickness of the addition, must be 4 inches long. Hence, their united length is $70 + 70 + 70 + 4 = 214$. This number is obtained in the operation by multiplying the 7 by 3, and annexing the 4 to the product, the result being written in column I, on the next line below the trial divisor. Multiplying 214, the length, by 4, the common width, we obtain 856, the area of the four blocks, which added to 14700, the trial divisor, makes 15556, the *complete divisor*; and multiplying this by 4, the second figure in the root, and subtracting the product from the dividend, we obtain a remainder of 8270 solid inches. With this remainder, for the same reason as before, we must proceed to make a new enlargement. But since we have already two figures

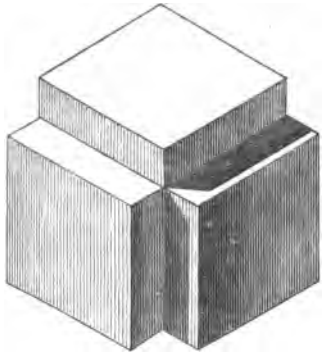


FIG. II.

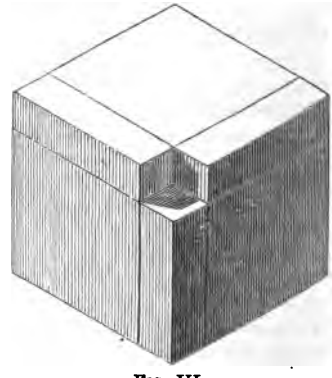


FIG. III.

in the root, corresponding to the two periods of the given number, the next figure of the root must be a decimal; and we therefore annex to the remainder a period of three decimal ciphers, making 8270.000 for a new dividend.

The trial divisor to obtain the thickness of this second enlargement, or the next figure of the root, will be the area of three new flat blocks to cover the three sides of the cube already formed; and this surface (Fig. IV) is composed of 1 face of each of the flat blocks already used, 2 faces of each of the oblong blocks, and 3 faces of the little cube. But we have in the complete divisor, 15556, 1 face of each of the flat blocks, oblong blocks, and little cube; and in the correction of the trial divisor, 856, 1 face of each of the oblong blocks and of the little cube; and in the square of the last root figure, 16, a third face of the little cube. Hence, 16 + 856 + 15556 = 16428, the significant figures of the new trial divisor. This number is obtained in the operation by adding the square of the last root figure mentally, and combining units of like

OPERATION — CONTINUED.

			413494 | 74
I.	II.		343
		14700	70494
214	856	15556	62224
			8270.000

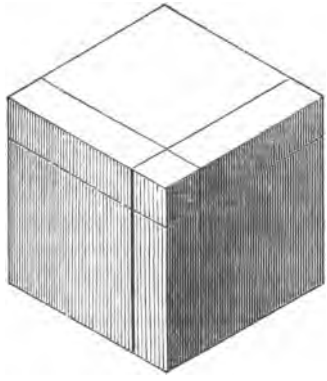


FIG. IV.

OPERATION — CONTINUED.

			413494 | 74.5
I.	II.		343
		14700	70494
214	856	15556	62224
		16428.00	8270.000
222.5	111.25	16539.25	8269.625
			.375

order, thus: 16, 6, and 6 are 28, and we write the unit figure in the new trial divisor; then 2 to carry, and 5 and 5 are 12, etc.

We annex 2 ciphers to this trial divisor, as to the former. Or we may obtain this trial divisor, in the same way as the first one, by taking 3 times the square of 74 (which we regard as the first part), with one cipher annexed. Dividing, we obtain 5, the third figure in the root. To complete the second trial divisor, after the manner of the first, the correction may be found by annexing .5 to 3 times the former figures, 74, and multiplying this number by .5. But as we have, in column I, 3 times 7, with 4 annexed, or 214, we need only multiply the last figure, 4, by 3, and annex .5, making 222.5, which multiplied by .5, gives 111.25, the correction required. Then we obtain the complete divisor, 16539.25, the product, 8269.625, and the remainder, .375, in the manner shown by the former steps.

The above is the geometrical explanation. The following is another explanation of the same process:

We find that the greatest cube in 413494 is 343000, whose cube root is 70, which we write at the right. Since the cube of a number divided into any two parts is equal to the cube of the first part, plus 3 times the square of the first by the second, plus 3 times the first by the square of the second, plus the cube of the second part (**424**), therefore, having found the cube of the first part 70, which is 343000, the remainder 70494 must be equal to 3 times the square of the first part by the second part (?), plus 3 times the first part by the square of the second part, plus the cube of the second part. Since we do not know the second part, we take as a trial divisor 3 times the square of the first, which is 14700, and we find that the second part is about 4. The product of the quotient by the trial divisor will be 3 times the square of the first by the second. To this must be added 3 times the first by the square of the second $= 3 \times 70 \times 4^2$ (but since the 4 in the quotient forms one of these factors we add to the trial divisor $3 \times 70 \times 4 = 840$), and the cube of the second, making the square 4×4 for addition to the trial divisor, and our complete divisor will be $14700 + 840 + 16 = 15556$, which multiplied by the quotient figure 4 gives a product of 62224. With the remainder, we proceed as before and the root is 74.5+.

RULE. — I. *Point off the given number into periods of three figures each, counting from units' place toward the left and right.*

II. *Find the greatest cube that does not exceed the left-hand period, and write its root for the first figure in the required root; subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.*

III. *At the left of the dividend write three times the square of the first figure of the root, and annex two ciphers, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for a trial figure in the root.*

IV. *Annex the trial figure to three times the former figure, and write the result in a column marked* I, *one line below the trial divisor; multiply this term by the trial figure, and write the product on the same line in a column marked* II; *add this term as a correction to the trial divisor, and the result will be the complete divisor.*

V. *Multiply the complete divisor by the trial figure, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

VI. *Add the square of the last figure of the root, the last term in column* II, *and the complete divisor together, and annex two ciphers, for a new trial divisor; with which obtain another trial figure in the root.*

VII. *Multiply the unit figure of the last term in column* I *by* 3, *and annex the trial figure of the root for the next term of column* I; *multiply this result by the trial figure of the root for the next term of column* II; *add this term to the trial divisor for a complete divisor, with which proceed as before.*

VIII. *If there is a remainder after the root of the last period is found, annex periods of ciphers and proceed as before. The figures of the root thus obtained will be decimals.*

1. If at any time the product is greater than the dividend, diminish the trial figure of the root, and correct the erroneous work.

2. If a cipher occurs in the root, annex two more ciphers to the trial divisor, and another period to the dividend; then proceed as before with column I, annexing both cipher and trial figure.

The cube root of a fraction may be found by extracting the cube root of the numerator and denominator.

In extracting the cube root of *decimal* numbers, begin at units' place and proceed both toward the left and the right, to separate into periods of *three* figures each.

2. What is the cube root of 79.112?

OPERATION.

79.112 | 4.2928 +, *Ans.*
64.

		4800	15112
122	244	5044	10088
		529200	5024000
1269	11421	540621	4865589
		55212300	158411000
12872	25744	55238044	110476088
		5526379200	47934912000
128768	1030144	5527409344	44219274752

3715637248, Rem.

3. What is the cube root of 84604519? *Ans.* 439.
4. What is the cube root of 2357947691? *Ans.* 1331.
5. What is the cube root of 10963240788375? *Ans.* 22215.
6. What is the cube root of 270671777032189896? *Ans.* 646866.
7. What is the cube root of .091125? *Ans.* .45.
8. What is the cube root of .000529475129? *Ans.* .0809.
9. What is the approximate cube root of .008649? *Ans.* .2052+.

Extract the cube roots of the following numbers:

10. $\sqrt[3]{2}$. *Ans.* 1.259921+.
11. $\sqrt[3]{3}$. *Ans.* 1.442249+.
12. $\sqrt[3]{4}$. *Ans.* 1.587401+.
13. $\sqrt[3]{5}$. *Ans.* 1.709975+.
14. $\sqrt[3]{6}$. *Ans.* 1.817120+.
15. $\sqrt[3]{7}$. *Ans.* 1.912931+.

APPLICATIONS OF CUBE ROOT.

441. The following principles of geometry afford applications in cube root.

PRINCIPLES. — I. *The ratio of two similar solids is equal to the cube of the ratio of any two like dimensions.*

II. *The ratio of any two like dimensions of similar solids is equal to the cube root of the ratios of the solids.*

EXAMPLES.

1. The lengths of two similar solids are 4 in. and 50 in. The first contains 16 cu. in. What does the second contain? *Ans.* 31250 cu. in.

2. If a ball 5 in. in diameter weighs 8 pounds, what will be the weight of a similar ball 10 in. in diameter?

3. I have two cubical boxes. The side of the smaller contains 6 in., and of the larger, 10 in. How many times would the smaller contain the larger?

4. What is the length of one side of a cistern of cubical form, containing 1331 solid feet? *Ans.* 11 feet.

5. The pedestal of a certain monument is a square block of granite, containing 373248 solid inches. What is the length of one of its sides? *Ans.* 6 feet.

6. A cubical box contains 474552 solid inches. What is the area of one of its sides? *Ans.* $42\frac{1}{4}$ sq. ft.

7. A man wishes to make a bin to contain 125 bushels, of equal width and depth, and length double the width. What must be its dimensions? *Ans.* Width and depth, 51.223+ inches; length, 102.446+ inches.

8. If a cable 4 in. in circum. will support a sphere 2 ft. in diam., what is the diam. of that sphere which will be supported by a cable 5 in. in circum.? *Ans.* 2.32+ ft.

ROOTS OF ANY DEGREE.

442. Any root whatever may be extracted by means of the square and cubic roots, as will be seen in the two cases which follow.

EXAMPLES.

443. When the index of the required root contains no other factor than 2 or 3.

We have seen that if we raise any power of a given number to any required power, the result will be the power of the given number denoted by the product of the two indices (**419**). Conversely, if we extract successively two or more roots of a given number, the result must be that root of the given number denoted by the product of the indices.

1. What is the 6th root of 2176782336?

OPERATION.

$6 = 2 \times 3.$

$\sqrt[2]{2176782336} = 46657.$

$\sqrt[3]{46656} \quad = 36,$ *Ans.*

Or,

$\sqrt[3]{2176782336} = 1296.$

$\sqrt[2]{1296} \quad = 36,$ *Ans.*

SOLUTION. — The index of the required root is $6 = 2 \times 3$; we therefore extract the square root of the given number, and the cube root of this result, and obtain 36, which must be the 6th root required. Or, we first find the cube root of the given number, and then the square root of the result, as in the operation.

RULE. — *Separate the index of the required root into its prime factors, and extract successively the roots indicated by the several factors obtained; the final result will be the required root.*

2. What is the 6th root of 6321363049? *Ans.* 43.
3. What is the 4th root of 5636405776? *Ans.* 274.
4. What is the 8th root of 1099511627776? *Ans.* 32.
5. What is the 6th root of 25632972850442049? *Ans.* 543.

6. What is the 9th root of 1.577635?

Ans. 1.051963+.

7. What is the 12th root of 16.3939? *Ans.* 1.2624+.

8. What is the 18th root of 104.9617? *Ans.* 1.2950+.

444. When the index of the required root is prime, or contains any other factor than 2 or 3.

To extract any root of a number is to separate the number into as many equal factors as there are units in the index of the required root; and it will be found that if by any means we can separate a number into factors nearly equal to each other, the *average* of these factors, or their sum divided by the number of factors, will be nearly equal to the root indicated by the number of factors.

1. What is the 7th root of 308?

OPERATION.

$\sqrt[6]{308} = 2.59+.$
$\sqrt[8]{308} = 2.04+.$
$2.59 + 2.04 = 4.63.$
$4.63 \div 2 = 2.31$, assumed root.

$2.31^6 = 151.93.$
$308 \div 151.93 = 2.0272+.$
$2.31 \times 6 + 2.0272 = 15.8872.$
$15.8872 \div 7 = 2.2696$, 1st approx.

$2.2696^6 = 136.6748.$
$308 \div 136.6748 = 2.253452+.$
$2.2696 \times 6 + 2.253452 = 15.871052.$
$15.871052 \div 7 = 2.267293$, 2d approx.

SOLUTION.—We first find the 6th root, and also the 8th root of 308; and since the 7th root must be less than the former and greater than the latter, we take the average of the two, or one half of their sums, 2.31, and call it the *assumed root.* We next raise the assumed root, 2.31, to the 6th power, and divide the given number, 308, by the result, and obtain 2.0272+ for a quotient; we thus separate 308 into 7 factors, 6 of which are equal to 2.31, and the other is 2.0272. As these 7 factors are nearly equal to each other, the average of them all must be a near approximation to the 7th root. Multiplying the 2.31 by 6, adding the 2.0272 to the product, and dividing this result by 7, we find the average to be 2.2696, which is the first approximation to the required root. We next divide 308 by the 6th power of 2.2696, and

obtain 2.253452+ for a quotient; and we thus separate the given number into 7 factors, 6 of which are each equal to 2.2696, and the other is 2.253452. Finding the average of these factors, as in the former steps, we have 2.267293, which is the 7th root of the given number, correct to 5 decimal places.

RULE. — I. *Find by trial some number nearly equal to the required root, and call this the assumed root.*

II. *Divide the given number by that power of the assumed root denoted by the index of the required root less* 1: *to this quotient add as many times the assumed root as there are units in the index of the required root less* 1, *and divide the amount by the index of the required root. The result will be the first approximate root required.*

III. *Take the last approximation for the assumed root, with which proceed as with the former, and thus continue till the required root is obtained to a sufficient degree of exactness.*

If the index of the required root contains the factors 2 or 3, we may first extract the square or cube root as many times, successively, as these factors are found in the index, after which we must extract that root of the result which is denoted by the remaining factor of the index. Thus, if the 15th root were required, we should first find the cube root, then the 5th root of this result.

2. What is the 20th root of 617?

OPERATION.

$$20 = 2 \times 2 \times 5.$$
$$\sqrt[2]{617} = 24.839485+.$$
$$\sqrt[2]{24.839485} = 4.983923+.$$
$$\sqrt[5]{4.983923} = 1.378206+. \quad \textit{Ans.}$$

3. What is the 5th root of 120?
4. What is the 7th root of 1.95678?
5. What is the 10th root of 743044?
6. What is the 15th root of 15?
7. What is the 25th root of 100?
8. What is the 5th root of 5?

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

445. An **Arithmetical Progression,** or **Series,** is a series of numbers increasing or decreasing by a constant common difference.

Thus, 3, 5, 7, 9, 11, etc., is an arithmetical progression with an *ascending series*, and 13, 10, 7, 4, etc., is an arithmetical progression with a *descending series.*

446. The **Terms** of a series are the numbers of which it is composed.

447. The **Extremes** are the first and last terms.

448. The **Means** are the intermediate terms.

449. The **Common Difference** is the difference between any two adjacent terms.

450. There are *five parts* in an arithmetical series, any *three* of which being given, the other *two* may be found. They are as follows: the *first term*, *last term*, *common difference*, *number of terms*, and *sum of all the terms.*

EXAMPLES.

451. To find one of the extremes when the other, the common difference, and number of terms are given.

1. Let 2 be the first term of an ascending series of 8 terms, and 3 the common difference. Find the 8th term. The series will be written 2, 5, 8, 11, 14, etc., or analyzed, thus, $2, 2+3, 2+3+3, 2+3+3+3, 2+3+3+3+3.$

OPERATION.

$3 \times 7 = 21.$

1st term, $2 + 21 = 23$, 8th term.

SOLUTION. — Since the series will be 2, 5, 8, 11, 14, or analyzed 2, $2 + 3$, $2 + 3 + 3$, $2 + 3 + 3 + 3$, $2 + 3 + 3 + 3 + 3$, etc., we see that in an *ascending* series, we obtain the *second* term by *adding* the common difference *once* to the first term; the *third* term, by adding the common difference *twice* to the first term; the *fourth* term, by adding the common difference *three times* to the first term, etc.; and, in general, we obtain *any* term by adding the common difference *as many times* to the first term as there are terms less *one*. Hence, the eighth term will be equal to the first term, 2, plus $7 \times 3 = 23$, *Ans.*

RULE. — *Multiply the common difference by the number of terms less* 1, *and add the product to the first term, if the series is ascending, and subtract it if the series is descending.*

2. The first term of an ascending series is 4, the common difference 3, and the number of terms 19. What is the last term? *Ans.* 58.

3. What is the 13th term of a descending series whose first term is 75, and common difference 5? *Ans.* 15.

4. A boy bought 18 hens, paying 2 cents for the first, 5 cents for the second, and 8 cents for the third, in arithmetical progression. What did he pay for the last hen?

5. What is the 40th term of the series $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, etc.? *Ans.* $10\frac{1}{4}$.

6. A man travels 9 days; the first day he goes 20 miles, the second 25 miles, increasing 5 miles each day. How far does he travel the last day of his journey?

Ans. 60 miles.

7. What is the amount of \$100, at 7%, for 45 years?

$\$100 + \$7 \times 45 = \$415$, *Ans.*

8. The first term of an arithmetical progression is 5, the common difference 4, and the number of terms 8. What is the last term? *Ans.* 33.

452. To find the common difference when the extremes and number of terms are given.

1. The first term is 2, the last term 23, and the number of terms 8. What is the common difference?

OPERATION.

Last term $23 - 1$st term $2 = 21$.
$8 - 1 = 7$. $21 \div 7 = 3$, Com. Dif.

SOLUTION. — Referring to the series, 2, 5, 8, 11, 14, analyzed in **451**, we readily see that, by subtracting the *first term* from any term, we have left the *common difference* taken as many times as there are terms less 1. Thus, by taking away 2 in the fifth term, $2 + 3 + 3 + 3 + 3$, we have left 12, which is 4 times 3. Hence, we divide the difference between 2 and 23, 21, by $8 - 1 = 7$, and find the common difference 3.

RULE. — *Divide the difference of the extremes by the number of terms less* 1.

2. The first term is 2, the last term is 17, and the number of terms is 6. What is the common difference?

3. A man has seven children whose ages are in arithmetical progression; the youngest is 2 years old, and the eldest 14. What is the common difference of their ages? *Ans.* 2 years.

4. The extremes of an arithmetical series are 1 and $50\frac{1}{2}$, and the number of terms is 34. What is the common difference?

5. An invalid commenced to walk for exercise, increasing the distance daily by a common difference; the first day he walked 3 miles, and the 14th day $9\frac{1}{2}$ miles. How many miles did he walk each day?

When we have found the common difference, we may add it once, twice, etc., to the first term, and we have the series, and consequently the *means*.

Ans. 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, etc.

6. The first term of an arithmetical progression is 5, the last term 54, and the number of terms 8. What is the common difference?

453. To find the number of terms when the extremes and common difference are given.

1. The extremes are 2 and 23, and the common difference 2. Find the number of terms.

OPERATION.

$23 - 2 = 21.$
$21 \div 3 = 7.$
$7 + 1 = 8$, No. Terms.

SOLUTION. — Examining the series, 2, 5, 8, 11, 14, analyzed in **451**, we see that after taking away the *first* term from any term we have left the common difference taken as many times as the number of terms less 1. Hence we take away 2, the first term, from 23, the last term, and the remainder 21 is 7 times 3. Since 7 is the number of terms less 1 the number is $7 + 1 = 8$.

RULE. — *Divide the difference of the extremes by the common difference, and add* 1 *to the quotient.*

2. The extremes are 7 and 43, and the common difference is 4. What is the number of terms? *Ans.* 10.

3. The first term is $2\frac{1}{2}$, the last term is 40, and the common difference is $7\frac{1}{2}$. What is the number of terms? *Ans.* 6.

4. A laborer agreed to build a fence on the following conditions: for the first rod he was to have 6 cents, with an increase of 4 cents on each successive rod; the last rod came to 226 cents. How many rods did he build?

Ans. 56 rods.

454. To find the sum of all the terms when the extremes and number of terms are given.

1. The extremes are 2 and 14 and the number of terms 5. What is the sum of the series?

OPERATION.

$2 + 5 + 8 + 11 + 14 = 40$, once the sum.
$14 + 11 + 8 + 5 + 2 = 40$, " " "
$16 + 16 + 16 + 16 + 16 = 80$, twice the sum.
$80 \div 2 = 40$, the sum.

SOLUTION. — To deduce a rule for finding the *sum of all the terms*, we will take the series 2, 5, 8, 11, 14, writing it under itself in an inverse order, and add each term.

Here we perceive that 16, the sum of the extremes, multiplied by 5, the number of terms, equals 80, which is *twice the sum of the series*. Dividing 80 by 2 gives 40, which is the sum required.

RULE. — *Multiply the sum of the extremes by the number of terms, and divide the product by 2.*

2. The extremes are 5 and 32, and the number of terms 12. What is the sum of all the terms? *Ans.* 222.

3. How many strokes does a common clock make in 12 hours? *Ans.* 78 strokes.

4. What debt can be discharged in a year by weekly payments in arithmetical progression, the first being $24, and the last $1224? *Ans.* $32448.

5. Suppose 100 apples were placed in a line 2 yards apart, and a basket 2 yards from the first apple. How far would a boy travel to gather them up singly, and return with each separately to the basket?

Ans. 20200 yards.

6. The first term of an arithmetical progression is 4, the common difference 5, and the number of terms 7. What is the sum of the series? *Ans.* 133.

NOTE. — First find the last term by **451**, and then proceed as in **454**.

7. The extremes of an arithmetical progression are 8 and 64, and the common difference is 8. What is the sum of the series? *Ans.* 288.

NOTE. — First find the number of terms by **453**, and then proceed as in **454**.

455. By reversing some one of the four problems now given, or by combining two or more of them, all of the sixteen remaining problems of Arithmetical Progression may be solved or analyzed.

GEOMETRICAL PROGRESSION.

456. A Geometrical Progression is a series of numbers increasing or decreasing by a constant multiplier.

When the multiplier is *greater* than a unit, the series is *ascending*.

When the multiplier is *less* than a unit, the series is *descending*.

Thus, 2, 6, 18, 54, 162, is an *ascending* series, in which 3 is the multiplier. 162, 54, 18, 6, 2, is a *descending* series, in which $\frac{1}{3}$ is the multiplier.

457. In every geometrical progression there are five *parts* to be considered, any *three* of which being given, the other *two* may be determined. They are as follows: The *first term*, *last term*, *ratio*, *number of terms*, and the *sum of all the terms*.

458. The first and last terms are the *extremes*, and the intermediate terms are the *means*.

EXAMPLES.

459. To find any term, the first term, the ratio, and number of terms being given.

1. The first term of a geometrical ratio is 2 and the multiplier or ratio is 3. What is the 4th term?

OPERATION.

$3^3 = 27$.

$2 \times 27 = 54$, 4th term.

SOLUTION. — The first term exists independently of the ratio. Since the number is multiplied by 3 the second term is 2×3, the third term $2 \times 3 \times 3$, or 2×3^2, the fourth term $2 \times 3 \times 3 \times 3$ or 2×3^3. Using the ratio *once* as a factor, gives the second term; using it *twice* or its *second power*, the *third* term; using it *three times* or its *third power*, the *fourth term*. The third power of 3 is 27 and the first term 2 *multiplied* by 27 gives the 4th term 54.

RULE. — *Multiply the first term by that power of the ratio denoted by the number of terms less* 1.

2. The first term of a geometrical series is 4, the ratio is 3. What is the 9th term? *Ans.* $4 \times 3^8 = 26244$.

3. The first term is 1024, the ratio $\frac{1}{4}$, and the number of terms 8. What is the last term? *Ans.* $\frac{1}{16}$.

4. The first term of a geometrical progression is 6, the ratio 10, and the number of terms 5. What is the last term? *Ans.* 60000.

5. A boy bought 9 oranges, agreeing to pay 1 mill for the first orange, 2 mills for the second, and so on. What did the last orange cost him? *Ans.* \$.256.

6. The first term is 7, the ratio $\frac{1}{7}$, and the number of terms 7. What is the last term? *Ans.* $\frac{1}{16807}$.

7. What is the amount of \$1 at compound interest for 5 years, at 7% per annum? *Ans.* \$1.40255+

In the above example the first term is \$1, the ratio is \$1.07, and the number of terms is 6.

8. A drover bought 7 oxen, agreeing to pay \$3 for the first ox, \$9 for the second, \$27 for the third, and so on. What did the last ox cost him? *Ans.* \$2187.

460. To find the sum of all the terms, the extremes and ratio being given.

1. Find the sum of a geometrical series in which the first term is 2, the last term 512, and the ratio 4.

OPERATION.

$8 + 32 + 128 + 512 + 2048 = 2728 =$ *Four times* the sum of all the terms.

But $2 + 8 + 32 + 128 + 512 = 682 =$ *Once* the sum of all the terms.

Hence, by subtracting, we get $2048 - 2 = 2046 =$ *Three times* the sum of all the terms.

Dividing by 3, the ratio less 1, $2046 \div 3 = 682 =$ *Once* the sum of all the terms.

$512 \times 4 = 2048 - 2 = 2046 \div 3 = 682$, sum. Ans.

SOLUTION.— If we take the series 2, 8, 32, 128, 512, in which the ratio is 4, multiply each term by the ratio, and add the terms thus multiplied, we shall have the result shown in the operation.

The subtraction is performed by taking the *lower line* or series from the *upper*. All the terms cancel except 2048 and 2. Taking their difference, which is 3 times the sum, and dividing by 3, the ratio less 1, we must have the sum of all the terms.

RULE. — *Multiply the greater extreme by the ratio, subtract the less extreme from the product, and divide the remainder by the ratio less* 1.

Let every *decreasing* series be inverted, and the first term called the last; then the ratio will be *greater* than a unit. If the series is *infinite*, the first term is a cipher.

2. The first term is 2, the last term 486, and the ratio 3. What is the sum of all the terms? *Ans.* 728.

3. The first term is 4, the last term 262144, and the ratio is 4. What is the sum of the series?

Ans. 349524.

4. The first term of a descending series is 162, the last term 2, and the ratio $\frac{1}{3}$. What is the sum? *Ans.* 242.

5. What is the value of $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$, etc., to infinity?

Ans. $\frac{1}{4}$.

6. The extremes are 3 and 384, and the ratio is 2. What is the sum of the series? *Ans.* 765.

7. If the extremes are 5 and 1080, and the ratio is 6, what is the sum of the series?

8. If the first term is $4\frac{4}{5}$, the last term $\frac{8}{405}$, and the ratio $\frac{1}{3}$, what is the sum of the series? *Ans.* $7\frac{77}{405}$.

461. Given the first term, the ratio, and the number of terms, to find the sum of the series.

1. The first term of a geometrical progression is 4, the ratio 3, and the number of terms 6. What is the sum of the series?

OPERATION.

$$4 \times 3^5 = \text{last term.}$$

$$\frac{4 \times 3^5 \times 3 - 4}{3-1} = \frac{4 \times 3^6 - 4}{3-1} = \frac{4(3^6 - 1)}{3-1} = 1456, \textit{ Ans.}$$

SOLUTION. — We have all the conditions of **460**, except that the last term is needed. Since the first term, the ratio, and number of terms are given, we find the last term by **459**. The problem is then similar to **460**, and is solved in the same way.

RULE. — *Raise the ratio to a power indicated by the number of terms, and subtract* 1 *from the result; then multiply this remainder by the first term, and divide the product by the ratio less* 1.

2. The first term is 7, the ratio 3, and the number of terms 4. What is the sum of the series? *Ans.* 280.

3. The first term is 375, the ratio $\frac{1}{5}$, and the number of terms 4. What is the sum of the series?

4. The first term is 175, the ratio 1.06, and the number of terms 5. What is the sum of the series?

Ans. 986.49+.

5. The first term is 4, the ratio 5, the number of terms 5. What is the sum of the series?

6. What yearly debts can be discharged by monthly payments, the first being $2, the second $6, and the third $18, and so on in geometrical progression?

Ans. $531440.

7. If a grain of wheat produces 7 grains, and these are sown the second year, each yielding the same increase, how many bushels will be produced at this rate in 12 years, if 1000 grains make a pint?

Ans. 252315 bu. $4\frac{1}{2}$ qt.

8. Six persons of the Morse family came to this country 200 years ago. Suppose that their number has *doubled every 20* years since, what would it be now?

COMPOUND INTEREST BY GEOMETRICAL PROGRESSION.

462. We have seen (**318**) that if any sum at compound interest is multiplied by the amount of $1 for the given interval, the product will be the amount of the given sum or principal at the end of the first interval; and that this amount constitutes a new principal for the second interval, and so on for a third, fourth, or any other interval. Hence,

463. A question in compound interest constitutes a geometrical progression, whose first term is the principal; the common multiplier or ratio is 1 plus the rate per cent for one interval; the number of terms is equal to the number of intervals + 1; and the last term is the amount of the given principal for the given time.

All the usual cases of compound interest, and discount computed at compound interest, can therefore be solved by the rules for geometrical progression.

EXAMPLES.

464. 1. Find the amount of $250 for 4 years, at 6% compound interest.

OPERATION.

$$\$250 \times 1.06^4 = \$250 \times 1.262477 = \$316.21925.$$

SOLUTION. — Here we have $250, the first term, 1.06, the ratio, and 5, the number of terms, to find the last term. Then by **459** we find the last term, which is the amount required.

2. What is the amount of $350 in 4 years, at 6% per annum, compound interest? *Ans.* $441.86.

3. Of what principal is $150 the compound interest for 2 years, at 7%?

4. In how many years will $40 amount to $53.24, at 10% compound interest? Ans. 3 years.

ANNUITIES.

465. An **Annuity** is literally a sum of money which is payable annually. The term is, however, applied to a sum which is payable at any equal intervals, as monthly, quarterly, semi-annually, etc. Annuities are of three kinds: *Certain*, *Contingent*, and *Perpetual*.

NOTE. — The term *interval* will be used to denote the time between payments.

466. A **Certain Annuity** is one whose period of continuance is definite or fixed.

467. A **Contingent Annuity** is one whose time of commencement, or ending, or both, is uncertain; and hence the period of its continuance is uncertain.

468. A **Perpetual Annuity** or **Perpetuity** is one which continues forever.

469. Each of these kinds is subject, in reference to its commencement, to the following conditions:

1. It may be *deferred*, i.e. it is not to be entered upon until after a certain period of time. 2. It may be *reversionary*, i.e. it is not to be entered upon until after the death of a certain person, or the occurrence of some certain event. 3. It may be *in possession*, i.e. it is to be entered upon at once.

470. An **Annuity in Arrears** or **Forborne** is one on which the payments were not made when due. Interest is to be reckoned on each payment of an annuity in arrears, from its maturity, the same as on any other debt.

ANNUITIES AT SIMPLE INTEREST.

471. In reference to an annuity at simple interest, we observe:

1. The first payment becomes due at the end of the first interval, and hence will bear interest until the annuity is settled.

2. The second payment becomes due at the end of the second interval, and hence will bear interest for one interval less than the first payment.

3. The third payment will bear interest for one interval less than the second; and so on to any number of terms. Hence,

4. All the payments being settled at one time, each will be less than the preceding, by the interest on the annuity for one interval.

Therefore, they will constitute a descending arithmetical progression, whose *first term* is the *annuity plus its interest* for as many intervals less one as intervene between the commencement and settlement of the annuity; the *common difference* is the *interest* on the annuity for one interval; the *number of terms* is the *number of intervals* between the commencement and settlement of the annuity; and the *last term* is the *annuity* itself.

472. The rules in Arithmetical Progression will solve all problems in annuities at simple interest.

EXAMPLES.

1. A man works for a farmer one year and six months, at $20 per month, payable monthly; and these wages remain unpaid until the expiration of the whole term of service. How much is due to the workman, allowing simple interest at 6% per annum?

OPERATION.

$\$20 + \$.10 \times 17 = \$21.70$, 1st term.

$\frac{\$20 + \$21.70}{2} \times 18 = \$375.30$, sum.

SOLUTION.— Here the last month's wages, $20, is the last term; the number of months, 18, is the number of *terms;* and the interest on 1 month's wages, $.10, is the common *difference;* and since the first month's wages has been on interest

17 months, the progression is a descending series. Then, by **451**, we find the first term, which is the amount of the first month's wages for 17 months; and by **454** we find the sum of the series, which is the sum of all the wages and interest.

2. A father deposits annually for the benefit of his son, commencing with his tenth birthday, such a sum that on his 21st birthday the first deposit at simple interest amounts to $210, and the sum due his son to $1860. How much is the deposit, and at what rate per cent is it deposited?

OPERATION.

$$\frac{\$1860 \times 2 - \$210 \times 12}{12} = \$100, \text{ deposit.}$$

$$\frac{210 - 100}{11} = 10\%, \text{ rate.}$$

SOLUTION. — Here the $210, the amount of the first deposit, is the first term; 12, the number of deposits, is the number of terms; and $1860, the amount of all the deposits and interests, is the sum of the series. By **454**, we find the last term to be $100, which is the annual deposit; and by **452** we find the common difference to be $10, which is the annual rate per cent.

3. What is the amount of an annuity of $150 for $5\frac{1}{2}$ years, payable quarterly, at $1\frac{1}{2}\%$ per quarter?

Ans. $3819.75.

4. In what time will an annual pension of $500 amount to $3450, at 6% simple interest?

Ans. 6 years.

5. Find the rate per cent at which an annuity of $6000 will amount to $59760 in 8 years, at simple interest?

Ans. 7%.

6. What is the present worth of an annuity of $300 for 5 years, at 6%?

7. What is the present worth of an annuity of $500 for 10 years, at 10%?

ANNUITIES AT COMPOUND INTEREST.

473. An Annuity at compound interest constitutes a geometrical progression whose *first term* is the *annuity* itself; the *common multiplier* is 1 *plus the rate per cent* for one interval expressed decimally; the *number of terms* is the *number of intervals* for which the annuity is taken; and the *last term* is the *first term multiplied by* 1 *plus the rate per cent for one interval raised to a power* 1 *less than the number of terms.*

474. The **Present Value of an Annuity** is such a sum as would produce, at compound interest, at a given rate, the same amount as the sum of all the payments of the annuity at compound interest.

Hence, to find the present value; — *First find the amount of the annuity at the given rate and for the given time, by* **461**; *then find the present value of this amount by dividing it by the amount of* $1 *at compound interest.*

The present value of a *reversionary annuity* is that principal which will amount, at the time the reversion expires, to what will then be the present value of the annuity.

The present value of a *perpetuity* is a sum whose interest equals the annuity.

475. Hence, it will be seen that all questions in Annuities at compound interest can be solved by the rules of Geometrical Progression, by substituting for *the terms* given the corresponding terms used in *Progression.*

EXAMPLES.

1. What is the amount of an annuity of $500 which is 7 years in arrears, at 6% compound interest?

OPERATION.

$$\frac{\$500 \times (1.06^7 - 1)}{1.06 - 1} = \$251.815 \div .06 = \$4196.91\tfrac{2}{3}, \textit{Ans.}$$

SOLUTION. — The payment now due, $500, is the first term of a geometrical ratio, 1.06, the amount of $1 for 1 year, is the ratio, and 7 the number of terms. Solving by **461**, we find the sum of the series, which is the amount of the annuity, to be $4196.91⅔.

2. An annual pension of $500 is in arrears 10 years. What is the amount now due, allowing 6% compound interest? *Ans.* $6590.40.

NOTE. — Consult the compound interest table. The amount of $1 at 6% for 10 years will be equal to the 10th power of 1.06.

3. What is the present worth of the annuity in Ex. 1?

OPERATION.

$$\$4196.91\tfrac{2}{3} \div \$1.50363 = \$2791.18+.$$

SOLUTION. — The amount of the annuity is $4196.91⅔. The amount of $1 for 7 years at 6% is 1.50363. Hence the present worth is $4196.91\tfrac{2}{3} \div 1.50363 = \$2791.18+$, *Ans.*

4. Find the annuity whose amount for 5 years, at 6% compound interest, is 2818.55. *Ans.* $500.

5. What is the present value of a reversionary lease of $100 commencing 14 years hence and to continue 20 years, compound interest at 5%. *Ans.* $629.426.

6. Allowing 6% compound interest on an annuity of $200, which is in arrears 20 years, what is its present amount? *Ans.* $7357.11.

7. What is the present worth of an annuity of $500 for seven years, at 6% compound interest?

8. An annuity of $200 for 12 years is in reversion 6 years. What is its present worth, compound interest at 6%? Ans. $1182.05+.

MENSURATION.*

476. Mensuration is the process of finding the number of units in extension.

477. Extension denotes that property of bodies, by virtue of which they occupy definite portions of space. Its dimensions are length, breadth, and thickness.

478. A point is that which has position only.

479. Direction is relative position of points.

480. A **line** has length, but neither breadth, nor thickness. A **surface** has length and breadth, but no thickness. A **solid** has length, breadth, and thickness.

LINES AND ANGLES.

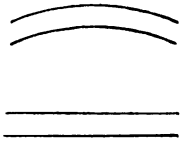
481. A Straight Line is a line that does not change its direction. It is the shortest distance between two points.

482. A broken or crooked line is one made up of two or more straight lines. It changes its direction at one or more points.

483. A Curved Line changes its direction at every point.

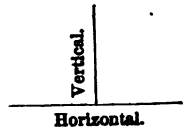
* *Some of* the problems under Mensuration have been explained before as *applications of* Denominate Numbers, Involution, Evolution, etc. They are *here repeated.*

484. Parallel Lines have the same direction; and being in the same plane and equally distant from each other, they can never meet.



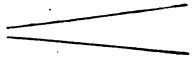
485. A Horizontal Line is a line parallel to the horizon or water level.

486. A Perpendicular Line is a straight line drawn to meet another straight line, so as to incline no more to the one side than to the other.

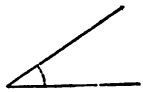


A perpendicular to a horizontal line is called a *vertical* line.

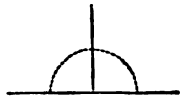
487. Oblique Lines approach each other, and will meet if sufficiently extended.



488. An **Angle** is the opening between two lines that meet each other in a common point, called the *vertex*.



489. A **Right Angle** is an angle formed by two lines perpendicular to each other.



A right angle is always equal to 90°.

490. An **Obtuse Angle** is greater than a right angle.

It may be equal to any number of degrees more than 90° and less than 180°. At 180° the two lines forming the angle merge into one, and become a straight line.

491. An **Acute Angle** is less than a right angle.

An acute angle may be any number of degrees less than 90°.

PLANE FIGURES.

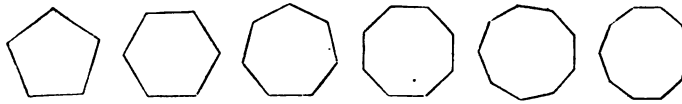
492. A **Plane Figure** is a portion of a plane surface bounded by straight or curved lines.

493. A **Polygon** is a plane figure bounded by straight lines.

494. The **Perimeter** of a polygon is the *sum* of its sides.

495. The **Area** of a plane figure is the surface included within the lines which bound it.

1. A *regular polygon* has all its sides and all its angles equal.
2. A polygon of three sides is called a triangle; of four sides, a quadrilateral; of five sides, a *pentagon*, etc.



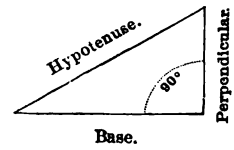
Pentagon. Hexagon. Heptagon. Octagon. Nonagon. Decagon.

TRIANGLES.

496. A **Triangle** is a plane figure bounded by three sides, and having three angles.

497. A **Right-angled Triangle** is a triangle having one right angle.

498. The **Hypotenuse** of a right-angled triangle is the side opposite the right angle.



499. The **Base** of a triangle, or of any plane figure, is the side on which it may be supposed to stand.

500. The **Perpendicular** of a right-angled triangle *is the* side which forms a right angle with the base.

501. The **Altitude** of a triangle is a line drawn perpendicular to the base from the angle opposite.

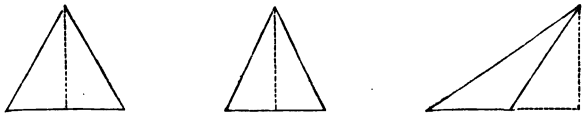
1. The dotted *vertical* lines in the figures represent the *altitude.*
2. Triangles are named from the relation both of their sides and angles.

502. An **Equilateral Triangle** has its three sides equal.

503. An **Isosceles Triangle** has only two of its sides equal.

504. A **Scalene Triangle** has all of its sides unequal.

FIG. 1. FIG. 2. FIG. 3.



Equilateral. Isosceles. Scalene.

505. An **Equiangular Triangle** has three equal angles. (Fig. 1.)

506. An **Acute-angled Triangle** has three acute angles. (Fig. 2.)

507. An **Obtuse-angled Triangle** has one obtuse angle. (Fig. 3.)

EXAMPLES.

508. The base and altitude of a triangle being given to find its area.

1. Find the area of a triangle whose base is 26 ft. and altitude 14.5 ft.

OPERATION.

$14.5 \times 26 \div 2 = 188\frac{1}{2}$ sq. ft. Or, $26 \times \frac{14.5}{2} = 188\frac{1}{2}$ sq. ft., *area.*

RULE. — *Divide the product of the base and altitude by 2, or multiply the base by one half the altitude.*

2. What is the area of a triangle whose altitude is 10 yd. and base 40 ft.? Ans. 600 sq. ft.

Find the area of a triangle:

3. Whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.

Ans. $42\frac{3}{16}$ sq. ft.

4. Whose base is 25.01 ch. and altitude 18.14 ch.

5. What will be the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch., at $60 an acre?

509. The area and one dimension being given to find the other dimension.

1. What is the base of a triangle whose area is 189 sq. ft. and altitude 14 ft.?

OPERATION.

189 sq. ft. $\times$ 2 $\div$ 14 = 27 ft., *base.*

RULE. — *Double the area, then divide by the given dimension.*

2. Find the altitude of a triangle whose area is $20\frac{1}{4}$ sq. ft. and base 3 yards.

Find the other dimension of the triangle:

3. When the area is 65 sq. in. and the altitude 10 in. *Ans.* 13 in.

4. When the base is 42 rd. and the area 588 sq. rd.

5. I paid $1050 for a piece of land in the form of a triangle, at the rate of $5\frac{1}{4}$ per square rod. If the base is 8 rd., what is its altitude? *Ans.* 50 rods.

510. The three sides of a triangle being given to find its area.

1. Find the area of a triangle whose sides are 30, 40, and 50 ft.

OPERATION.

$(30 + 40 + 50) \div 2 = 60$; $60 - 30 = 30$; $60 - 40 = 20$;

$60 - 50 = 10$. $\sqrt{60 \times 30 \times 20 \times 10} = 600$ sq. ft., *area.*

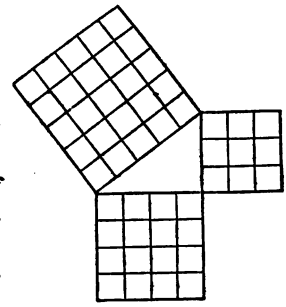
RULE. — *From half the sum of the three sides subtract each side separately; multiply the half-sum and the three remainders together; the square root of the product is the area.*

2. What is the area of an isosceles triangle whose base is 20 ft., and each of its equal sides 15 ft.? Ans. 111.8 sq. ft.

3. How many acres are there in a field in the form of an equilateral triangle whose sides measure 70 rods? *Ans.* 13 A. 41.76 P.

4. The roof of a house 30 ft. wide has the rafters on one side 20 ft. long, and on the other 18 ft. long. How many square feet of boards will be required to board up both gable ends?

511. The following principles relating to *right-angled* triangles have been established by Geometry:



1. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

2. *The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypotenuse diminished by the square of the other side.*

512. To find the hypotenuse.

1. The base of a right-angled triangle is 12, and the perpendicular 16. What is the length of the hypotenuse?

OPERATION.

$12^2 + 16^2 = 400$ (**511**, 1). $\sqrt{400} = 20$, *hypotenuse.*

RULE. — *Extract the square root of the sum of the squares of the base and the perpendicular; and the result is the hypotenuse.*

2. The foot of a ladder is 15 ft. from the base of a building, and the top reaches a window 36 ft. above the base. What is the length of the ladder? *Ans.* 39 ft.

3. If the gable end of a house 40 ft. wide is 16 ft. high, what is the length of the rafters?

4. A park 25 ch. long and 23 ch. wide has a walk running through it from opposite corners in a straight line. What is the length of the walk? *Ans.* 33.97+ ch.

5. A room is 20 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from one of the lower corners to the opposite upper corner? Ans. 28 ft. 3.36 in.

513. To find the base or perpendicular.

1. The hypotenuse of a right-angled triangle is 35 ft., and the perpendicular 28 ft. What is the base?

OPERATION.

$35^2 - 28^2 = 441$ (511, 2). $\sqrt{441} = 21$ ft., *base.*

RULE. — *Extract the square root of the difference between the square of the hypotenuse and the square of the given side; and the result is the required side.*

2. The hypotenuse of a right-angled triangle is 53 yd., and the base 84 ft. Find the perpendicular.

3. A line reaching from the top of a precipice 120 ft. high, on the bank of a river, to the opposite side is 380 ft. long. How wide is the river? *Ans.* 360 ft. 6+ in.

4. A ladder 52 ft. long stands against the side of a building. How many feet must it be drawn out at the bottom that the top may be lowered 4 ft.? *Ans.* 20 ft.

QUADRILATERALS.

514. A **Quadrilateral** is a plane figure bounded by four straight lines, and having four angles.

There are three kinds of quadrilaterals, the *Parallelogram*, *Trapezoid*, and *Trapezium*.

515. A **Parallelogram** is a quadrilateral which has its opposite sides parallel.

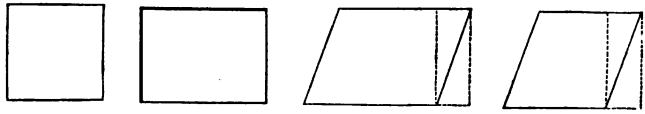
There are four kinds of parallelograms, the *Square*, *Rectangle*, *Rhomboid*, and *Rhombus*.

516. A **Rectangle** is any parallelogram having its angles right angles.

517. A **Square** is a rectangle whose sides are equal.

518. A **Rhomboid** is a parallelogram whose opposite *sides only* are equal, but whose angles are not right *angles*.

519. A **Rhombus** is a parallelogram whose sides are *all* equal, but whose angles are not right angles.



Square. Rectangle. Rhomboid. Rhombus.

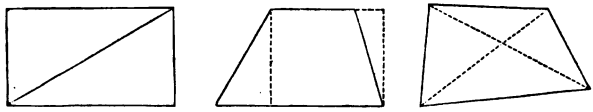
520. A **Trapezoid** is a quadrilateral, two of whose sides are *parallel* and two *oblique*.

521. A **Trapezium** is a quadrilateral having no two sides parallel.

522. The **Altitude** of a parallelogram or trapezoid is the perpendicular distance between its *parallel* sides.

The *vertical* dotted lines in the figures represent the altitude.

523. A **Diagonal** of a plane figure is a straight line joining the vertices of two angles not adjacent.



Parallelogram. Trapezoid. Trapezium.

EXAMPLES.

524. To find the area of any parallelogram.

1. Find the area of a parallelogram whose base is 16.25 ft. and altitude 7.5 ft.

OPERATION.

$16.25 \times 7.5 = 121.875$ sq. ft., *area*.

RULE. — *Multiply the base by the altitude.*

2. The base of a rhombus is 10 ft. 6 in., and its altitude 8 ft. What is its area?

3. How many acres are there in a piece of land in the form of a rhomboid, the base being 8.75 ch. and altitude 6 ch.? Ans. $5\frac{1}{4}$ A.

525. To find the area of a trapezoid.

1. Find the area of a trapezoid whose parallel sides are 23 ft. and 11 ft., and the altitude 9 ft.

OPERATION.

$\overline{23 \text{ ft.} + 11 \text{ ft.}} \div 2 = 17 \text{ ft.}$; $17 \text{ ft.} \times 9 = 153$ sq. ft., *area.*

RULE. — *Multiply one half the sum of the parallel sides by the altitude.*

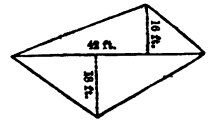
2. What is the area of a trapezoid whose parallel sides are 178 ft. and 146 ft., and the altitude 69 ft. *Ans.* 11178 sq. ft.

3. How many square feet are there in a board 16 ft. long, 18 in. wide at one end and 25 in. wide at the other end?

4. One side of a quadrilateral field measures 38 rd.; the side opposite and parallel to it measures 26 rd., and the distance between the two sides is 10 rd. Find the area. *Ans.* 2 A.

526. To find the area of a trapezium.

1. Find the area of a trapezium whose diagonal is 42 ft. and perpendiculars to this diagonal, as in the diagram, are 16 ft. and 18 ft.



OPERATION.

$(\overline{18 \text{ ft.} + 16 \text{ ft.}} \div 2) \times 42 = 714$ sq. ft., *area.*

RULE. — *Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of the opposite angles.*

2. Find the area of a trapezium whose diagonal is 35 ft. 6 in., and the perpendiculars to this diagonal 9 ft. and 12 ft. 6 in.

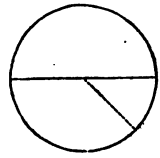
3. How many acres are there in a quadrilateral field whose diagonal is 80 rd. and the perpendiculars to this diagonal 20.453 and 50.832 rd.?

1. To find the area of any *regular* polygon, multiply its perimeter, or the sum of its sides, by the perpendicular falling from its center to one of its sides.

2. To find the area of an *irregular* polygon, divide the figure into triangles and trapeziums, and find the area of each separately. The sum of these areas will be the area of the whole polygon.

CIRCLES.

527. A **Circle** is a plane figure bounded by a curved line, called the *circumference*, every point of which is equally distant from a point within called the *center*.



528. The **Diameter** of a circle is a line passing through its center, and terminated at both ends by the circumference.

529. The **Radius** of a circle is a line extending from its center to any point in the circumference. It is one half the diameter.

EXAMPLES.

530. When either the diameter or the circumference of a circle is given, to find the other dimension of it.

1. Find the circumference of a circle whose diameter is 20 in.

OPERATION. — 20 in. × 3.1416 = 62.832 in. = 5 ft. 2.832 in., *circumference.*

2. Find the diameter of a circle whose circumference is 62.832 ft.

OPERATION. — 62.832 ft. ÷ 3.1416 = 20 ft., *diameter.*

RULE. — I. *Multiply the diameter by* 3.1416; *the product is the circumference.*

II. *Divide the circumference by* 3.1416; *the quotient is the diameter.*

3. What is the circumference of a wheel 5 ft. 6 in. in diameter?
4. Find the diameter of a wheel whose circumference is 50 ft.
5. What is the diameter of a tree whose girt is 18 ft. 6 in.?
6. Find the length of a tire that will band a wheel 7 ft. 9 in. in diameter. *Ans.* 24 ft. 4+ in.
7. The diameter of a cylinder is 8 ft. 6 in. Find its girt.
8. What is the radius of a circle whose circumference is 31.416 ft.?

531. To find the area of a circle, when both its diameter and circumference are given, or when either is given.

1. Find the area of a circle whose diameter is 10 ft. and circumference 31.416 ft.

OPERATION. — 31.416 ft. $\times \overline{10 \div 4} = 78.54$ sq. ft., *area.*

2. Find the area of a circle whose diameter is 10 ft.

OPERATION. — $(10 \text{ ft.})^2 \times .7854 = 78.54$ sq. ft., *area.*

3. Find the area of a circle whose circumference is 31.416 ft.

OPERATION. — 31.416 ft. $\div 3.1416 = 10$ ft., *diam.;* $(10 \text{ ft.})^2 \times .7854 = 78.54$ sq. ft., *area.*

RULE. — I. *Multiply the square of its diameter by .7854.*

II. *Multiply $\frac{1}{4}$ of its diameter by the circumference.*

4. What is the area of a circular pond whose circumference is 200 chains? *Ans.* 318.3 A.

5. The distance around a circular park is $1\frac{1}{2}$ miles. How many acres does it contain? *Ans.* 114.59 A.

6. Find the area of the largest circle that can be drawn by using as a radius a string 20 in. long.

532. To find the diameter or circumference of a circle, when the area is given.

1. What is the diameter of a circle whose area is 1319.472?

OPERATION. — $1319.472 \div .7854 = 1680$; $\sqrt{1680} = 40.987+$, *diam.*

2. What is the circumference of a circle whose area is 19.635?

OPERATION. — $19.635 \div 3.1416 = 6.25$; $\sqrt{6.25} = 2.5$, *radius;* $2.5 \times 2 \times 3.1416 = 15.708$, *circumference.*

RULE. — I. *Divide the area by .7854 and extract the square root of the quotient; the result is the diameter.*

II. *Divide the area by 3.1416 and extract the square root of the quotient; the result is the radius. The circumference is obtained by* **530.** Or,

III. Divide the area by .07958, and extract the square root of the quotient.

3. The area of a circular lot is 38.4846 square rods. What is its diameter?

4. The area of a circle is 286.488 square feet. What are its diameter and the circumference?

5. The area of a circular lot is 1 acre. What is its diameter?

SUMMARY OF CIRCLES.

1. The *diameter* of any circle

Multiplied / Divided } by { 3.1416, the product / .3183, the quotient } = the *circumference.*

2. The *radius* of any circle

Multiplied / Divided } by { 6.28318, the product / .15915, the quotient } = the *circumference.*

3. The *square* of the diameter of any circle

Multiplied / Divided } by { .7854, the product / 1.2732, the quotient } = the *area.*

4. The *circumference* of any circle

Multiplied / Divided } by { .3183, the product / 3.1416, the quotient } = the *diameter.*

5. The *square* of the circumference of any circle

Multiplied / Divided } by { .07958, the product / 12.5663, the quotient } = the *area.*

6. The *area* of any circle

Multiplied / Divided } by { 1.2732, the product / .7854, the quotient } = the *square of the diam.*

7. { The square of the radius of any circle × 3.1416 / Half the circumference of a circle × ½ its diameter / *Square* of the circumference of a circle × .07958 } = *area.*

SOLIDS.

533. A **Solid** or **Body** has three dimensions, length, breadth, and thickness.

The planes which bound it are called its *faces*, and their intersections, its *edges*.

PRISMS AND CYLINDERS.

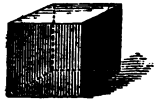
534. A **Prism** is a solid whose ends are equal and parallel polygons, and its sides parallelograms.

Prisms take their names from the *forms* of their bases, as *triangular*, *quadrangular*, *pentagonal*, etc.

535. The **Altitude** of a prism is the perpendicular distance between its bases.

Parallelopipedon.

536. A **Parallelopipedon** is a prism bounded by six parallelograms, the opposite ones being parallel and equal.



Cube.

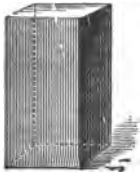
537. A **Cube** is a parallelopipedon whose faces are all equal squares.

538. A **Cylinder** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

1. A cylinder is conceived to be generated by the revolution of a rectangle about one of its sides as an axis.

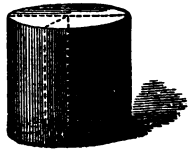
2. The line joining the centers of the bases, or ends, of the cylinder is its *altitude*, or *axis*.

Triangular Prism.



Quadrangular Prism.

Pentagonal Prism.

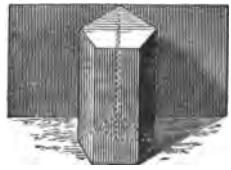


Cylinder.

EXAMPLES.

539. To find the convex surface of a prism or cylinder.

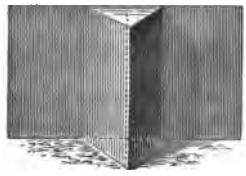
1. Find the area of the convex surface of a prism whose altitude is 7 ft., and its base a pentagon, each side of which is 4 ft.



OPERATION.

4 ft. × 5 = 20 ft., *perimeter.*
20 ft. × 7 = 140 sq. ft., *convex surface.*

2. Find the area of the convex surface of a triangular prism whose altitude is 8½ ft., and the sides of its base 4, 5, and 6 ft., respectively.



OPERATION.

4 ft. + 5 ft. + 6 ft. = 15 ft., *perimeter.*
15 ft. × 8½ = 127½ sq. ft., *convex surface.*

3. Find the area of the convex surface of a cylinder whose altitude is 2 ft. 5 in., and the circumference of its base 4 ft. 9 in.

OPERATION.

2 ft. 5 in. = 29 in.; 4 ft. 9 in. = 57 in.
57 × 29 = 1653 sq. in. = 11 sq. ft. 69 sq. in., *convex surface.*

RULE. — I. *To find the convex surface, multiply the perimeter of the base by the altitude.*

II. *To find the* ENTIRE *surface, add the area of the bases or ends.*

4. If a gate 8 ft. high and 6 ft. wide revolves upon a point in its center, what is the entire surface of the cylinder described by it?

5. Find the superficial contents, or entire surface, of a parallelopipedon 8 ft. 9 in. long, 4 ft. 8 in. wide, and 3 ft. 3 in. high.

6. What is the entire surface of a cylinder formed by the revolution about one of its sides of a rectangle that is 6 ft. 6 in. long and 4 ft. wide? *Ans.* 263.89 sq. ft.

7. Find the entire surface of a prism whose base is an equilateral triangle, the perimeter being 18 ft., and the altitude 15 ft.

540. To find the volume of any prism or cylinder.

1. Find the volume of a triangular prism whose altitude is 20 ft., and each side of the base 4 ft.

OPERATION.

The area of the base is 6.928 sq. ft.
6.928 sq. ft. × 20 = 138.56 cu. ft., *volume.*

2. Find the volume of a cylinder whose altitude is 8 ft. 6 in., and the diameter of its base 3 ft.

OPERATION.

$3^2 \times .7854 = 7.0686$ sq. ft., *area of base.*
7.0686 sq. ft. × 8.5 = 60.083 cu. ft., *volume.*

RULE. — *Multiply the area of the base by the altitude.*

3. What is the volume of a parallelopipedon whose base is 9.8 ft. by 7.5 ft., and its height 5 ft. 3 in.?

4. What is the volume of a log 18 ft. long and $1\frac{1}{2}$ ft. in diameter?

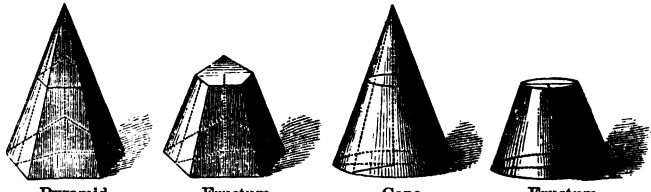
5. Find the solid contents of a cube whose edges are 6 ft. 6 in.

6. Find the cost of a piece of timber 18 in. square and 40 ft. long, at $.30 a cubic foot. *Ans.* $27.

PYRAMIDS, CONES, AND SPHERES.

541. A **Pyramid** is a body having for its base a polygon, and for its other faces three or more triangles, which terminate in a common point called the *vertex.*

Pyramids, like prisms, take their names from their bases, and are called *triangular, square, or quadrangular, pentagonal,* etc.



Pyramid. Frustum. Cone. Frustum.

542. A **Cone** is a body having a circular base, and whose convex surface tapers uniformly to the vertex.

A cone is a body conceived to be formed by the revolution of a right-angled triangle about one of its sides containing the right angle.

543. The **Altitude** of a pyramid or of a cone is the perpendicular distance from its vertex to the plane of its base.

544. The **Slant Height** of a *pyramid* is the perpendicular distance from its vertex to one of the sides of the base; of a *cone*, a straight line from the vertex to the circumference of the base.

545. The **Frustum** of a pyramid or cone is that part which remains after cutting off the top by a plane parallel to the base.

546. A **Sphere** is a body bounded by a uniformly curved surface, all the points of which are equally distant from a point within called the *center*.

547. The **Diameter** of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by its surface.

548. The **Radius** of a sphere is a straight line drawn from the center to any point in the surface.

EXAMPLES.

549. To find the convex surface of a pyramid or cone.

1. Find the convex surface of a triangular pyramid, the slant height being 16 ft., and each side of the base 5 ft.

OPERATION.

$(5 \text{ ft.} + 5 \text{ ft.} + 5 \text{ ft.}) \times \overline{16 \text{ ft.} \div 2} = 120$ sq. ft., convex surface.

2. Find the convex surface of a cone whose diameter is 17 ft. 6 in., and the slant height 30 ft.

OPERATION.

17.5 ft. × 3.1416 = 54.978 ft., *circumference;*
54.978 ft. × $\overline{30 \div 2}$ = 824.67 sq. ft., *convex surface.*

RULE. — I. *To find the convex surface multiply the perimeter or circumference of the base by one half the slant height.*

II. *To find the entire surface, add to this product the area of the base.*

3. Find the entire surface of a pyramid whose base is 8 ft. 6 in. square, and its slant height 21 ft.

4. Find the entire surface of a cone, the diameter of whose base is 6 ft. 9 in., and the slant height 45 ft. *Ans.* 512.9 sq. ft.

5. Find the cost of painting a church spire, at $.25 a sq. yd., whose base is a hexagon 5 ft. on each side, and the slant height 60 ft.

550. To find the volume of a pyramid or of a cone.

1. What is the volume, or solid contents, of a square pyramid whose base is 6 ft. on each side, and its altitude 12 ft. ?

OPERATION.

6 ft. × 6 ft. × $\overline{12 \text{ ft.} \div 3}$ = 144 cu. ft., *volume.*

2. Find the volume of a cone, the diameter of whose base is 5 ft., and its altitude 10½ ft.

OPERATION.

5^2 ft. × .7854 × $\overline{10\frac{1}{2} \div 3}$ = 68.72¼ cu. ft., *volume.*

RULE. — *Multiply the area of the base by one third the altitude.*

3. Find the solid contents of a cone whose altitude is 24 ft., and the diameter of its base 30 inches.

4. What is the cost of a triangular pyramid of marble, whose altitude is 9 ft., each side of the base being 3 ft., at $2½ per cubic foot? *Ans.* $29.23.

5. Find the volume and the entire surface of a pyramid whose *base is a* rectangle 80 ft. by 60 ft., and the edges which meet at the *vertex* are 130 ft. Ans. 192000 cu. ft., volume.

551. To find the convex surface of a frustum of a pyramid or cone.

1. What is the convex surface of a frustum of a square pyramid whose slant height is 7 ft., each side of the greater base 4 ft., and of the less base 18 in.?

OPERATION.

The *perimeter* of the greater base is 16 ft., of the less 6 ft.

$\overline{16 \text{ ft.} + 6 \text{ ft.}} \times \overline{7 \div 2} = 77$ sq. ft., *convex surface.*

RULE. — I. *To find the convex surface, multiply the sum of the perimeters, or circumferences, by one half the slant height.*

II. *To find the entire surface, add to this product the areas of both ends, or bases.*

2. Find the convex surface of a frustum of a cone whose slant height is 15 ft., the circumference of the lower base 30 ft., and of the upper base 16 ft.

3. How many square yards are there in the convex surface of a frustum of a pyramid, whose bases are heptagons, each side of the lower base being 8 ft., and of the upper base 4 ft., and the slant height 55 ft.? *Ans.* $256\frac{2}{3}$ sq. yd.

552. To find the volume of a frustum of a pyramid or cone.

1. Find the volume of a frustum of a square pyramid whose altitude is 10 ft., each side of the lower base 12 ft., and of the upper base 9 ft.

OPERATION. — $12^2+9^2=225$; $(225+\sqrt{144\times81})\times\overline{10\div3}=1110$ cu. ft.

RULE. — *To the sum of the areas of both bases add the square root of the product, and multiply the result by one third of the altitude.*

2. How many cubic feet are there in the frustum of a cone whose altitude is 6 ft., and the diameters of its bases 4 ft. and 3 ft.?

3. How many cubic feet are there in a piece of timber 30 ft. long, the greater end being 15 in. square, and the less 12 in.?

4. How many cubic feet are there in the mast of a ship, its height being 50 ft., the circumference at one end 5 ft. and at the other 3 ft.?

553. To find the surface of a sphere.

1. Find the surface of a sphere whose diameter is 9 in.

OPERATION. — 9 in. × 3.1416 = 28.2744 in., *circumference.*
28.2744 in. × 9 = 254.4696 sq. in., *surface.*

RULE. — *Multiply the diameter by the circumference of a great circle of the sphere.*

2. What is the surface of a globe 16 in. in diameter?

3. Find the area of the surface of a sphere whose circumference is 31.416 ft. *Ans.* 314.16 sq. ft.

554. To find the volume of a sphere.

1. Find the volume of a sphere whose diameter is 18 in.

OPERATION. — 18 in. × 3.1416 = 56.5488 in., *circumference.*
56.5488 in. × 18 = 1017.8784 sq. in., *surface.*
1017.8784 sq. in. × $\overline{18 \div 6}$ = 3053.6352 cu. in., *volume.*

RULE. — *Multiply the surface by $\frac{1}{6}$ of the diameter, or $\frac{1}{3}$ of the radius.*

2. Find the volume of a globe whose diameter is 30 in.

3. Find the volume of a globe whose circumference is 31.416 ft.

SUMMARY OF SPHERES.

1. The *Surface* = { Circumference × its diam. / Radius² × 12.5664. / Diameter² × 3.1416. / Circumference² × .3183. }

2. The *Volume* = { Surface × $\frac{1}{6}$ its diameter. / Radius³ × 4.1888. / Diameter³ × .5236. / Circumference³ × .0169. }

3. The *Diameter* = { $\sqrt{\text{Of surface}}$ × .5642. / $\sqrt[3]{\text{Of volume}}$ × 1.2407. }

4. The *Circumference* = { $\sqrt{\text{Of surface}}$ × 1.77255. / $\sqrt[3]{\text{Of volume}}$ × 3.8978. }

5. The *Radius* = { $\sqrt{\text{Of surface}}$ × .2821. / $\sqrt[3]{\text{Of volume}}$ × .6204. }

PROMISCUOUS EXAMPLES.

1. One half the sum of two numbers is 800, and one half the difference of the same numbers is 200. What are the numbers? *Ans.* 1000 and 600.

2. What number is that to which, if you add $\frac{2}{5}$ of $\frac{8}{11}$ of itself, the sum will be 61? *Ans.* 55.

3. What part of a day is 3 h. 21 min. 15 sec.? *Ans.* $\frac{161}{1152}$.

4. A commission merchant received 70 bags of wheat, each containing 3 bu. 3 pk. 3 qt. How many bushels did he receive?

5. Four men, A, B, C, and D, are in possession of $1100; A has a certain sum, B has twice as much as A, C has $300, and D has $200 more than C. How many dollars has A? *Ans.* $100.

6. At a certain election, 3000 votes were cast for three candidates, A, B, and C; B had 200 more votes than A, and C had 800 more than B. How many votes were cast for A? *Ans.* 600.

7. What part of $17\frac{1}{2}$ is $3\frac{1}{4}$? *Ans.* $\frac{13}{70}$.

8. A merchant bought a hogshead of liquor for $28.35. How much water must be added to reduce the first cost to 35 cents per gallon? *Ans.* 18 gal.

9. A and B traded with equal sums of money; A gained a sum equal to $\frac{1}{5}$ of his stock; B lost $200, and then he had $\frac{1}{2}$ as much as A. How much was the original stock of each? *Ans.* $500.

10. The difference between $\frac{6}{7}$ and $\frac{7}{8}$ of a number is 10. What is the number? *Ans.* 560.

11. A farmer sold 17 bu. of barley, and 13 bu. of wheat for $31.55; he received for the wheat 35 cents a bushel more than for the barley. What was the price of each per bushel? *Ans.* Barley, $.90; wheat, $1.25.

12. What is the interval of time between March 20, 21 minutes past 3 o'clock P.M., and April 11, 5 minutes past 7 o'clock A.M.? *Ans.* 21 da. 15 h. 44 min.

13. What o'clock is it when the time from noon is $\frac{9}{11}$ of the time to midnight? *Ans.* 5 o'clock 24 min. P.M.

14. What is the least number of gallons that can be shipped in either hogsheads, tierces, or barrels, just filling the vessels, without deficit or excess? *Ans.* 126 gal.

15. A ferryman has four boats: one will carry 8 barrels, another 9, another 15, and another 16. What is the smallest number of barrels that will make full freight for any one and all of the boats?

16. A and B have the same income; A saves $\frac{1}{8}$ of his, but B, by spending $30 a year more than A, at the end of four years finds himself $40 in debt. What is their income, and how much does each spend a year?

Ans. Inc. $160. A spends $140. B, $170.

17. If a load of plaster weighing 1825 lb. costs $2.19, how much is that per ton of 2000 lb.? *Ans.* $2.40.

18. If $2\frac{1}{2}$ yards of cloth $1\frac{2}{5}$ yd. wide cost 3.37\frac{2}{5}$, what will be the cost of $36\frac{1}{2}$ yd. $1\frac{1}{2}$ yd. wide? *Ans.* $52.779.

19. I lend my neighbor $200 for 6 months. How long ought he to lend me $1000 to balance the favor?

20. I bought railroad stock to the amount of $2356.80, and found that the sum invested was 40% of what I had left. What sum had I at first? *Ans.* $8248.80.

21. 20% of $\frac{3}{5}$ of a number is what per cent of $\frac{3}{5}$ of it?

Ans. $12\frac{1}{2}$%.

22. Divide a prize of $10200 among 60 privates, 6 subaltern officers, 3 lieutenants, and a commander, giving to each subaltern double the share of a private, to each lieutenant 3 times as much as to the subaltern, and to the commander double that of a lieutenant. How much is each man's share? *Ans.* Com. $1200; each man, $100.

23. A is 51 miles in advance of B, who is in pursuit of him; A travels 16 miles per hour, and B 19 miles. In how many hours will B overtake A?

24. A mixes 3 gal. water with 12 gal. vinegar, at 50 cts. a gal. What is 1 gal. of the mixture worth? *Ans.* 40 cts.

25. If 240 bushels of wheat are purchased at the rate of 18 bushels for $22½, and sold at the rate of 22½ bushels for $33.75, what is the profit on the whole?

Ans. $60.

26. My horse, wagon, and harness together are worth $169; the wagon is worth 4 times the harness, and the horse is worth double the wagon. What is the value of each? *Ans.* Horse, $104. Wagon, $52. Harness, $13.

27. The shadow of a tree measures 42 ft.; a staff 40 in. in length casts a shadow 18 in. at the same time. What is the height of the tree? *Ans.* 93⅓ ft.

28. If a piece of land 40 rd. long and 4 rd. wide makes an acre, how wide must it be to contain the same if it is but 25 rd. long? *Ans.* 6⅖ rd.

29. A, B, and C are employed to do a piece of work for $26.45; A and B together are supposed to do ⅛⅛ of it; A and C ⅕, and B and C ⅛⅛. If they are paid proportionally, how much must each receive?

Ans. A, $11.90¼; B, $5.29; C, $9.25¾.

30. A certain principal, at compound interest for 5 years, at 6%, will amount to $669.113. In what time will the same principal amount to the same sum at 6% simple interest? *Ans.* 5 yr. 7 mo. 19.3+ da.

31. If 12 oz. of wool make $2\frac{1}{2}$ yd. of cloth that is 6 quarters of a yard wide, how many pounds of wool will it take for 150 yd. of cloth 1 yd. wide?

32. Six persons, A, B, C, D, E, and F, are to share among them \$6300; A is to have $\frac{1}{7}$ of it, B $\frac{1}{5}$, C $\frac{2}{9}$, D is to have as much as A and C together, and the remainder is to be divided between E and F in the proportion of 3 to 5. How much does each one receive?

33. What is the amount of \$200 for 8 yr. at 6% compound interest? *Ans.* \$318.769.

34. A garrison, consisting of 360 men, was provisioned for 6 months; but at the end of 5 months they dismissed so many of the men that the remaining provision lasted 5 months longer. How many men were sent away?

35. I paid \$148.352 for 9728 ft. of pine lumber. How much was that per thousand?

36. Comparing two numbers, 483 was found to be their least common multiple, and 23 their greatest common divisor. What is the product of the numbers compared?

Ans. 11109.

37. Eight workmen, laboring 7 h. a day for 15 da., were able to execute $\frac{1}{8}$ of a job; in how many days can they complete the rest, by working 9 h. a day, if 4 workmen are added to their number? *Ans.* $15\frac{5}{9}$ da.

38. A, B, and C traded in company; A put in \$1 as often as B put in \$3, and B put in \$2 as often as C put in \$5; B's money was in twice as long as C's, and A's twice as long as B's; they gained \$52.50. How much was each man's share of the gain?

Ans. A's, \$12. B's, \$18. C's, \$22.50.

39. If a hall 36 ft. long and 9 ft. wide requires 36 yd. of carpeting 1 yd. wide to cover the floor, how many yards $1\frac{1}{4}$ yd. wide will cover a floor 60 ft. long and 27 ft. wide? *Ans.* 144 yd.

40. A and B found a watch worth $45, and agreed to divide the value of it in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$. How much was each one's share? *Ans.* A's, $20. B's, $25.

41. A man received $33.25 interest on a sum of money, loaned 5 yr. previous, at 7%. What was the sum loaned? *Ans.* $95.

42. The diameter of a ball weighing 32 lb. is 6 in. What is the diam. of a ball weighing 4 lb.? *Ans.* 3 in.

43. Divide $360 in the proportion of 2, 3, and 4.

44. If by working $6\frac{3}{5}$ h. a day a man can accomplish a job in $12\frac{1}{2}$ da., how many days will be required if he works $8\frac{1}{3}$ h. per day? *Ans.* $9\frac{9}{10}$ da.

45. An open court contains 40 sq. yd. How many stones, 9 in. sq., will be required to pave it? *Ans.* 640.

46. A drover paid $76 for calves and sheep, paying $3 for calves and $2 for sheep; he sold 3 of his calves and $\frac{2}{5}$ of his sheep for $23, and by so doing lost 8% on their cost. How many of each did he purchase?

47. If a cistern, $17\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. broad, and 13 ft. deep, holds 546 bbl., how many barrels will a cistern hold which is 16 ft. long, 7 ft. broad, and 15 ft. deep?

48. If 12 men, working 9 h. a day for $15\frac{5}{8}$ da., were able to execute $\frac{3}{8}$ of a job, how many men may be withdrawn, and the rest be finished in 15 da. more, if the laborers are employed only 7 h. a day? *Ans.* 4 men.

49. A general formed his men into a square, that is, an equal number in rank and file, and found that he had 59 men over; and increasing the number in both rank and file by 1 man, he wanted 84 men to complete the square. How many men had he? *Ans.* 5100.

50. Divide $630 among 3 persons, so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two. What is the share of each?

Ans. 1st, $240. 2d, $180. 3d, $210.

51. I bought wheat at $1.50 per bushel, corn at $.75 per bushel, and barley at $.60 per bushel; the wheat cost twice as much as the corn, and the corn twice as much as the barley; of the sum paid, $243 and $\frac{1}{4}$ of the whole was for the wheat, and $153 and $\frac{1}{12}$ of the whole was for the corn. How many bushels of grain did I purchase? *Ans.* 756.

52. I bought a hogshead of molasses for $28, and 7 gal. leaked out. At what rate per gallon must the remainder be sold to gain 20%?

53. 20% of $\frac{4}{5}$ of a number is how many per cent of 2 times $\frac{3}{4}$ of $1\frac{1}{2}$ times the number? *Ans.* $7\frac{1}{9}$.

54. B and C, trading together, find their stock to be worth $3500, of which C owns $2100; they have gained 40% on their first capital. What did each put in?

Ans. B, $1000. C, $1500.

55. If the ridge of a building is 8 ft. above the beams, and the building is 32 ft. wide, what must be the length of the rafters?

56. I paid $375, at the rate of $2\frac{1}{2}$%, for insurance on a cotton factory and the machinery. For what amount was the policy given?

57. If 12 workmen, in 12 da., working 12 h. a day, can make up 75 yd. of cloth, $\frac{3}{4}$ of a yard wide, into articles of clothing, how many yards, 1 yd. wide, can be made up into like articles, by 10 men, working 9 da., 8 h. each day? *Ans.* $23\frac{7}{16}$ yd.

58. In 1 yr. 4 mo. $311.50 amounted to $336.42, at simple interest. What was the rate per cent? *Ans.* 6%.

59. Three persons engage to do a piece of work for $20; A and B estimate that they do $\frac{5}{6}$ of it, A and C that they do $\frac{1}{2}$ of it, and B and C that they do $\frac{2}{3}$ of it. According to this estimate, what part of the $20 should each man receive? *Ans.* A, $6\frac{2}{3}$; B, $10; C, $3\frac{1}{3}$.

60. A grocer sells a farmer 100 lb. of sugar, at $.12 a pound, and makes a profit of 9%; the farmer sells him 100 lb. of beef, at $.06 a pound, and makes a profit of 10%. Who gains more by the trade, and how much more? *Ans.* The grocer gains $.445+ more.

61. A merchant bought goods in Boston to the amount of $1000, and gave his note, dated Jan. 1, 1890, on interest after 3 mo.; six months after the note was given he paid $560, and 5 months after the first payment he paid $406. What was due Aug. 23, 1892?

Ans. $66.63+.

62. If $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's, and $\frac{3}{7}$ of B's is equal to $\frac{2}{5}$ of C's, and $\frac{3}{4}$ of C's is equal to $\frac{4}{5}$ of D's, and D has $45 more than C, how much has each?

Ans. A, $378; B, $336; C, $360; D, $405.

63. A owed B $900, to be paid in 3 years; but at the expiration of 9 mo. A agreed to pay $300 if B would wait long enough for the balance to compensate for the advance. How long should B wait after the expiration of the 3 years? *Ans.* $13\frac{1}{2}$ mo.

64. A certain clerk receives $800 a year; his expenses equal $\frac{5}{11}$ of what he saves. How much of his salary does he save yearly?

65. A merchant sold cloth at $1 per yard, and made 10% profit. What would have been his gain or loss had he sold it at $.87$\frac{1}{2}$ per yard? *Ans.* Loss, $3\frac{3}{4}$%.

66. What is the cube of $\frac{21\frac{9}{11}}{29\frac{1}{11}}$? *Ans.* $\frac{27}{64}$.

67. What is the cube root of $\frac{63}{149\frac{1}{8}}$? *Ans.* $\frac{3}{4}$.

68. On a certain day the mercury in the thermometer stood as follows: from 6 till 10 A.M., at 63°; from 10 A.M. till 1 P.M., 70°; from 1 till 3 P.M., 75°; from 3 till 7 P.M., 73°; from 7 P.M. till 6 A.M. of the next day, 55°.

What was the mean temperature of the day, from sunrise to sunrise? *Ans.* $62\frac{7}{8}°$.

69. A man owes $6480 to his creditors; his debts are in arithmetical progression, the least being $40, and the greatest $500. What is the number of his creditors, and what the common difference between the debts?

Ans. 24 creditors, $20 difference.

70. Two ships sail from the same port; one goes due north 128 miles, and the other due east 72 miles. How far are the ships from each other? *Ans.* 146.86+ miles.

71. If 10 lb. cheese are equal in value to 7 lb. of butter, and 11 lb. of butter to 2 bu. of corn, and 14 bu. of corn to 8 bu. of rye, and 4 bu. of rye to 1 cd. of wood, how many pounds of cheese are equal in value to 10 cd. of wood? *Ans.* 550.

72. A and B traded until they gained 6% on their stock; then $\frac{2}{5}$ of A's gain was $18. If A's stock was to B's as $\frac{2}{5}$ to $\frac{1}{8}$, how much did each gain, and what was the original stock of each?

Ans. { A's gain, $45; stock, $750.
B's " $37.50; " $625.

73. If 20 men, in 21 da., by working 10 h. a day, can dig a trench 30 ft. long, 15 ft. wide, and 12 ft. deep, when the ground is estimated at 3 degrees of hardness, how many men, in 25 da., by working 8 h. a day, can dig another trench 45 ft. long, 16 ft. wide, and 18 ft. deep, when the ground is estimated at 5 degrees of hardness?

Ans. 84.

74. Wishing to know the height of a certain steeple, I measured the shadow of the same on a horizontal plane, $27\frac{1}{2}$ ft.; I then erected a 10-ft. pole on the same plane, and it cast a shadow of $2\frac{3}{8}$ ft. What was the *height* of the steeple? *Ans.* $103\frac{1}{8}$ ft.

75. A can do a piece of work in 3 da., B can do 3 times

as much in 8 da., and C 5 times as much in 12 da. In what time can they all do the first piece of work?

Ans. $\frac{8}{9}$ da.

76. A person sold two farms for $1890 each; for one he received 25% more than its true value, and for the other 25% less than its true value. Did he gain or lose by the sale, and how much? *Ans.* Lost $252.

77. Three men paid $100 for a pasture; A put in 9 horses, B 12 cows for twice the time, and C some sheep for $2\frac{1}{2}$ times as long as B's cows; C paid one half the cost. How many sheep had he, and how much did A and B each pay, provided 6 cows eat as much as 4 horses, and 10 sheep as much as 3 cows?

Ans. C had 25 sheep; A paid $18; B paid $32.

78. A man purchased goods for $10500, to be paid in three equal installments, without interest; the first in 3 months, the second in 4 months, the third in 8 months. How much ready money will pay the debt, money being worth 7%? *Ans.* $10203.94+.

79. A fountain has 4 receiving pipes, A, B, C, and D; A, B, and C will fill it in 6 hours, B, C, and D in 8 hours, C, D, and A in 10 hours, and D, A, and B in 12 hours; it has also 4 discharging pipes, W, X, Y, and Z; W, X, and Y will empty it in 6 hours, X, Y, and Z in 5 hours, Y, Z, and W in 4 hours, and Z, W, and X in 3 hours. Suppose the pipes are all open, and the fountain full, in what time would it be emptied? *Ans.* $6\frac{6}{19}$ h.

80. A farmer sold 50 fowls, consisting of geese and turkeys; for the geese he received $.75 apiece, and for the turkeys $1.25 apiece, and for the whole he received $52.50. How many were there of each?

Ans. 20 geese, 30 turkeys.

81. There is an island 73 miles in circumference, and 3 footmen start together and travel around it in the same

direction; A goes 5 miles an hour, B 8, and C 10. In what time will they all come together again if they travel 12 hours a day? *Ans.* 6 da. 1 h.

82. A, B, and C are to share $100000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively; but C dying, it is required to divide the whole sum proportionally between the other two. How much is each one's share?

Ans. A's, 57142.85\frac{5}{7}$; B's, 42857.14\frac{2}{7}$.

83. A, B, and C have 135 sheep; A's plus B's are to B's plus C's as 5 to 7, and C's minus B's to C's plus B's as 1 to 7. How many has each?

Ans. A, 30; B, 45; C, 60.

84. A man sold one hog, weighing 250 lb., at 4 cents per lb.; a second, weighing 300 lb., at $4\frac{1}{2}$ cents; and a third, weighing 369 lb., at 5 cents. What was the average price per lb. for the whole? *Ans.* $4\frac{51}{91}$ cents.

85. How many building lots, each 75 ft. by 125 ft., can be laid out on 1 A. 46 P. $18\frac{1}{2}$ sq. yd.? *Ans.* 6.

86. A man bought a house, and agreed to pay for it $1 on the first day of January, $2 on the first day of February, $4 on the first day of March, and so on, in geometrical progression, through the year. What was the cost of the house, and what the average time of payment?

Ans. $4095; average time, Nov. 1.

87. A man sold a rectangular piece of ground, measuring 44 ch. 32 l. long by 36 ch. wide. How many acres did it contain? *Ans.* 159 A. 88.32 P.

88. What number is that which being increased by its half, its third, and 18 more, will be doubled.

Ans. 108.

89. A man owes a debt to be paid in 4 equal installments at 4, 9, 12, and 20 months, respectively; discount being allowed at 5%, he finds that $750 ready money will *pay the* debt. How much does he owe? *Ans.* $784.74+.

90. A merchant has 200 lb. of tea, worth \$.62½ per pound, which he will sell at \$.56 per pound, provided the purchaser will pay in coffee at \$.22, which is worth \$.25 per pound. Does the merchant gain or lose by the sale of the tea, and how much per cent?

Ans. He gains $1\frac{9}{11}\%$.

91. A and B traded upon equal capitals; A gained a sum equal to $\frac{2}{5}$ of his capital, and B a sum equal to $\frac{11}{40}$ of his; B's gain was \$500 less than A's. What was the capital of each? *Ans.* \$4000.

92. I purchase goods in bills as follows: June 4, 1891, \$240.75; Aug. 9, 1891, \$137.25; Aug. 29, 1891, \$65.64; Sept. 4, 1891, \$230.36; Nov. 12, 1891, \$36. If the merchant agrees to allow credit of 6 mo. on each bill, when may I settle by paying the whole amount?

Ans. Feb. 1, 1892.

93. Find the dimensions of a bin that holds 450 bu. of grain, if the width and depth are equal, and the length 3 times the width.

94. What is the diameter of a circular island containing $1\frac{1}{4}$ sq. miles? *Ans.* 403.7 rd.

95. If a marble column 10 in. in diameter contains 27 cu. ft., what is the diameter of a column of equal length that contains 81 cubic feet?

96. What must be the inner edge of a cubical bin to hold 1250 bushels of wheat? *Ans.* 11 ft. 7 in.

97. A man bought a piece of land for \$3000, agreeing to pay 7% interest, and to pay principal and interest in 5 equal annual installments. How much was the annual payment? *Ans.* \$731.67 +.

98. I have three notes payable as follows: one for \$200, due Jan. 1, 1891; another for \$350, due Sept. 1, 1891; and another for \$500, due April 1, 1892. What is the average of maturity? *Ans.* Oct. 24, 1891.

99. A man held three notes, the first for $600, due July 7, 1890; the second for $530, due Oct. 4, 1890; and the third for $400, due Feb. 20, 1891; he made an equitable exchange of these with a speculator for two other notes, one of which was for $730, due Nov. 15, 1890. What was the face of the other, and when due?

Ans. Face, $800; due Aug. 29, 1890.

100. A house that is 50 ft. long and 40 ft. wide has a square or pyramidal roof, whose height is 15 ft. Find the length of a rafter reaching from a corner of the building to the vertex of the roof.

101. Find the length of a rafter reaching from the middle of one side. *Ans.* 25 ft.

102. Find the cost, at 18 cents a square foot, of paving a space in the form of a rhombus, if its sides are 15 ft., and a perpendicular drawn from one oblique angle will meet the opposite side 9 feet from the adjacent angle.

Ans. $32.40.

103. How much less will the fencing of 20 A. cost in the square form than in the form of a rectangle whose breadth is $\frac{1}{5}$ the length, the price being $2.40 per rod?

Ans. $185.54.

104. A young man inherited a fortune, $\frac{1}{4}$ of which he spent in 3 mo., and $\frac{3}{7}$ of the remainder in 10 mo., when he had only $2524 left. How much had he at first?

Ans. $5889.33+.

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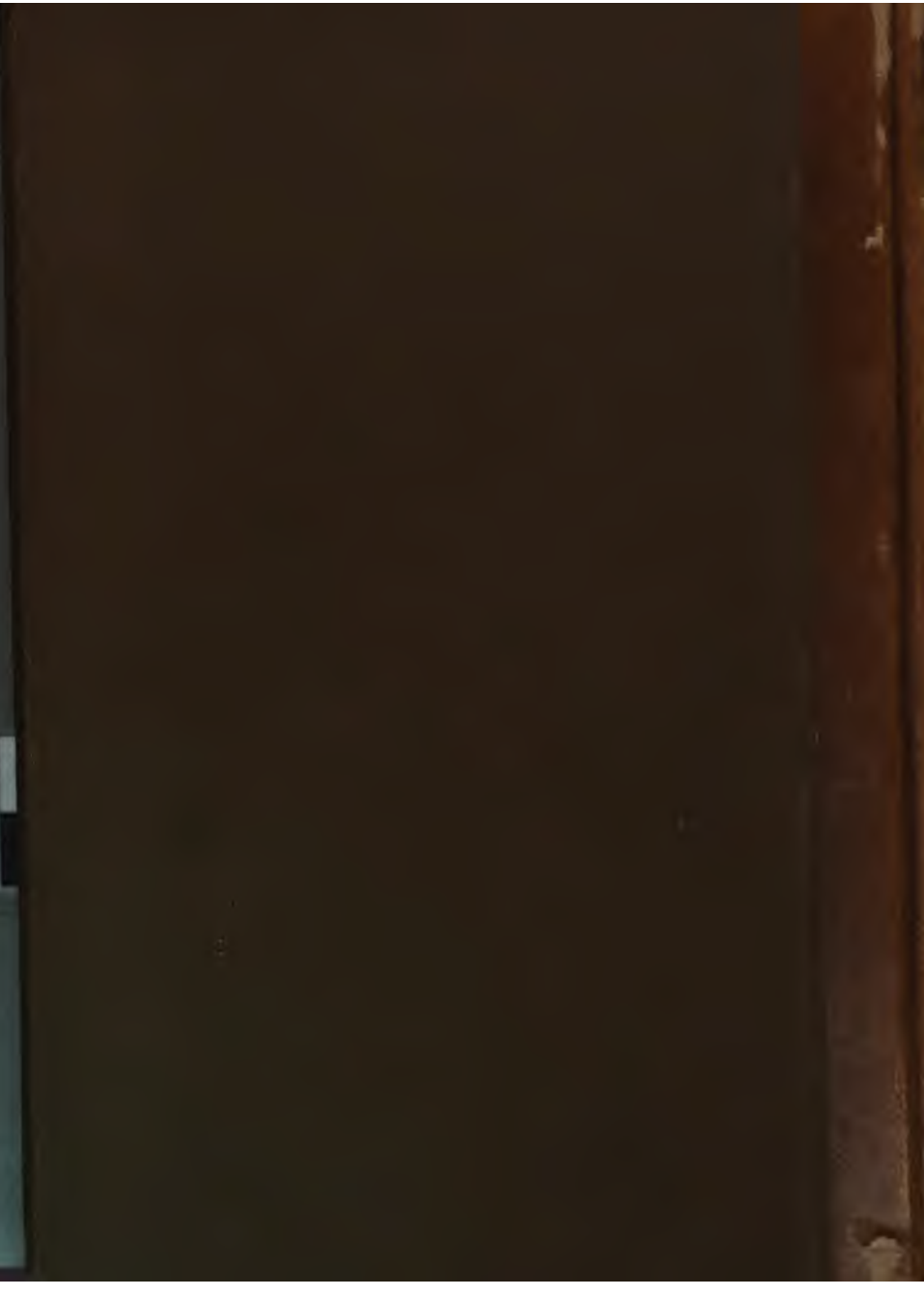
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